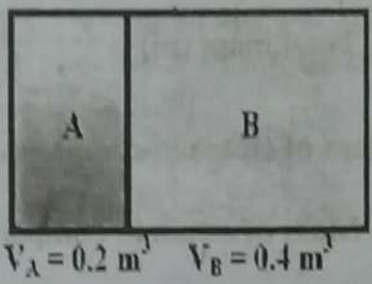
Chapter 1 Introduction

1.1. Numerical Problems

1. A container has two compartments as shown in figure below. Specific volume of steam in compartment A and compartment B are 5 m³/kg and 10 m³/kg respectively. If the membrane breaks and steam comes to uniform state, determine the resulting specific volume.



Solution:

Given, Volume of compartment A (V_A) = 0.2 m^3

Specific volume of compartment A (ν_A) = 5 m^3/kg

Volume of compartment B (V_B) = 0.4 m^3

Specific volume of compartment B (ν_B) = 10 m^3/kg

Then, mass of compartment A $(m_A) = \frac{V_A}{\nu_A} = \frac{0.2}{5} = 0.04$ kg

Mass of compartment B $(m_B) = \frac{V_B}{\nu_B} = \frac{0.4}{10} = 0.04$ kg

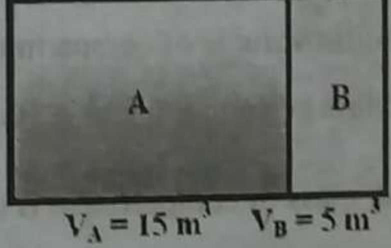
For final state,

Total volume (V) = $V_A + V_B = 0.2 + 0.4 = 0.6\ m^3$

Total mass (m) = $m_A + m_B = 0.04 + 0.04 = 0.08$ kg

$\therefore$ Resulting specific volume $(\nu) = \frac{V}{m} = \frac{0.6}{0.08} = 7.5\ m^3/kg$

2. A container having two compartments contains steam as shown in figure below. The specific volume of steam in compartment B is 5 m³/kg. The membrane breaks and the resulting specific volume is 8 m³/kg. Find the original specific volume of steam in compartment A.



Solution:

For initial state,

Specific volume of steam in compartment B (v_B) = 5 m^3/kg

Volume of compartment A (V_A) = 15 m^3

Volume of compartment B (V_B) = 5 m^3

For final state,

Resulting specific volume (v) = 8 m^3/kg

Total volume (V) = $V_A + V_B = 15 + 5 = 20\ m^3$

$\therefore$ Total mass (m) $= \frac{V}{v} = \frac{20}{8} = 2.5$ kg

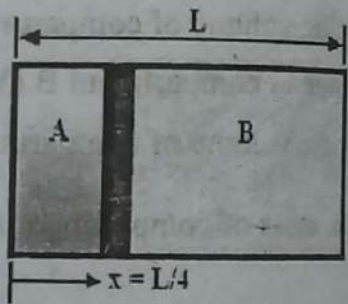
Mass of steam in compartment B (m_B) $= \frac{V_B}{v_B} = \frac{5}{5} = 1$ kg

$\therefore$ Mass of steam in compartment A (m_A) $= m - m_B = 2.5 - 1 = 1.5$ kg

Hence, initial specific volume of steam in compartment A (v_A) $= \frac{V_A}{m_A} = \frac{15}{1.5}$

$= 10\ m^3/kg$

3. **A cylinder with a total volume of 1 m³ has a movable piston as shown in figure below. When the piston is at one fourth of the length, both sides have same specific volume of 4 m³/kg. Determine the specific volumes of both sides when the piston is at middle of the cylinder.**



Solution:

Given, Total volume in the cylinder (V) = 1 m^3

When piston is at $\frac{1}{4}^{th}$ of the length,

Volume of compartment A (V_A) = 0.25 m^3

Volume of compartment B (V_B) = 0.75 m^3

Specific volume of compartment A (v_A) = 4 m^3/kg

Specific volume of compartment B (v_B) = 4 m^3/kg

$\therefore$ Mass of compartment A (m_A) $= \frac{V_A}{v_A} = \frac{0.25}{4} = 0.0625$ kg

And, mass of compartment B (m_B) $= \frac{V_B}{v_B} = \frac{0.75}{4} = 0.1875$ kg

When the piston is at the middle of the cylinder,

$V_A = V_B = 0.5\ m^3$

$\therefore$ Specific volume of compartment A $(\nu_A) = \frac{V_A}{m_A} = \frac{0.5}{0.0625} = 8\ m^3/kg$

And specific volume of compartment A $(\nu_B) = \frac{V_B}{m_B} = \frac{0.5}{0.1875} = 2.667\ m^3/kg$

4. An oxygen cylinder having a volume of 10 m³ initially contains 5 kg of oxygen. Determine the specific volume of oxygen in the cylinder initially. During certain process 3 kg of oxygen is consumed, determine the final specific volume of oxygen in the cylinder. Also plot the amount of oxygen that has been consumed versus the specific volume of the remaining in the cylinder.

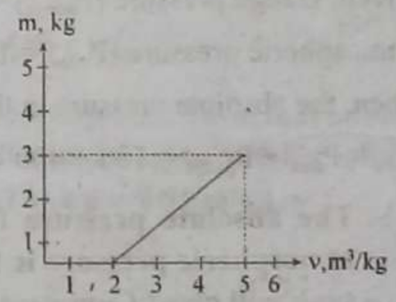
Solution:

Given, Initial volume of oxygen $(V_1) = 10\ m^3$

Initial mass of oxygen $(m_1) = 5$ kg

Final mass of oxygen $(m_2) = 2$ kg

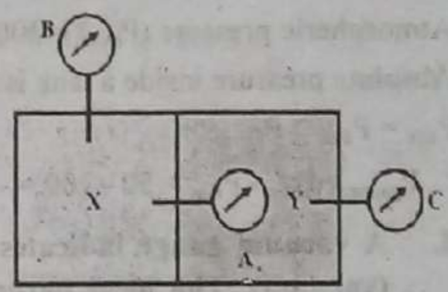
Final volume of oxygen $(V_2) = 10$ kg



Then, initial specific volume of oxygen $(\nu_1) = \frac{V_1}{m_1} = \frac{10}{5} = 2\ m^3/kg$

And, final specific volume of oxygen $(\nu_2) = \frac{V_2}{m_2} = \frac{10}{2} = 5\ m^3/kg$

5. A large chamber is separated into two compartments which aremaintained different pressures, as shown figure below. Pressure gauge A reads 180 kPa, and pressure gauge B reads 120 kPa. If the barometric pressure is 100 kPa, determine the absolute pressure existing in the compartments and the reading of gauge C.



Solution:

Given, atmosphere pressure $(P_{atm}) = 100$ kPa

Gauge pressure for Gauge A $(P_A) = 180$ kPa

Gauge pressure for Gauge B $(P_B) = 120$ kPa

Here, pressure gauge C measures pressure of Y relative to atmosphere, pressure gauge A measures pressure of compartment X and pressure gauge B measures the pressure of compartment X relative to atmosphere.

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Hence, Absolute pressure of compartment X is given by

$(P_{abs})_X = P_{atm} + P_B = 100 + 120 = 220$ kPa

Similarly, absolute pressure of compartment Y is given by

$(P_{abs})_Y = (P_{abs})_X - P_A = 220 - 180 = 40$ kPa

For pressure gauge C,

$(P_{abs})_y = P_{atm} + P_C$

$\therefore\ P_C = (P_{abs})_Y - P_{atm} = 40 - 100 = -60$ kPa

6. **A pressure gauge connected to a cylinder reads 400 kPa at a location where the atmospheric pressure is 100 kPa. Determine the absolute pressure in the cylinder.**

Solution:

Given, Gauge pressure (P_{gauge}) = 400 kPa

Atmospheric pressure (P_{atm}) = 100 kPa

Then, the absolute pressure in the cylinder is given by

$P_{abs} = P_{atm} + P_{gauge} = 100 + 400 = 500$ kPa

7. **The absolute pressure inside a tank is 50 kPa and the surrounding atmospheric pressure is 100 kPa. What reading a gauge mounted in the tank will give? Comment upon the result.**

Solution:

Given, Absolute pressure inside a tank (P_{abs}) = 50 kPa

Atmospheric pressure (P_{atm}) = 100 kPa

Absolute pressure inside a tank is given as

$P_{abs} = P_{atm} + P_{gauge}$

$\therefore\ P_{gauge} = P_{abs} - P_{atm} = 50 - 100 = -50$ kPa (Vacuum gauge)

8. **A vacuum gauge indicates the pressure of air in a cylinder is 15 kPa (vacuum). The local barometer reads 750 mm of Hg. Determine the absolute pressure inside the cylinder. [Take $\rho_{Hg} = 13600$ kg/m^3 and g = 9.81 m/s^2]**

Solution:

Given, Gauge pressure of air inside a cylinder (P_{gauge}) = - 15 kPa

Barometer reading (Z_{baro}) = 750 mm of H_g

Atmospheric pressure (P_{atm}) = $\rho_{Hg} g Z_{baro} = 13600 \times 9.81 \times 750 \times 10^{-3} = 100.062$ kPa

Then, absolute pressure inside the cylinder is given by

$P_{abs} = P_{atm} + P_{gauge} = 100.002 - 15 = 85.062$ kPa

9. At the inlet and exhaust of a turbine the absolute steam pressure are 6000 kPa 4.0 cm of Hg, respectively. Barometric pressure is 75 cm of Hg. Calculate the gauge pressure for the entering steam and the vacuum gauge pressure for the exhaust steam. [Take ρ_{Hg} = 13600 kg/m^3 and g = 9.81 m/s^2]

Solution:

Given, Absolute pressure at turbine inlet $(P_{abs})_{inlet}$ = 6000 kPa

Absolute pressure at turbine exhaust $(P_{abs})_{exhaust}$ = $13600 \times 9.81 \times 4 \times 10^{-2}$ = 5.337 kPa

Barometer reading (Z_{baro}) = 75 cm of Hg

Atmospheric pressure (P_{atm}) = $13600 \times 9.81 \times 75 \times 10^{-2}$ = 100.062 kPa

Then, gauge pressure at turbine inlet is given as

$(P_{guage})_{inlet} = (P_{abs})_{inlet} - P_{atm} = 6000 - 100.062 = 5899.94$ kPa

Also, gauge pressure at turbine exhaust is given as

$(P_{guage})_{exhaust} = (P_{abs})_{exhaust} - P_{atm} = 5.337 - 100.062 = 94.725$ kPa

10. During the operation of the lift, it can be subjected to a maximum pressure of 500 kPa. If it is designed to lift a mass upto 900 kg, what should be diameter of the piston/cylinder? [Take g = 9.81 m/s^2]

Solution:

Given, Maximum pressure subjected during the operation of lift (P_{max}) = 500 kPa

Mass to be lifted (m) = 900 kg

Diameter of piston/cylinder (d_p) = ?

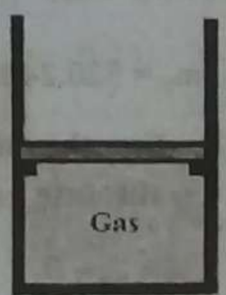
Maximum pressure subjected during the operation of lift is given as

$$P = \frac{W}{A}$$

$$\text{or, } P_{max} = \frac{mg}{\frac{\pi d_p^2}{4}} = \frac{900 \times 9.81}{\frac{\pi (d_p^2)}{4}}$$

$$\therefore\ d_p = \sqrt{\frac{900 \times 9.81 \times 4}{\pi \times 500 \times 10^3}} = 0.15 \text{ m}$$

11. A piston cylinder arrangement shown in figure below has a cross sectional area of 0.01 m^2 and a piston mass of 80 kg. If atmospheric pressure is 100 kPa, what should be the gas pressure to lift the piston? [Take g = 9.81 m/s^2]



Solution:

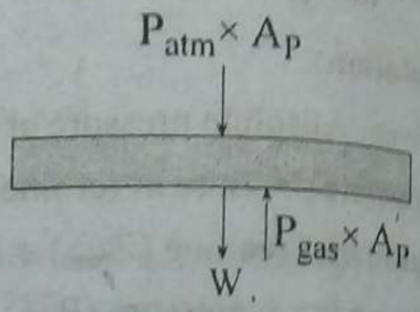
Given, Cross-sectional area of piston (A_p) = 0.01 m^2

Mass of piston (m_p) = 80 kg

Atmospheric pressure (P_{atm}) = 100 kPa

Absolute pressure of gas to lift the piston (P_{gas}) = ?

Referring to the free body diagram of the piston we can write the equilibrium equation as



$$P_{gas} \times A_p = P_{atm} \times A_p + W$$

$$\therefore P_{gas} = P_{atm} + \frac{m_p g}{A_p} = 100 + \frac{80 \times 9.81}{0.01} \times 10^{-3}$$

$= 178 .48$ kPa

12. A pisotn cylinder has a diameter of 0.2 m. with an outside atmospheric pressure of 100 kPa, determine the piston mass that will create an inside pressure of 500 kPa. [Take g = 9.81 m/s²]

Solution:

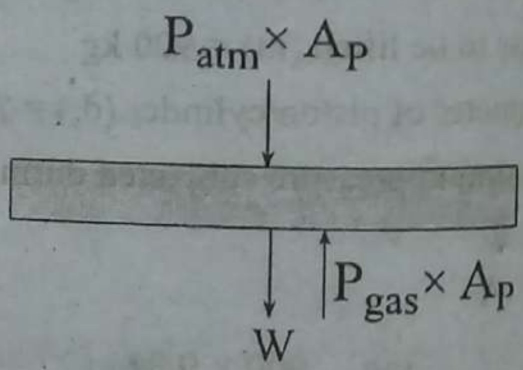
Given, Diameter of the piston (d_p) = 0.1 m

Absolute pressure of the gas (P_{gas}) = 500 kPa

Atmospheric pressure (P_{atm}) = 100 kPa

Mass of piston (m_p) = ?

Referring to the free body diagram of the piston we can write the equilibrium equation as



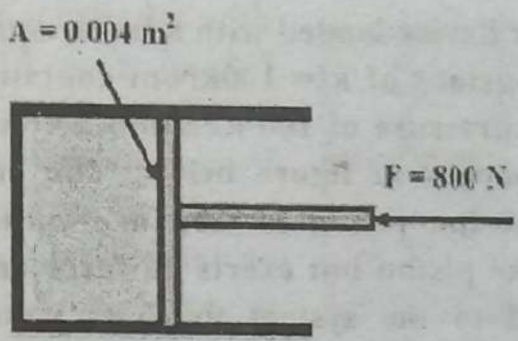
$$P_{gas} \times A_p = P_{atm} \times A_p + W$$

$$\text{or,} \quad P_{gas} = P_{atm} + \frac{m_p . g}{\frac{\pi(d_p^2)}{4}}$$

$$\text{or, } 500 \times 10^3 = 100 \times 10^3 + \frac{m_p \times 9.81 \times 4}{\pi \times (0.1)^2}$$

$\therefore$ $m_p = 320.24$ kg

13. For the piston cylinder device shown in figure below, determine the absolute pressure inside the device. [Take P_{atm} = 101.3 kPa]



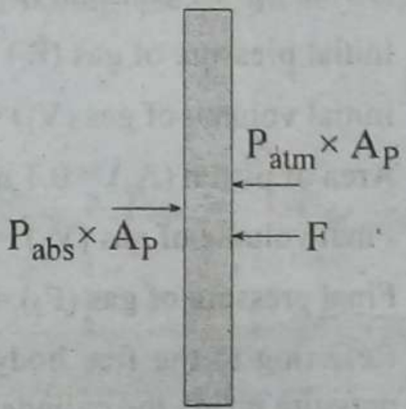
Solution:

Given, Atmospheric pressure (P_{atm}) = 101.3 kPa

Area of piston (A_p) = 0.004 m²

Force applied to the piston (F) = 800 N

Referring to the free body diagram of the piston we can write the equilibrium equation as

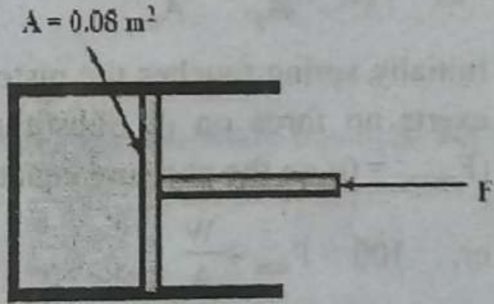


$P_{abs} \times A_p = P_{atm} \times A_p + F$

$\therefore P_{abs} = P_{atm} + \dfrac{F}{A_p}$

$= 101.3 + \dfrac{800}{0.004 \times 1000} = 301.3$ kPa

14. For the piston cylinder device shown in figure below, determine the force necessary to produce an absolute pressure of 500 kPa within the device. [Take P_{atm} = 100 kPa]



Solution:

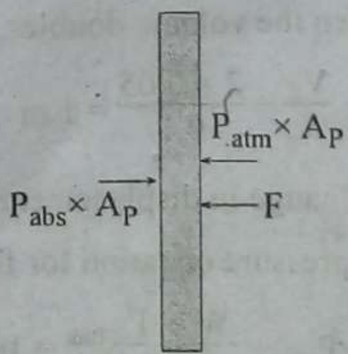
Given, Absolute pressure (P_{abs}) = 500 kPa

Atmospheric pressure (P_{atm}) = 100 kPa

Area of piston (A_p) = 0.08 m²

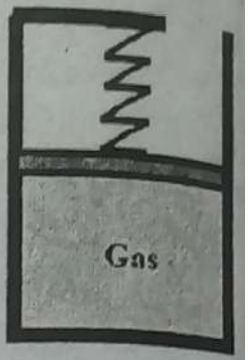
Force (F) = ?

Referring to the free body diagram of the piston we can write the equilibrium equation as



$P_{abs} \times A_p = P_{atm} \times A_p + F$

$\therefore F = (P_{abs} - P_{atm}) \times A_p = (500 - 100) \times 0.08 = 32$ kN



15. A piston cylinder device loaded with a linear spring with a spring constant of k = 100kN/m contains a gas initially at a pressure of 100 kPa and a volume of 0.05 m³, as shown in figure below. The cross sectional area of the piston is 0.01 m². Initially spring touches the piston but exerts no force on it. Heat is supplied to the system until its volume doubles, determine the final pressure. ***(IOE 2068 Bhadra)***

Solution:

Given, Spring constant (k) = 100 kN/m

Initial pressure of gas (P_1) = 100 kPa

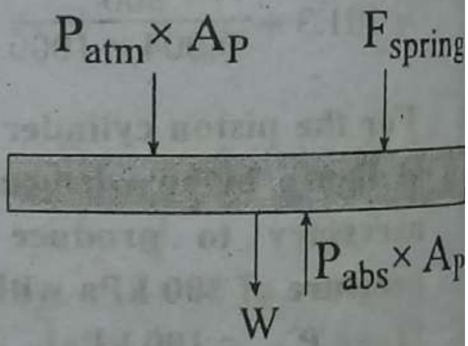
Initial volume of gas (V_1) = 0.05 m^3

Area of piston (A_p) = 0.1 m^2

Final volume of gas (V_2) = $2V_1 = 2 \times 0.05 = 0.1\ m^3$

Final pressure of gas (P_2) = ?

Referring to the free body diagram of the piston we can write equation for the pressure inside the cylinder as



$$P_{abs} = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p}$$

Initially spring touches the piston the piston but exerts no force on it. Substituting initial state ($F_{spring} = 0$) on the pressure equation, we get

or, $$100 = P_{atm} + \frac{W}{A_p}$$

Here, $$x_1 = \frac{V_1}{A_p} = \frac{0.05}{0.1} = 0.5\ m$$

When the volume doubles,

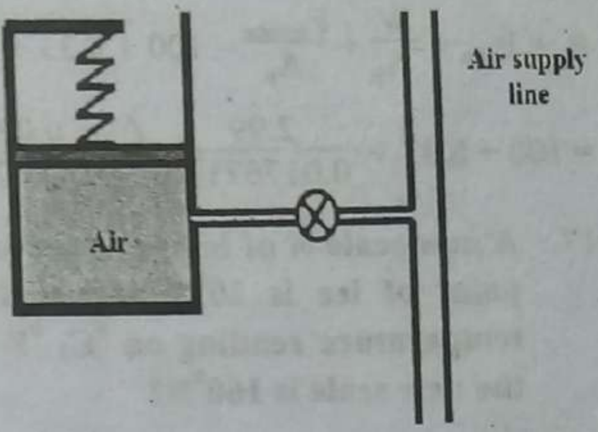
$$x_2 = \frac{V_2}{A_p} = \frac{2 \times 0.05}{0.1} = 1\ m$$

∴ Change in displacement of piston (x) = $x_2 - x_1 = 1 - 0.5 = 0.5$ m

So, pressure equation for final state is given as

$$P_2 = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p} = 100 + \frac{kx}{A_p} = 100 + \frac{100 \times 0.5}{0.1} = 600\ kPa$$

16. **A 15 kg piston in a cylinder with diameter of 0.15 m is loaded with a linear spring and the outside atmospheric pressure of 100 kPa, as shown in figure below. The spring exerts no force on the piston when it is at the bottom of the cylinder and for the state shown, the pressure is 300 kPa with volume of 0.02 m^3. The valve is opened to let some air in, causing the piston to rise 5 cm. Find the new pressure. [Take g = 9.81 m^2/s]**



Solution:

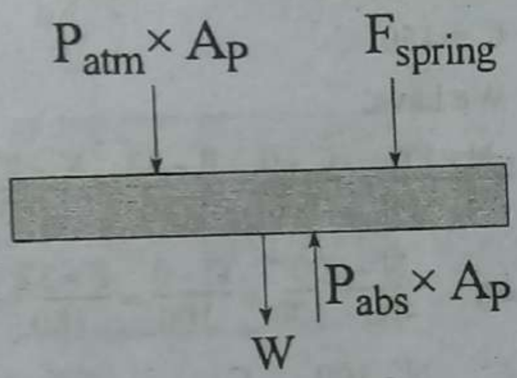
Given, Mass of piston (m_p) = 15 kg

Diameter of piston (d_p) = 0.15 m

Atmospheric pressure (P_{atm}) = 100 kPa

Initial state: P_1 = 300 kPa, V_1 = 0.02 m^3

Final state: x_2 = 5 cm = 0.05m



$$\text{Area of piston } (A_p) = \frac{\pi d_p^2}{4} = \frac{\pi\,(0.15)^2}{4}$$

$$= 0.01767146 \text{ m}^2$$

Referring to the free body diagram of the piston we can write equation for the pressure inside the cylinder as

$$P_{abs} = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p} = P_{atm} + \frac{m_p g}{A_p} + \frac{kx}{A_p}$$

Substituting initial state (P_1 = 300 kPa, V_1 = 0.02 m^3) on the pressure equation, we get

$$300 = 100 + \frac{m_p g}{A_p} + \frac{k \times V_1}{(A_p)^2}$$

$$\text{or,}\quad 200 = \frac{15 \times 9.81 \times 10^{-3}}{0.01767146} + \frac{k \times 0.02}{(0.01767146)^2}$$

$$\text{or,}\quad 200 = 8.33 + \frac{0.02 \times k}{3.123 \times 10^{-4}}$$

$\therefore$ k = 2.99 kN/m

Now, pressure equation at final state is given as

$$P_2 = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p} = 100 + 8.33 + \frac{k\,(x_1 + x_2)}{A_p}$$

$$= 100 + 8.33 + \frac{2.99}{0.01767146}\left(\frac{0.02}{0.01767146} + 0.05\right) = 308.7 \text{ kPa}$$

17. A new scale N of temperature is deviced in such a way that the freezing point of ice is 20⁰N and boiling point is 200⁰N. What will be the temperature reading on ⁰C, ⁰F and K scales when the temperature on the new scale is 160⁰N?

Solution:

Given, For new scale:

Freezing point (F.P) = 100^0 N

Boiling point (B.P) = 400^0 N

C = 150^0 C

We have,

$$\frac{N - FP}{BP - FP} = \frac{C - 0}{100} = \frac{F - 32}{180} = \frac{K - 273}{100}$$

or, $$\frac{N - 100}{400 - 100} = \frac{C - 0}{100} = \frac{F - 32}{180} = \frac{K - 273}{100}$$

or, $$\frac{N - 100}{300} = \frac{C}{100}$$

or, $N = 3 \times 150 + 100 = 550$ N

Also, $$\frac{N - 100}{300} = \frac{C}{100}$$

When both the scale reading will be same, we get

N - 100 = 3N

or, 2N = - 100

∴ N = - 50

18. The temperature of a system drops by 36⁰F during a process. Express this drop in temperature in ⁰C and K.

Solution:

Given, F = 36^0 F

We know,

$$\frac{C-0}{100} = \frac{F - 32}{180} = \frac{K - 273}{100}$$

Chapter 2 Energy and Energy Transfer

2.1 Numerical problems

1. A gas is contained in a piston cylinder device initially at a pressure of 150 kPa and a volume of 0.04 m³. Calculate the work done by the gas when it undergoes the following processes to a final volume of 0.1 m³.

(a) constant pressure

(b) constant temperature

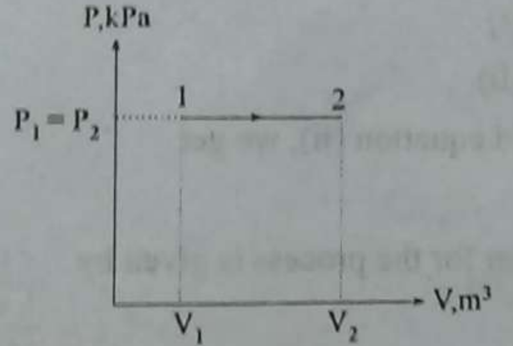
(c) $PV^{1.35}$ = constant. *(IOE 2070 Bhadra)*

Solution:

Given, Initial state: $P_1 = 150$ kPa, $V_1 = 0.04$ m³

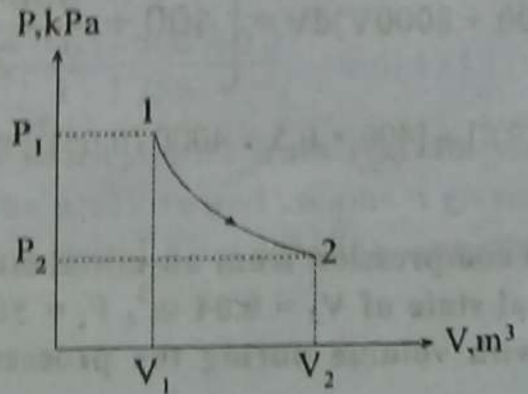
Final state: $V_2 = 0.1$ m³

a) For constant pressure expansion to final volume of 0.1m³, work transfer is given by



$$W = P_1 (V_2 - V_1) = 150 (0.1 - 0.04) = 9 \text{ kJ}$$

b) For constant temperature process, work done is given by



$$W = P_1V_1 \ln\left(\frac{V_2}{V_1}\right) = 150 \times 0.04 \ln\left(\frac{0.1}{0.04}\right) = 5.498 \text{ kJ}$$

c) For polytropic process ($PV^{1.35}$ = constant)

$$P_1V_1^{1.35} = P_2V_2^{1.35}$$

$$\therefore\ P_2 = P_1\left(\frac{V_1}{V_2}\right)^{1.35} = 150\left(\frac{0.04}{0.1}\right)^{1.35} = 43.538 \text{ kPa}$$

Then, work done during polytropic expansion is given by

$$W = \frac{P_1V_1 - P_2V_2}{1 - n} = \frac{150 \times 0.04 - 43.538 \times 0.1}{1 - 1.35} = 4.703 \text{ kJ}$$

2. **A spring loaded piston cylinder device contains gas initially at pressure of 800 kPa and a volume of 0.05 m^3. Pressure – volume relationship for the set up is given by P = a + bV, where a and b are constants. Heat is added to the system till its final state P_2 = 2000 kPa and V_2 = 0.2 m^3 is reached. Determine the work transfer during the process.**

Solution:

Given, Initial stage: P_1 = 800 kPa, V_1 = 0.05 m^3

Final stage: P_2 = 2000 kPa, V_2 = 0.2 m^3

Substituting initial and final states on given process relation P = a + bV, we get

800 = a + 0.05b …… (i)

2000 = a + 0.2b …… (ii)

Solving equation (i) and equation (ii), we get

a = 400, b = 8000

Hence, the P - V relation for the process is given by

P = 400 + 2000 V

Then, work transfer during the process is given by

$$W = \int_{V_2}^{V_1} PdV = \int_{0.05}^{0.2} (400 + 8000V)dV = \left[400 + \frac{8000V^2}{2}\right]_{0.05}^{0.2}$$

$$= [400 \times 0.2 + 4000\,(0.2)^2] - [400 \times 0.5 + 4000\,(0.05)^2] = 240 - 30 = 210 \text{ kJ}$$

3. **A gas undergoes compression from an initial state of V_1 = 0.1 m^3, P_1 = 200 kPa to a final state of V_2 = 0.04 m^3, P_2 = 500 kPa. If the pressure varies linearly with volume during the process, determine the work transfer.**

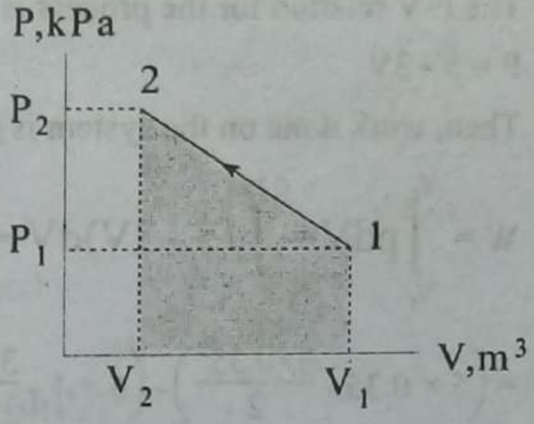
Solution:

Given, Initial state: V_1 = 0.1 m^3, P_1 = 200 kPa

Final state: V_2 = 0.04 m^3, P_2 = 500 kPa

Pressure varies linearly with volume during the process. So, work transfer during the process is given by

Work transfer = Area under the curve = Area of trapezium



$$= \frac{1}{2}(P_2 + P_1)(V_2 - V_1) = \frac{1}{2}(500 + 200) \times (0.04 - 0.1) = -21 \text{ kJ}$$

4. **A gas undergoes a ploytropic process from an initial state of 500 kPa and 0.02 m^3 to a final state of 100 kPa and 0.05 m^3. Determine the worktransfer.**

Solution:

Given, Initial state: P_1 = 500 kPa, V_1 = 0.02 m^3

Final state: P_2 = 100 kPa, V_2 = 0.05 m^3

For polytropic process (PV^n = constant)

$$P_1V_1^n = P_2V_2^n$$

$$\text{or, } \left(\frac{V_1}{V_2}\right)^n = \frac{P_2}{P_1}$$

$$\text{or, } n \ln\left(\frac{V_1}{V_2}\right) = \ln\left(\frac{P_2}{P_1}\right)$$

$$\therefore\ n = \frac{\ln\left(\frac{P_2}{P_1}\right)}{\ln\left(\frac{V_1}{V_2}\right)} = \frac{\ln\left(\frac{100}{500}\right)}{\ln\left(\frac{0.02}{0.05}\right)} = 1.756$$

Then, work done during polytropic expansion is given by

$$W = \frac{P_2V_2 - P_1V_1}{1 - n} = \frac{100 \times 0.05 - 500 \times 0.02}{1 - 1.756} = 6.614 \text{ kJ}$$

5. **Air is compressed in a cylinder from 1 m^3 to 0.35 m^3 by a piston. The relation between the pressure and volume is given by $P = 5 - 3V$, where P is in bar and V is in m^3. Compute the magnitude of the work dne n the system in kJ.**

Solution:

Given, Initial state: V_1 = 1 m^3

Final state: $V_2 = 0.35\ m^3$

The P-V relation for the process is

$P = 5 - 3V$

Then, work done on the system is given by

$$W = \int_{V_1}^{V_2} pdV = \int_{1}^{0.35} (5 - 3V)dV = \left[5V - \frac{3V^2}{2}\right]_1^{0.35}$$

$$= \left(5 \times 0.35 - \frac{3\times 0.35^2}{2}\right) - \left(5 \times 1 - \frac{3 \times 1^2}{2}\right) = -1.93375 \times 10^2 = -193.375\ kPa$$

6. **In a non flow process, a gas expands from volume 0.1 m^3 to a volum[e] 0.2 m^3 according to the law**

$$P = \frac{2}{V} + 1.5$$

where P is the pressure in bar, and V is the volume in m^3. Determin[e] the pressure at the end of the expansion and (ii) the work done by [the] gas in the expansion process, in kJ.

Solution:

Given, Initial state: $V_1 = 0.1\ m^3$

Final state: $V_2 = 0.2\ m^3$

The P-V relation for the process is given as

$$P = \frac{2}{V} + 1.5$$

i) Pressure at the final state is given as

$$P_2 = \left(\frac{2}{V_2} + 1.5\right) \times 10^2\ kPa = \left(\frac{2}{0.2} + 1.5\right) \times 10^2\ kPa = 1150\ kPa$$

ii) Work done by the gas in the expansion process is given by

$$W = \int_{0.1}^{0.2} PdV = \int_{0.1}^{0.2} \left(\frac{2}{V} + 1.5\right) dV = [2\ln V + 1.5V]_{0.1}^{0.2}$$

$$= [2\ \ln(0.2) + 1.5\ (0.2)] - [2\ \ln(0.1) + 1.5\ (0.1)] = 1.536 \times 10^2\ kJ = 153.63\ kJ$$

7. **A non flow reversible process occurs for which pressure and volume [are] correlated by the expression**

$$P = V^2 + \frac{6}{V}$$

where P is in bar and V is in m^3. What amount of work will be do[ne] whenvolume changes from 2 to 4 m^3?

Solution:

Given, Initial state: $V_1 = 2\ m^3$

Final state: $V_2 = 4\ m^3$

The P-V relation for the process is given as

$$P = V^2 + \frac{6}{V}$$

Then, work done during the process is given by

$$W = \int_{V_1}^{V_2} pdV = \int_2^4 (V^2 + \frac{6}{V})dV = \left[\frac{V^3}{3} + 6\ln V\right]_2^4$$

$$= \left(\frac{(4)^3}{3} + 6\ln 4\right) - \left(\frac{(2)^3}{3} + 6\ln 2\right) = 22.83 \times 10^2\ kPa = 2283\ kJ$$

8. **An ideal gas undergoes an isothermal compression from $V_1 = 3\ m^2$ to $P_2 = 100$ kPa and $V_2 = 2\ m^3$. It is further compressed at constant pressure until its volume reduces to 1 m^3. Determine the total work transfer for the process**

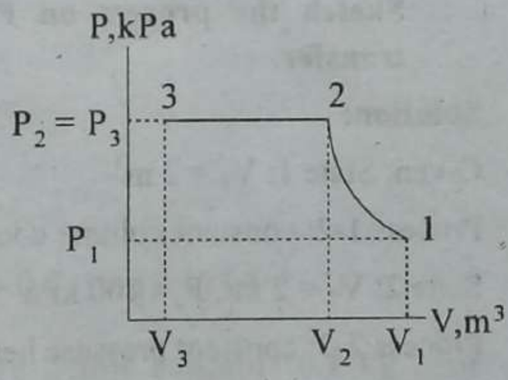
Solution:

Given, Initial state: $V_1 = 3\ m^3$

State 2: $P_2 = 100$ kPa, $V_2 = 2\ m^3$

Final state: $V_3 = 1\ m^3$

Process 1 -2 is isothermal process, so pressure at state 1 is given as



$$P_1V_1 = P_2V_2$$

$$P_1 = P_2\left(\frac{V_2}{V_1}\right) = 100 \times \left(\frac{2}{3}\right) = 66.667\ kPa$$

Then, total work transfer for the process is given by

$$W = W_{12} + W_{23} = P_1V_1\ \ln\frac{V_2}{V_1} + P_2\ (V_2 - V_2)$$

$$= 66.667 \times 3\ \ln\left(\frac{2}{3}\right) + 100\ (2\text{-}3) = -\ 81.093 - 100 = -181.093\ kJ$$

9. **An ideal gas undergoes two processes in series**

   Process 1-2: an expansion from 0.1 m^3 to 0.2 m^3 at constant pressure of 200 kPa.

Process 2-3: an from expansion 0.2 m^3 to 0.4 $m^3$with linear rising pressure from 200 kPa to 400 kPa.

Sketch the process on P-V diagram and determine the total work transfer.

Solution:

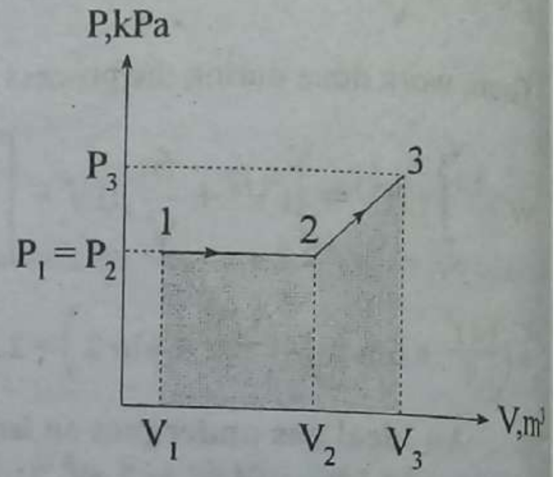
Given, State 1: $P_1 = 200$ kPa, $V_1 = 0.1\ m^3$

Process 1-2: constant pressure expansion

State 2: $P_2 = 200$ kPa, $V_2 = 0.2\ m^3$

State 3: $P_3 = 400$ kPa, $V_3 = 0.4\ m^3$

Then, total work transfer during the process is given by



W = Area under the curve = $W_{12} + W_{23}$

$$= P_1 (V_2 - V_1) + \frac{1}{2} (P_3 + P_2) (V_3 - V_2)$$

$$= 200 (0.2 - 0.1) + \frac{1}{2} (400 + 200) (0.4 - 0.2) = 80\text{kJ}$$

10. Two kgs of gas undergoes following process in series to from a cycle:

Process 1-2: constant volume cooling, $V_1 = V_2 = 2\ m^3$

Process 2-3: constant pressure heating, $P = 100$ kPa, $V_3 = 10\ m^3$

Process 3-1: isothermal compression

Sketch the process on P-V diagram and determine the total work transfer.

Solution:

Given, State 1: $V_1 = 2\ m^3$

Process 1-2: constant volume cooling

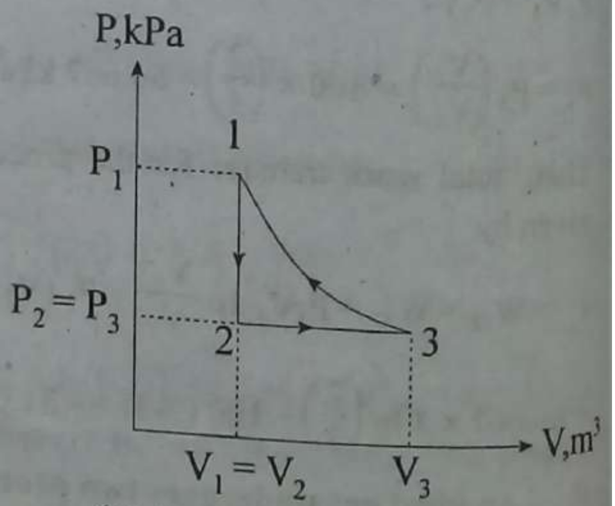
State 2: $V_2 = 2\ m^3$, $P_2 = 100$ kPa

Process 2-3: constant pressure heating

State 3: $P_3 = 100$ kPa, $V_3 = 10\ m^3$

Process 3-1: isothermal compression

Then, net work transfer during the process is given by



$$W = W_{12} + W_{23} + W_{31} = P_2 (V_3 - V_2) + P_3 V_3 \ln\left(\frac{V_1}{V_3}\right)$$

$= 100\,(10 - 2) + 100 \times 10 \ln\left(\frac{2}{10}\right) = -\,809.438$ kJ

11. Air undergoes three process in series to from a cycle:

Process 1-2: compression with $PV^{1.3}$ = constant form $P_1 = 100$ kPa, $V_1 = 0.04\ m^3$ to $V_2 = 0.02\ m^3$

Process 2-3: constant pressure process to $V_3 = V_1$

Process 3-1: constant volume

Sketch the process on P-V diagram and determine the total work transfer.

Solution:

Given, State 1: $P_1 = 100$ kPa, $V_1 = 0.04\ m^3$

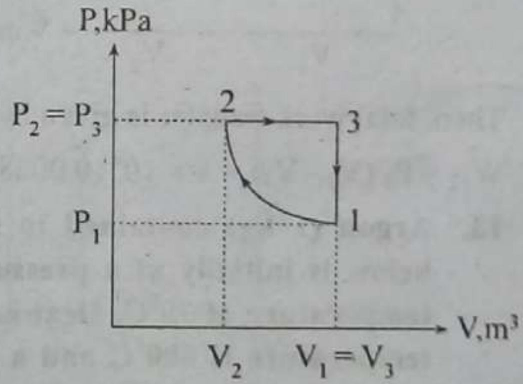
Process 1-2: compression with $PV^{1.3}$ = constant

State 2: $V_2 = 0.02\ m^3$

Process2-3: constant pressure

State 3: $V_3 = 0.04\ m^3$

Process 3-1: constant volume



For process 1 – 2,

$P_1V_1^{1.3} = P_2V_2^{1.3}$

$\therefore P_2 = P_1\left(\frac{V_1}{V_2}\right)^{1.3} = 100\left(\frac{0.04}{0.02}\right)^{1.3}$

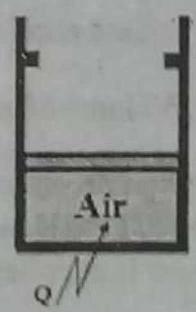
$= 246.229$ kPa

Then, total work transfer is given by

$= W_{12} + W_{23} + W_{31} = \frac{P_2 V_2 - P_1V_1}{1 - n} + P_2\,(V_3 - V_2)$

$= \frac{246.229 \times 0.02 - 100 \times 0.04}{1 - 1.3} + 246.229\,(0.04 - 0.02) = 1.843$ kJ

12. A piston cylinder device shown in figure below contains 0.1 kg f air initially at a pressure of 4 MPa and temperature of 200°C. Heat is added o the system until the pressure is 8 MPa and the temperature is 800°C. Sketch the process on P-V diagram and T-V diagrams and determine the total work transfer.



Solution:

Given, Mass of air (m) = 0.1 kg

Initial state: $P_1 = 4$ MPa, $T_1 = 200^\circ C = 473K$

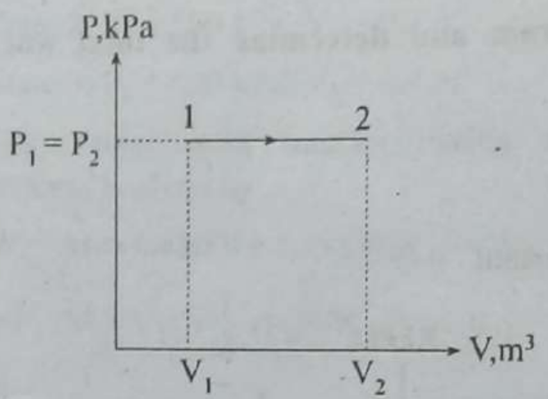
Final state: $P_2 = 8$ MPa, $T_{final} = 800^0C = 1073$ K

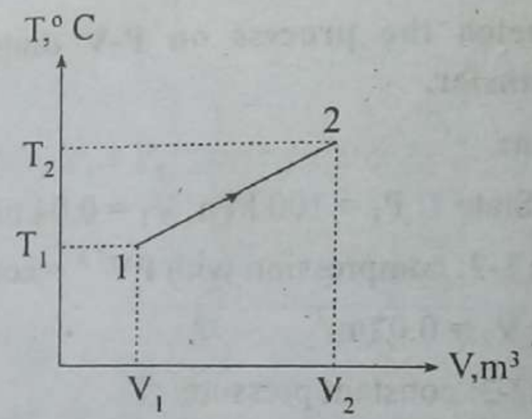
$\therefore$ Volume of air at initial state, $V_1 = \frac{mRT_1}{P_1} = \frac{0.1 \times 287 \times 473}{4 \times 10^6}$

$= 0.003394\ m^3$

And, Volume of air at final state, $V_2 = \frac{mRT_2}{P_2} = \frac{0.1 \times 287 \times 1073}{8 \times 10^6}$

$= 0.003849\ m^3$

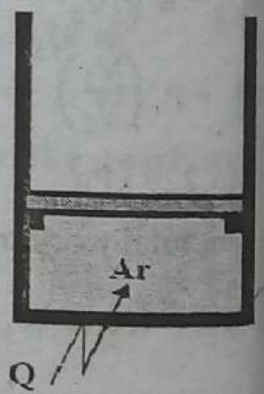




Then, total work transfer is given by

$W_{12} = P_2 (V_2 - V_1) = 8 \times 10^3 (0.003849 - 0.003394) = 3.64$ kJ

13. Argon (1 kg) contained in a piston cylinder device shown in figure below is initially at a pressure of 500 kPa and a temperature of 70°C. Heat is added until the final temperature is 600°C and a pressure of 1 MPa is required to lift the piston form the stops. Sketch the process on P-V and T-V diagrams and determine the total work transfer. [Take R = 208 J/kg K]



Solution:

Given, Mass of argon (m) = 1 kg

Initial state: $P_1 = 500$ kPa, $T_1 = 70^0C = 343$ K

Final state: $T_{final} = 600^0C = 873$ K

Pressure required to lift the piston, $P_{lift} = 1$ MPa = 1000 kPa

$\therefore$ Volume of argon at initial state, $V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 208 \times 343}{500 \times 10^3} = 0.14269\ m^3$

Initial pressure of the system is 500 kPa and pressure required to lift the piston is 1000 kPa. Hence, during initial stage of heating piston remain stationary although heat is supplied to the system, so process is constant volume heating (Process 1-

2). During constant volume heating, pressure of the system increases from 500 kPa to 1000 kPa. Hence, we can define state 2 as

State 2: $P_2 = 1000$ kPa, $V_2 = 0.14269$ m^3

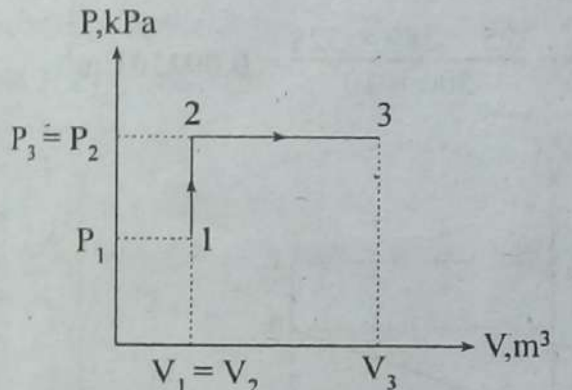
$\therefore$ Temperature of argon at state 2, $T_2 = \frac{P_2V_2}{mR} = \frac{1000\times10^3 \times 0.14269}{1\times208} = 686$ K

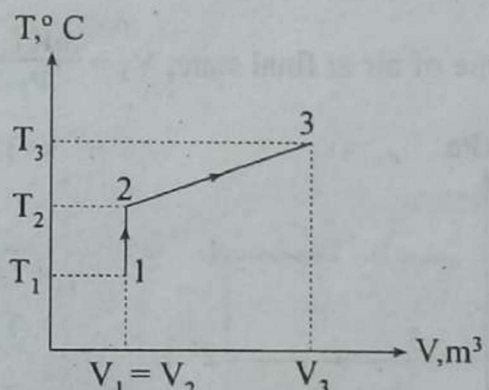
$= 413^0$C

But the required final temperature is 800^0 C, hence it should be further heated to increase the temperature from 413^0C to 800^0 C and the process occurs at constant pressure of 1000 kPa (Process 2-3). Hence, we can define state 3 as

State 3: $P_3 = 1000$ kPa, $T_3 = 600^0$C

$\therefore$ Volume of argon at final state, $V_3 = \frac{mRT_3}{P_3} = \frac{1 \times 208 \times 873}{1000\times10^3} = 0.18158$ m^3

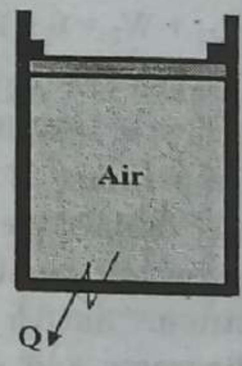




Then, total work transfer is given by

$W = W_{12} + W_{23} = 0 + P_2 (V_3 - V_2) = 1000\ (0.18158 - 14269) = 38.89$ kJ

14. Air (0.5 kg) in the piston cylinder device shown in figure below has an initial pressure and temperature of 1 MPa and 500^{0}C respectively. The system is cooled until the temperature reaches 50^{0}C. It takes a pressure of 0.5 MPa to support the piston. Sketch the process on P-V and T-V diagrams and determine the total work transfer. [Take R = 287 J/kg K]



Solution:

Given, Mass of air (m) = 0.5 kg

Initial state: $P_1 = 1$ MPa $= 1000$ kPa, $T_1 = 500^0$C $= 773$K

Final state: $T_{final} = 50^0$C $= 323$ K

Pressure required to support the piston, $P_{support} = 0.5$ MPa $= 500$ kPa

$\therefore$ Volume of air at initial state, $V_1 = \frac{mRT_1}{P_1} = \frac{0.5 \times 287 \times 773}{1000\times10^3} = 0.11093$ m^3

Initial pressure of the system is 1000 kPa and pressure required to support the piston is 500 kPa. Hence, during initial stage of cooling piston remain stationary although heat is removed from the system, so process is constant volume cooling (Process 1-2). During constant volume cooling, pressure of the system decreases from 1000 kPa to 500 kPa. Hence, we can define state 2 as

State 2: $P_2 = 500$ kPa, $V_2 = 0.11093\ m^3$

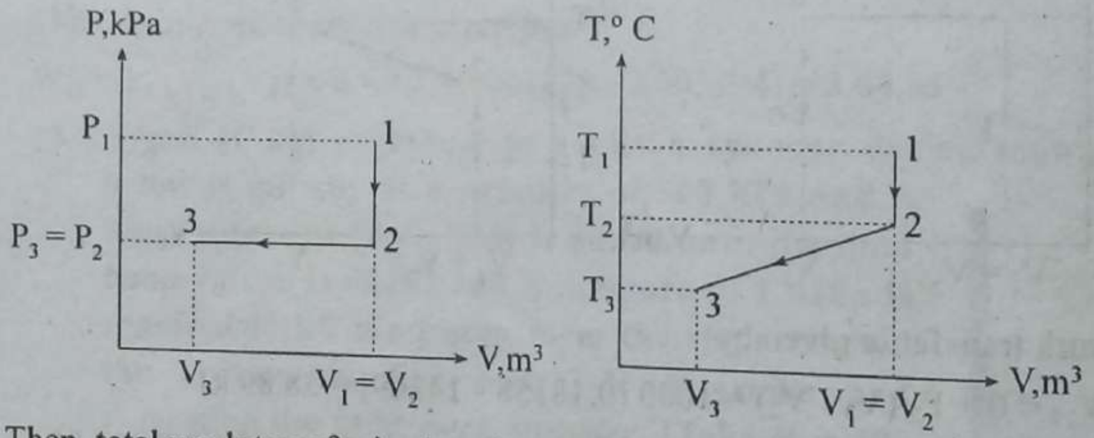
$\therefore$ Temperature of air at state 2, $T_2 = \dfrac{P_2V_2}{mR} = \dfrac{500\times10^3 \times 0.11093}{0.5\times287} = 386.52$ K

$= 113.52^0$ C

But the required final temperature is 50^0 C, hence it should be further cooled to decrease the temperature from 113.52^0C to 50^0 C and the process occurs at constant pressure of 500 kPa (Process 2-3). Hence, we can define state 3 as

State 3: $P_3 = 500$ kPa, $T_3 = 50^0$C

$\therefore$ Volume of air at final state, $V_3 = \dfrac{mRT_3}{P_3} = \dfrac{0.5 \times 287 \times 323}{500 \times 10^3} = 0.092701\ m^3$



Then, total work transfer is given by

$W = W_{12} + W_{23} = 0 + P_2 (V_3 - V_2) = 500 (0.092701 - 0.11093) = - 9.115$ kJ

15. Oxygen (3.6 kg) contained in a piston cylinder device shown in figure below is initially at a pressure of 200 kPa and a temperature of 50°C. Heat is added until the piston just reaches the upper stops where the total volume is 3 m³. It requires a pressure of 500 kpa to lift the piston. Sketch the process on P-V and T-V diagrams and determine the total work transfer. [Take R = 260 J/kg K]

Solution:

Given, Mass of oxygen (m) = 3.6 kg

Initial state: $P_1 = 200$ kPa, $T_1 = 50^0C = 323$ K

Final state: $V_{final} = 3\ m^3$

Pressure required to lift the piston (P_{lift}) = 100 kPa

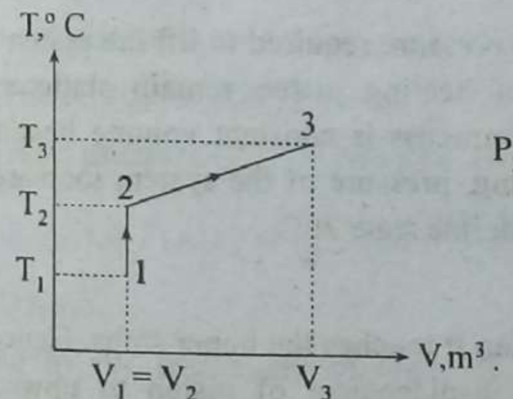
$\therefore$ Volume of oxygen at initial state, $V_1 = \frac{mRT_1}{P_1} = \frac{3.6 \times 260 \times 323}{200 \times 10^3} = 1.51164\ m^3$

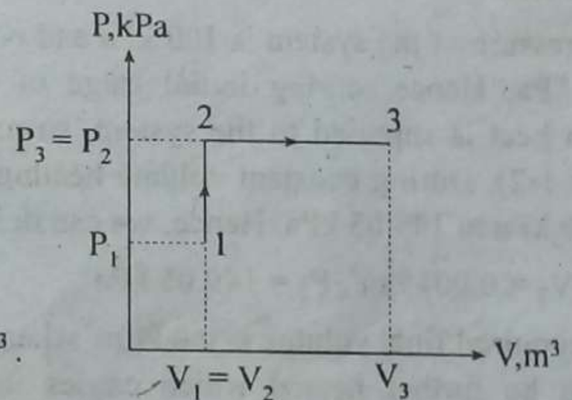
Initial pressure of the system is 200 kPa and pressure required to lift the piston is 500 kPa. Hence, during initial stage of heating piston remain stationary although heat is supplied to the system, so process is constant volume heating (Process 1-2). During constant volume heating, pressure of the system increases from 200 kPa to 500 kPa. Hence, we can define state 2,

State 2: $P_2 = 500$ kPa, $V_2 = 1.51164\ m^3$

But the required final volume is $3\ m^3$ when it touches the upper stops. Hence, it should be further heated which causes displacement of piston to upward direction and the process occurs at constant pressure of 500 kPa (Process 2-3) until it just touches the upper stops. Hence, we can define state 3,

State 3: $P_3 = 500$ kPa, $V_3 = 3\ m^3$

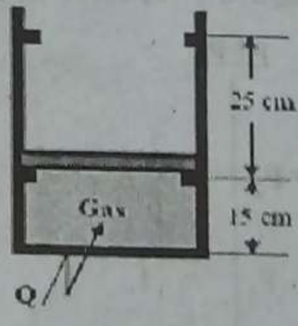




Then, total work transfer is given by

$W = W_{12} + W_{23} = P_2 (V_3 - V_2) = 500 (3 - 1.51164) = 744.18$ kJ

16. A piston cylinder arrangement shown in figure below contains gas initially at $P_1 = P_{atm} = 100$ kPa and $T_1 = 20^0C$. Piston with a cross sectional area of 0.01 m^2 has a mass of 50 kg ad is initially resting on the bottom stops. Heat is added to the system until it touches the upper stops.



(a) Sketch the process on P-V and T-V diagrams.

(b) Determine the total work transfer.

(c) Determine the final temperature of the gas. [Take g = 9.81 m/s²]

Solution:

Given, Mass of piston (m_p) = 50 kg

Initial state: $P_1 = P_{atm} = 100$ kPa, $T_1 = 20^0C = 293$ K

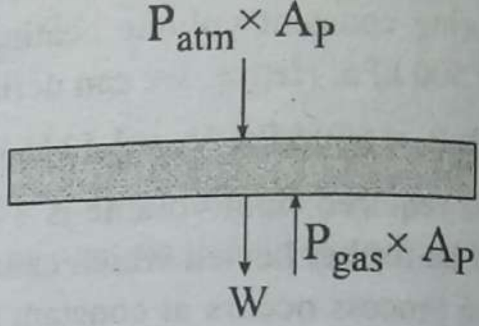
Cross sectional area of piston $(A_p) = 0.01\ m^2$

$\therefore$ Volume of gas at initial state, $V_1 = A_p \times x_1 = 0.01 \times 0.15 = 0.0015\ m^3$

And, Volume of gas at final state, $V_{final} = A_p\,(x_1 + x_2) = 0.01 \times (0.15 + 0.25)$

$= 0.004\ m^3$

Referring to the free body diagram of the piston we can write equation for the pressure inside the cylinder as



$$P_{gas} \times A_p = P_{atm} \times A_p + W$$

$$\text{or, } P_{gas} = P_{atm} + \frac{m_p g}{A_p}$$

$\therefore$ Pressure required to lift the piston,

$$P_2 = P_{atm} + \frac{m_p g}{A_p} = 100 + \frac{50 \times 9.81}{0.01}$$

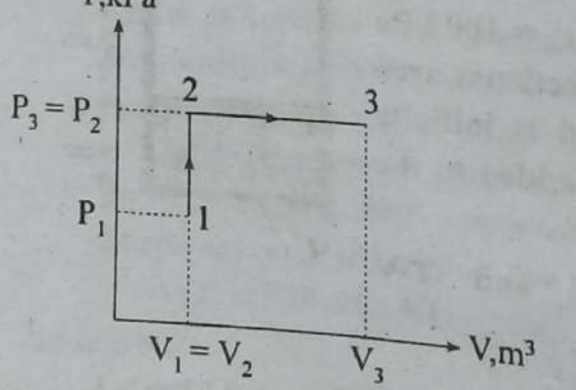
$= 149.05$ kPa

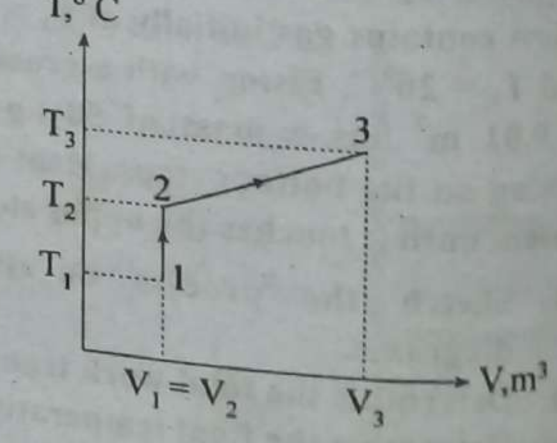
Initial pressure of the system is 100 kPa and pressure required to lift the piston is 149.05 kPa. Hence, during initial stage of heating piston remain stationary although heat is supplied to the system, so process is constant volume heating (Process 1-2). During constant volume heating, pressure of the system increases from 100 kPa to 149.05 kPa. Hence, we can define state 2,

State 2: $V_2 = 0.0015\ m^3$, $P_2 = 149.05$ kPa

But the required final volume is $0.004\ m^3$ when it touches the upper stops. Hence, it should be further heated which causes displacement of piston to upward direction and the process occurs at constant pressure of 149.05 kPa (Process 2-3) until it just touches the upper stops. Hence, we can define state 3,

State 3: $P_3 = 149.05$ kPa, $V_3 = 0.004\ m^3$





Then, total work transfer is given as

$$W = W_{12} + W_{23} = 0 + P_2\,(V_3 - V_2) = 149.05\,(0.004 - 0.0015) \times 10^3 = 372.625\ J$$

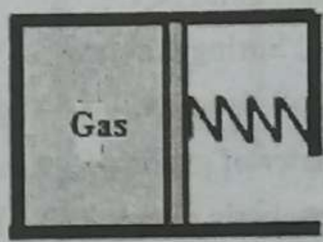
Now, using ideal gas equation

$$\frac{P_1V_1}{T_1} = \frac{P_3V_3}{T_3}$$

or, $$\frac{100 \times 0.0015}{293} = \frac{149.05 \times 0.004}{T_3}$$

$\therefore\ T_3 = 1164.58\ K = 891.58^0C$

17. An unstretched spring (k = 1 kN/m) is attached to a piston cylinder device as shown in figure below. Heat is added until the gas pressure inside the cylinder is 400 kPa. If the diameter of the piston is 50 mm, determine the work done by the gas on the piston. [Take P_{atm} = 100 kPa]



Solution:

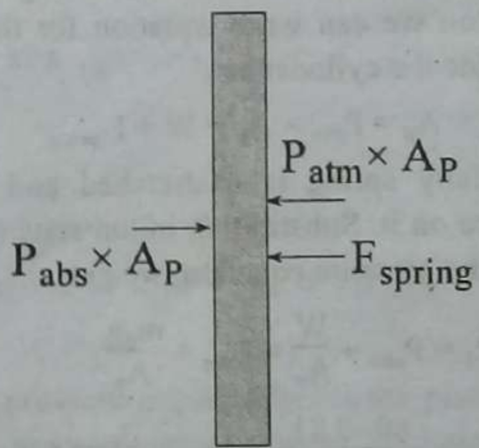
Given, Spring constant(k) = 1 kN/m

Diameter of piston (d_p) = 50 mm = 0.05 m

Final pressure, P_{final} = 400 kPa = P_2

Atmospheric pressure (P_{atm}) = 100 kPa

Area of piston (A_p) $= \frac{\pi d_p^2}{4} = \frac{\pi(0.05)^2}{4} =$ 0.0019635 m^2



Referring to the free body diagram of the piston we can write equation for the pressure inside the cylinder as

$$P_{abs} \times A_p = P_{atm} \times A_p + F_{spring}$$

or, $$P_2 = P_{atm} + \frac{F_{spring}}{A_p} = P_{atm} + \frac{kx}{A_p}$$

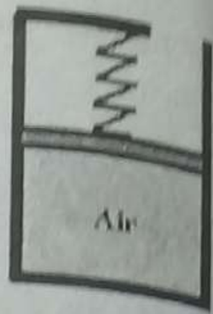
or, $$400 = \frac{1 \times x}{0.0019635} + 100$$

$\therefore\ x = 0.589$ m

Then, work done by the gas on the pistonis given by

$$W = \frac{1}{2}(P_1 + P_2) \times (V_2 - V_1) = \frac{1}{2}(P_1 + P_2) \times A_p \times x$$

$$= \frac{1}{2} \times (100 + 400) \times 0.0019635 \times 0.589 = 0.289\ kJ$$



18. A piston cylinder arrangement loaded with a linear spring (k = 2 kN/m) as shown in figure below contains air. Spring is initially unstretched and undergoes a compression of 40 mm during a process. If the mass of the piston is 80 kg and piston diameter is 0.1 m, determine the total work transfer. [Take P_{atm} = 100 kPa and g = 9.81 m/s²]

Solution:

Given, Spring constant (k) = 2 kN/m

Mass of piston (m_p) = 80 kg

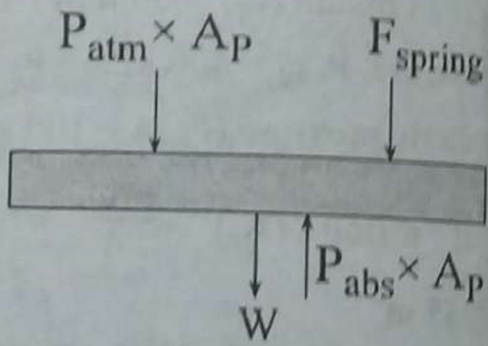
Diameter of piston (d_p) = 0.1m

Atmospheric pressure (P_{atm}) = 100 kPa

Displacement of spring (x) = 40 mm = 0.04 m

$$\therefore \text{Area of piston } (A_p) = \frac{\pi d_p^2}{4} = \frac{\pi \times 0.1^2}{4} = 0.007854 \text{ m}^2$$

Referring to the free body diagram of the piston we can write equation for the pressure inside the cylinder as



$$P_{abs} \times A_p = P_{atm} \times A_p + W + F_{spring}$$

Initially spring is unstretched and exerts no force on it. Substituting initial state (F_{spring} = 0) on the pressure equation, we get

$$\therefore\ P_1 = P_{atm} + \frac{W}{A_p} = P_{atm} + \frac{m_p g}{A_p}$$

$$= 100 + \frac{80 \times 9.81}{0.007854} = 100 + 99.924 = 199.924 \text{ kPa}$$

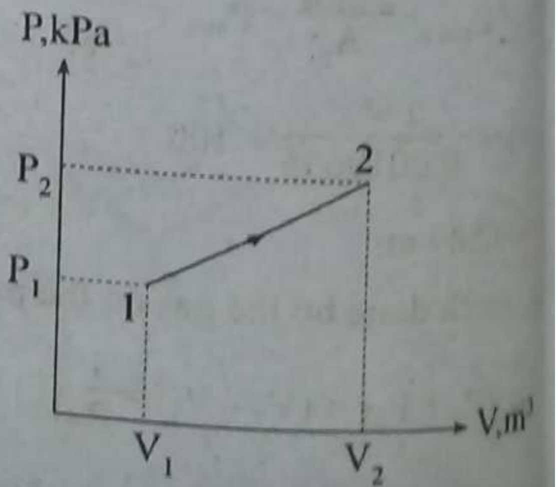
Now, after spring gets compressed by 40 mm during a process, pressure inside the cylinder is given as

$$P_2 = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p} = 199.924 + \frac{kx}{A_p}$$

$$= 100 + 99.924 + \frac{2 \times 0.04}{0.007854} = 210.11 \text{ kPa}$$

Now, total work transfer is given as

$$W_{12} = \frac{1}{2}(P_2 + P_1)(V_2 - V_1)$$

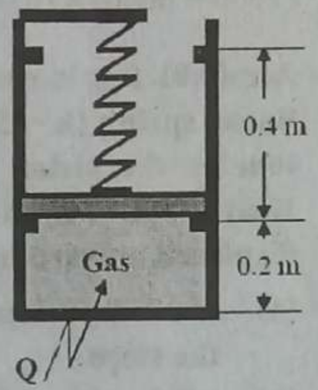


$= \frac{1}{2} \times (P_2 + P_1) \times A_p \times x$

$= \frac{1}{2} \times (199.924 + 210.11) \times 0.007854 \times 0.04$

$= 0.06441$ kJ = 64.41 J

19. A piston cylinder arrangement with two set of stops is restrained by a linear spring (k = 12 kN/m) as shown in figure below. The initial pressure of the gas is 500 kPa and the pressure required to lift the piston is 1000 kPa. Cross-sectional area of the piston is 0.05 m². Heat is supplied to the gas until its pressure reaches 6000 kPa. Sketch the process on P-V diagram and determine the total work transfer.



Solution:

Given, Spring constant (k) = 12 kN/m

Initial state: $P_1 = 500$ kPa

Pressure required to lift the piston (P_{lift})= 1000 kPa

Final pressure, $P_{final} = 6000$ kPa

Area of the piston (A_p) = 0.05 m²

∴ Volume of gas at initial state, $V_1 = A_p \times x_1 = 0.05 \times 0.2 = 0.01$ m³

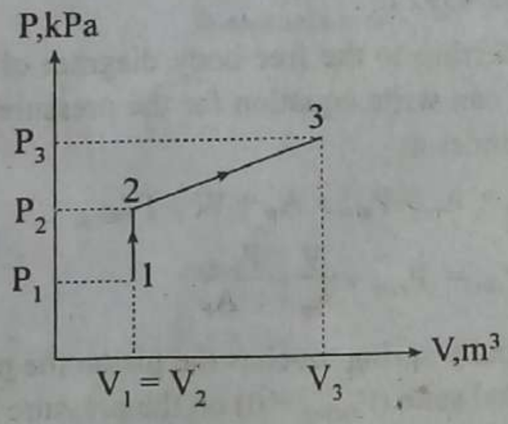
And, Volume of gas at final state, $V_{final} = A_p (x_1 + x_2) = 0.05 \times (0.2 + 0.4)$

= 0.03 m³

Initial pressure of the system is 500 kPa and pressure required to lift the piston is1000 kPa. Hence, during initial stage of heating piston remain stationary although heat is supplied to the system, so process is constant volume heating (Process 1-2). During constant volume heating, pressure of the system increases from 100 kPa to 1000 kPa. Hence, we can define state 2,

State 2: $P_2 = 1000$ kPa, $V_2 = 0.01$ m³

But the required final pressure is 6000 kPa. Hence, it should be further heated which causes displacement of piston to upward direction until pressure reaches to 6000 kPa (Process 2-3). Hence, we can define state 3 as,



State 3: $P_3 = 6000$ kPa, $V_3 = 0.03$ m³

Then, total work transfer is given as

$W = W_{12} + W_{23}$

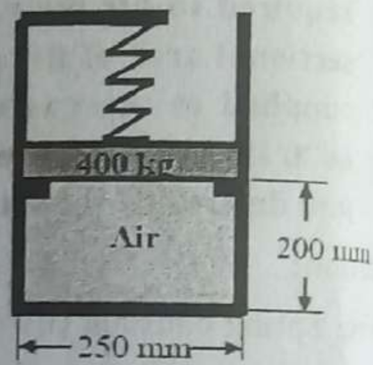
$= 0 + \frac{1}{2}(P_2 + P_3)(V_3 - V_2)$

$= \frac{1}{2}(1000 + 6000) \times (0.03 - 0.01) = 70$ kJ

20. Air (0.01 kg) is contained in a piston cylinder device restrained by a linear spring (k = 500 kN/m) as shown in figure below. Spring initially touches the piston but exerts no force on it. Heat is added to the system until the piston is displaced upward by 80 mm. determine

(a) the temperature at which piston leaves the stops

(b) work done by the air [Take R = 287J/kgK, P_{atm} = 100 kPa and g = 9.81 m/s²] ***(IOE 2070 Ashad)***



Solution:

Given, Mass of air (m) = 0.01 kg

Spring constant (k) = 500 kN/m

Displacement of spring (x) = 80 mm = 0.08 m

Atmospheric pressure (P_{atm}) = 100 kPa

Mass of piston (m_p) = 400 kg

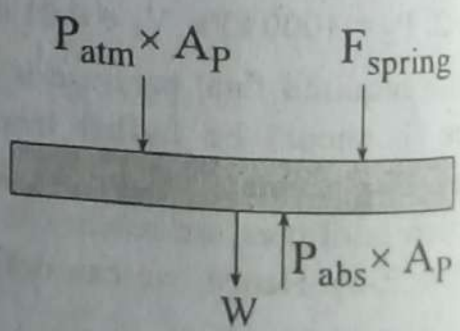
Diameter of piston (d_p) = 250 mm = 0.25m

$\therefore$ Area of piston $(A_p) = \frac{\pi(d_p)^2}{4} = \frac{\pi}{4} \times 0.25^2 = 0.0491\ m^2$

Volume of air at initial state, $V_1 = A_p \times x_1 = 0.0491 \times 0.2 = 0.00982\ m^3$

And, Volume of air at final state, $V_{final} = A_p \times (x_1 + x) = 0.0491 \times (0.2 + 0.08)$
$= 0.01392\ m^3$

Referring to the free body diagram of the piston we can write equation for the pressure inside the cylinder as



$P_{abs} \times A_p = P_{atm} \times A_p + W + F_{spring}$

$\therefore P_{abs} = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_P}$

Initially spring touches the piston the piston but exerts no force on it. Substituting initial state ($F_{spring} = 0$) on the pressure equation, we get

$\therefore\ P_1 = P_{atm} + \frac{W}{A_p} = P_{atm} + \frac{m_p g}{A_p} = 100 + \frac{400 \times 9.81 \times 10^{-3}}{0.0491} = 100 + 79.92$

$= 179.92$ kPa

Then, temperature at which piston leaves the stop, $T_1 = \frac{P_1V_1}{mR}$

$= \frac{179.92 \times 10^3 \times 0.00982}{0.01 \times 0.287} = 615.615\ K = 342.615°C$

After spring gets compressed by 80 mm during a process, pressure inside the cylinder is given as

$P_2 = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_P} = P_1 + \frac{kx}{A_p} = 179.92 + \frac{500 \times 0.08}{0.0491} = 179.92 + 814.664$

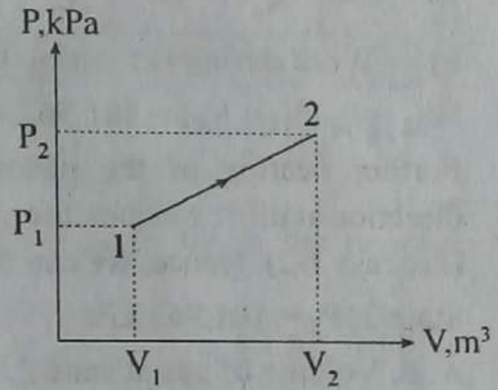
$= 994.584$ kPa

Then, work done by air is given by

$W = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$

$= \frac{1}{2}(994.584 + 179.92)(0.01392 - 0.00982)$

$= 2.408$ kJ

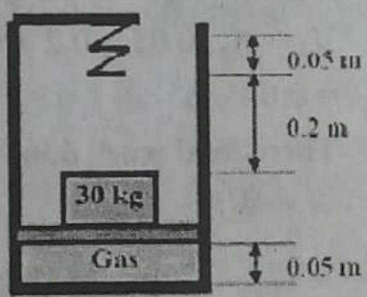


21. A gas enclosed by a piston shown in figure below starts to expand due to heating. The initial movement of 0.2 m is restrained by a fixed mass of 30 kg and the final 0.05 m is restrained both by the mass and a spring of stiffness 10 kN/m. The cross sectional area of the piston is 0.15 m² and the atmospheric pressure is 100 kPa.

(a) Neglecting the mass of the spring and the piston, sketch a P-V diagram of the process.

(b) Calculate the work during the initial 0.2 m movement.

(c) Calculate the total work done.



Solution:

Given, Mass of gas (m) = 30 kg

Spring constant (k) = 10 kN/m

Cross sectional area of piston (A_p) = 0.15 m²

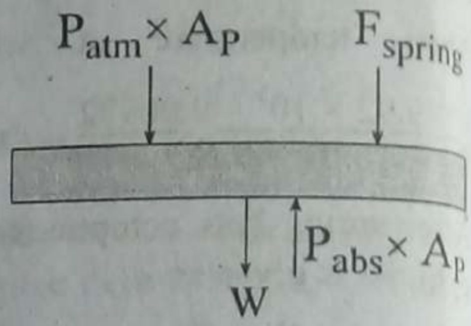
Atmospheric pressure (P_{atm})= 100 kPa

Initial displacement of piston (x_1) = 0.2 m

Final displacement of piston (x_2) = 0.05 m

∴ Volume of gas at initial state, $V_1 = A_p \times 0.05 = 0.15 \times 0.05 = 0.0075\ m^3$

a) Referring to the free body diagram of the piston we can write equation for the pressure inside the cylinder as



$$P_{gas} = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p}$$

Initially, spring is not in contact with the piston so it exerts no force on it. Substituting initial state ($F_{spring} = 0$) on the pressure equation, we get

$$P_1 = P_{atm} + \frac{W}{A_p} = P_{atm} + \frac{m_p g}{A_p} = 100 + \frac{30 \times 9.81}{0.15} = 101\ .962\ kPa$$

b) Work during the initial 0.2 m movement is given as

$W_{12} = P_2 (V_2 - V_1) = 101.962 \times (0.0375 - 0.0075) = 3.059$ kJ

Further heating of the piston causes the displacement of piston to upward direction until it touches the spring and the process is constant pressure heating (Process 1-2). Hence, we can define state 2 as

State 2: $P_2 = 101.962$ kPa

And, Volume of gas at state 2, $V_2 = A_p (0.05 + 0.2) = 0.15 \times 0.25 = 0.0375\ m^3$

When spring gets compressed, the pressure inside the cylinder is given as

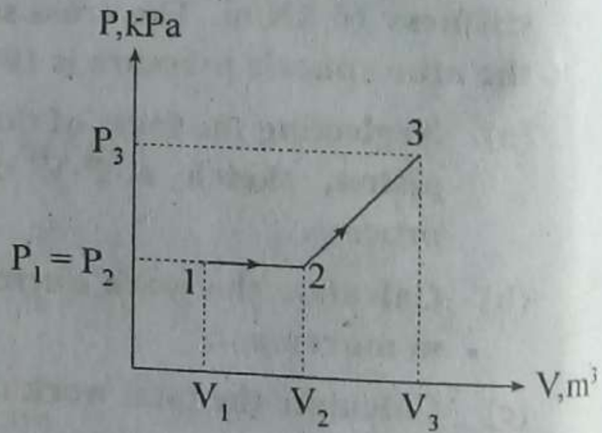
$$P_3 = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p} = P_2 + \frac{F_{spring}}{A_p} = P_2 + \frac{kx}{A_p} = 101.962 + \frac{10 \times 0.05}{0.15}$$

$= 105.295$ kPa

And, Volume of the gas of final state,

$V_3 = A_p (0.05 + 0.2 + 0.05) = 0.15 \times 0.3$

$= 0.045\ m^3$



Then, total work done by gas is given as

$W = W_{12} + W_{23}$

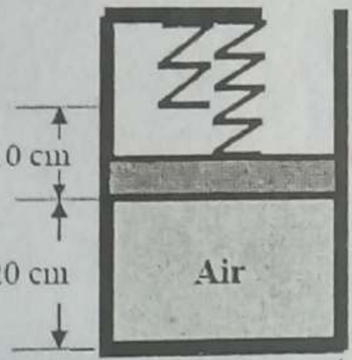
$= W_{12} + \frac{1}{2} (P_2 + P_3) (V_3 - V_2)$

$= 3.059 + \frac{1}{2}\ (101.962 + 105.295)\ (0.045 - 0.0375)$

$= 3.059 + 0.777 = 3.836$ kJ

22. A piston cylinder arrangement shown in figure below is restrained by two linear springs as shown. The system contains air initially at a pressure of 150 kPa and a volume of 0.002 m^3. Heat is added to the system until its volume doubles; determine the total work transfer. Also

sketch the process on P-V diagram. Both springs have spring constant of 100 kN/m.



Solution:

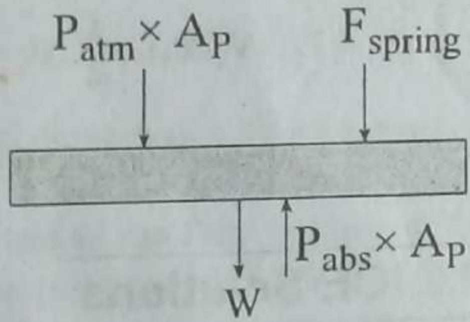
Given, Spring constant, $k_1 = k_2 = 100$ kN/m

Initial state: $V_1 = 0.002\ m^3$, $P_1 = 150$kPa

Final state: $V_{final} = 2V_1 = 0.004\ m^3$

$\therefore$ Area of piston, $A_p = \frac{V_1}{x_1} = \frac{0.002}{0.2} = 0.01\ m^2$

Referring to the free body diagram of the piston we can write equation for the pressure inside the cylinder as



$$P_{abs} \times A_p = P_{atm} \times A_p + W + F_{spring}$$

$$\therefore P_{abs} = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p}$$

Initially spring touches the piston the piston but exerts no force on it. Substituting initial state ($F_{spring} = 0$) on the pressure equation, we get

$$\therefore P_1 = P_{atm} + \frac{W}{A_p} = 150\ \text{kPa}$$

When the first spring gets compressed by 10 cm, the pressure inside the cylinder is given as

$$P_2 = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p} = 150 + \frac{k_1x_2}{A_p} = 150 + \frac{100 \times 0.1}{0.01} = 150 + 1000 = 1150\ \text{kPa}$$

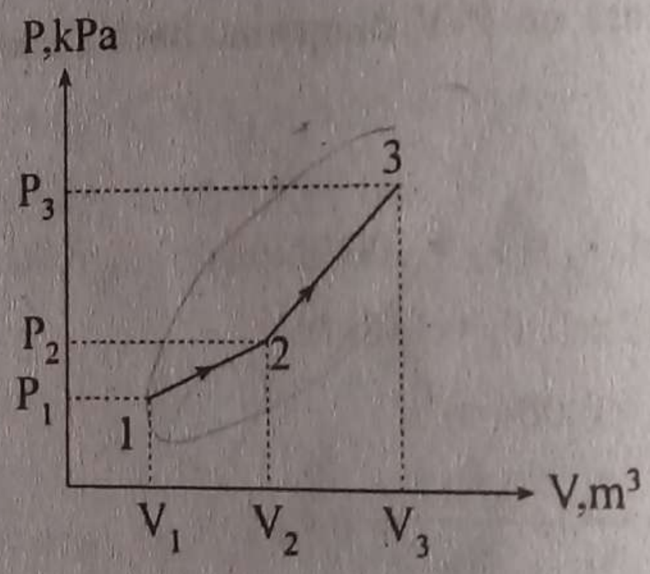
$\therefore$ Volume of air at state 2, $(V_2) = A_p \times (x_1 + x_2) = 0.01 \times (0.2 + 0.1) = 0.003\ m^3$

Final compression of the spring, $x_3 = \frac{V_3 - V_2}{A_p} = \frac{0.004 - 0.003}{0.01} = 0.1$ m

When both the springs get compressed by 0.1 m, then pressure of air inside the cylinder is given as

$$P_3 = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p} = 150 + \frac{k_1x_2}{A_p} + \frac{k_1x_3}{A_p} + \frac{k_2x_3}{A_p} = 1150 + \frac{k_1x_3}{A_p} + \frac{k_2x_3}{A_p}$$

$$= 1150 + \frac{100 \times 0.1}{0.01} + \frac{100 \times 0.1}{0.01} = 1150 + 1000 + 1000 = 3150\ \text{kPa}$$



Then, total work transfer, $W = W_{12} + W_{23s}$

$$= \frac{1}{2}(P_1 + P_2)(V_2 - V_1) + \frac{1}{2}(P_2 + P_3)(V_3 - V_2)$$

$$= \frac{1}{2}(150 + 1150)(0.003 - 0.002) + \frac{1}{2}(1150 + 3150)(0.004 - 0.003) = 2.8\ kJ$$

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Chapter 3

Properties of Common Substances

3.1 Numerical Problems

1. **Fill in the blanks in the following table with the corresponding properties of water or by the symbol ×, when it is no relevant or meaningless or by the symbol —, when it is indeterminate.**

S.N.	P, kPa	T, °C	x, %	v, m^3/kg
1	300	200		
2	300		65	
3		200		0.1050
4	10000			0.04863
5	20000	120		
6	101.325	100		

Solution:

State 1 ($P_1 = 300$ kPa, $T_1 = 200^\circ C$)

Referring to Table A2.1, $T_{sat}(300^\circ C) = 133.56^0 C$. Here T > Tsat, hence it is superheated vapor. Then referring to Table A2.4, $v_1 = 0.7163\ m^3/kg$, $h_1 = 2865.1$ kJ/kg and Degree of superheat = 200 - 133.56 = $66.44^0 C$. Quality is meaningless for the superheated vapor.

State 2 ($P_2 = 300$ kPa, $x_2 = 65\%$)

Quality of 65% means given state is a two phase mixture with 65% saturated vapor and remaining 35% saturated liquid. Then referring to Table A2.1, $T_2 = T_{sat}$ (300 kPa) = $133.56^0 C$. Specific volume and specific enthalpy are then given by

$v_2 = v_f + x_2 v_{fg} = 0.001073 + 0.65 \times 0.6048 = 0.3942\ m^3/kg$

$h_2 = h_f + x_2 h_{fg} = 561.61 + 0.65 \times 2163.7 = 1970.615$ kJ/kg

State 3 ($T_3 = 200^0 C$, $v_3 = 0.1050\ m^3/kg$)

Referring to Table A2.2, $v_f(200^\circ C) = 0.001156\ m^3/kg$

$v_f < v < v_g$, hence it is a two phase mixture. Then, its pressure quality and specific enthalpy are given by

$P_3 = P_{sat}(200^0 C) = 1553.6$ kPa

$$x_3 = \frac{v_3 - v_l}{v_{lg}} = \frac{0.1050 - 0.001156}{0.1261} = 0.8235 = 82.35\%$$

$h_3 = h_l + x_3 h_{lg} = 852.38 + 0.8235 \times 1940.1 = 2450.05$ kJ/kg

State 4 ($P_4 = 10000$ kPa, $v_4 = 0.04863$ m^3/kg)

Referring to Table A2.1, T_{sat} (10000 kPa) = 311.03^0 C, v_l (10000 kPa) = 0.0014 m^3/kg, v_{lg} = 0.01658 m^3/kg, v_g = 0.01803 m^3/kg. Here, $v > v_g$, hence, it is superheated vapor. Then referring to Table A2.4, T_4 = 800^0 C, h_4 = 4113.5 kJ/kg
Degree of superheat = 800 - 311.03 = 488.97^0 C

State 5 ($P_5 = 20000$ kPa, $T_5 = 120^0$ C)

Referring to Table A2.1, T_{sat}(20000 kPa) = 365.8^0 C

Here, $T < T_{sat}$, hence it is a compressed liquid. Properties of compressed liquid 120^0 C are not given in the provided table (Table A2.3). Hence to determine properties at 120^0 C for 20000 kPa, we have to use the interpolation technique. For this, we list the required properties for the interval which includes 120^0 C.

T,^{0}C	v_l, m^3/kg	h_l, kJ/kg	
110	0.001041	475.87	(a)
130	0.001058	559.90	(b)

Applying linear interpolation for specific volume and specific enthalpy,

$$v_5 - v_a = \frac{v_b - v_a}{T_b - T_a}(T_5 - T_a)$$

$$\therefore v_5 = v_a + \frac{v_b - v_a}{T_b - T_a}(T_5 - T_a)$$

$$= 0.001041 + \frac{0.001058 - 0.001041}{130 - 110}(120 - 110) = 0.0010495 \text{ m}^3/\text{kg}$$

$$\text{And, } h_5 = h_a + \frac{h_b - h_a}{T_a - T_a}(T_5 - T_a)$$

$$= 475.87 + \frac{559.90 - 475.87}{130 - 110}(120\text{-}110) = 517.87 \text{ kJ/kg}$$

State 6 ($P_6 = 101.325$ kPa, $T_5 = 100^0$ C)

Referring to Table A2.1, T_{sat}, (101.325 kPa) = 100^0 C. Here $T = T_{sat}$, hence it within saturation region. But within the saturation region we cannot fix the state of the substance with the help of pressure and temperature because they dependent within the saturation region. Hence, given state and required properties are indeterminate.

State	P kPa	T, ^{0}C	x%	v, m^3/kg	h, kJ/kg	Degree of superheat,^{0}C
1	300	200	×	0.7163	2865.1	66.44
2	300	300	65	0.3942	1970.615	×
3	1553.6	200	82.35	0.1050	2450.05	
4	10000	800	×	0.04863	4113.5	488.97
5	20000	120	×			
6	101.325	100	-	-	-	-

2. Determine the pressure for water at 250^0 C with the specific volume of 0.25 m^3/kg.

Solution:

Given, $T = 250^0 C$, $v = 0.25\ m^3/kg$

Referring to Table A2.2, $v_f(250^0 C) = 0.001251\ m^3/kg$ and $v_g(250^0 C) = 0.05011\ m^3/kg$. Here, $v > v_g$, hence it is a superheated vapor. Now referring to Table A2.4 at 250^0 C, specific volume of saturated vapor which includes the specific volume 0.25 m^3/kg and corresponding pressure are listed as:

P, kPa	v_g, m^3/kg	
800	0.2931	(a)
1000	0.232	(b)

Then applying linear interpolation for pressure,

$$P - P_a = \frac{P_b - P_a}{(v_g)_b - (v_g)_a} [v - (v_g)_a]$$

$$\therefore P = P_a + \frac{P_b - P_a}{(v_g)_b - (v_g)_a} [v - (v_g)_a]$$

$$= 800 + \frac{1000 - 800}{0.2326 - 0.2931}(0.25 - 0.2931) = 942.479 \text{ kPa}$$

3. Determine the temperature and quality (if needed) for water at a pressure of 200 kPa and having a specific volume of

(a) 0.8 m^3/kg

(b) 1.25 m^3/kg

Solution:

a) $P = 200$ kPa, $v = 0.8$ m^3/kg

Referring to Table A2.1, v_l (200 kPa) = 0.001060 m^3/kg and v_g (200k Pa) = 0.8859 m^3/kg, v_{lg} (200^0 C) = 0.8848 m^3/kg. Here, $v_l < v < v_g$, hence it is a two phase mixture.

$\therefore$ $T = T_{sat}$ (200 kPa) = 124.01^0C

$$x = \frac{v - v_l}{v_{lg}} = \frac{0.8 - 0.001060}{0.8848} = 0.90296$$

b) $P = 200$ kPa, $v = 1.25 m^3/kg$

Referring to Table A2.1, v_l (200 kPa) = 0.001060m^3/kg, v_g (200 kPa) = 0.8859 m^3/kg.Here $v > v_g$, hence it is a superheated vapor. Referring to Table A2.4, specific volume of saturated vapor which includes the specific volume 1.25 m^3/kg and corresponding temperatures for 200 kPa are listed as:

T^0, C	v_g, m^3/kg	
250	1.1988	(a)
300	1.3162	(b)

Applying linear interpolation for temperature,

$$T - T_a = \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v - (v_g)_a]$$

$$\therefore T = T_a + \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v - (v_g)_a]$$

$$= 250 + \frac{300 - 250}{1.3162 - 1.1988}(1.25 + 1.1988) = 271.806^0 C$$

4. **A rigid vessel contains 8 kg of water at 120^0C. If 5 kg of the water is in the liquid form and the rest in the vapor form. Determine:**

   (a) the pressure in the vessel,

   (b) the volume of the tank,

   (c) the volume of saturated liquid and saturated vapor respectively, and

   (d) the specific enthalpy of H_2O.

Solution:

Given, Mass of water (m) = 8 kg

Temperature of water (T) = 120^0 C

Mass of liquid (m_l) = 5 kg

Mass of vapor (m_g) = 3 kg

Since it is a two phase mixture, referring to table A2.1, P_{sat} (120^0 C) = 198.48 kPa, v_l (120^0 C) = 0.001060 m^3/kg, v_g (120^0 C) = 0.8922 m^3/kg, v_{lg} (120^0 C) = 0.8911 m^3/kg

a) Pressure of mixture is given as

$P = P_{sat}$ (120^0C) = 198.48 kPa

$\therefore$ Quality of two phase mixture (x) $= \frac{m_g}{m} = \frac{3}{8} = 0.375$

b) Volume of tank (V) = $v_l \times m_l + v_g \times m_g$

$= 0.001060 \times 5 + 0.8922 \times 3 = 26819$ m^3

c) Volume of saturated liquid (V_l) = $v_l \times m_l = 0.001060 \times 5 = 0.0053$ m^3

And, Volume of saturated vapor (v_g) = $v_g \times m_g = 0.8922 \times 3 = 2.6766$ m^3

d) The specific enthalpy of H_2O (h) = $h_l + xh_{lg}$

$= 50378 + 0.375 \times 2202.4 = 1329.68$ kJ/kg

5. **A two phase mixture of H_2O has a temperature of 200^0C and occupies a volume of 0.05 m^3. The mass of saturated liquid is 1 kg and saturated vapor is 3 kg. Determine the pressure and specific volume of the mixture.**

Solution:

Given, Temperature of H_2O (T) = 200^0 C

Volume of H_2O (V) = 0.05 m^3

Mass of saturated liquid (m_l) = 1 kg

Mass of saturated vapor (m_g) = 3 kg

Since it is a two phase mixture, referring to Table A2.1,P_{sat} (200^0 C) = 1553.6 kPa v_l (200^0 C) = 0.001156 m^3/kg, v_{lg} (200^0 C) = 0.1261 m^3/kg

Quality of mixture is given as

$$x = \frac{m_g}{m_l + m_g} = \frac{3}{1+3} = 0.75$$

Then, pressure of mixture is given as

$P = P_{sat}$ (200^0 C) = 1553.6 kPa

And specific volume of mixture, $v = v_l + xv_{lg}$

$= 0.001156 + 0.75 \times 0.1261 = 0.095731$ m^3/kg

6. A 0.3 m3 rigid vessel contains 5 kg of water at 150 kPa. Determine:

(a) the temperature,

(b) the mass of each phase, and

(c) the specific enthalpy

Solution:

Given, Volume of vessel (V) = 0.3 m^3

Mass of water (m) = 5 kg

Pressure (P) = 150 kPa

Specific volume of water (v) $= \frac{V}{m} = \frac{0.3}{5} = 0.06\ m^3/kg$

Referring to Table A2.1, T_{sat} (150 kPa) = 111.38^0 C, v_l (150 kPa) = 0.001 m^3/kg, v_{lg} (150 kPa) = 1.1584 m^3/kg, v_g (150 kPa) = 1.1595 m^3/kg, h_l(150 kPa) 467.18 kJ/kg, h_{lg} (150 kPa) = 2226.2 kJ/kg. Here, $v_l < v < v_g$, hence it is a phase mixture. Quality of mixture is given as

$$\therefore\ x = \frac{v - v_l}{v_{lg}} = \frac{0.06 - 0.001053}{1.1584} = 0.05088\ m^3/kg$$

a) Temperature of mixture is given as

$T = T_{sat}\ (150\ kPa) = 111.38^0 C$

b) Mass of saturated vapor (m_g) = = x × m = 0.05088 × 5 = 0.2544 kg

And, mass of saturated liquid (m_l) = m - m_g = 5 - 0.2544 = 4.7456 kg

c) Specific enthalpy of mixture is given as

$h = h_l + xh_{lg} = 467.18 + 0.05088 \times 2226.2 = 580.4491$ kJ/kg

7. A closed vessel contains 0.1 m^3 of saturated liquid and 0.9 m^3 saturated vapor in equilibrium at 200 ^{0} C. Determine its quality.

Solution:

Given, Volume of saturated liquid (V_l) = 0.1 m^3

Volume of saturated vapor (V_g) = 0.9 m^3

Temperature of water (T) = 200^{0}C

Referring to Table A. 2.2, v_l (200^{0}C) = 0.001156 m3/kg, v_g (200^{0}C) = 0.1 m^3/kg

Then,

Mass of saturated liquid (m_l) $= \frac{V_l}{v_l} = \frac{0.1}{0.001156} = 86.51$ kg

Mass of saturated vapor $(m_g) = \frac{V_g}{v_g} = \frac{0.9}{0.1273} = 7.07$ kg

$\therefore$ Quality of mixture, $x = \frac{m_g}{m_l + m_g} = \frac{7.07}{86.51 + 7.07} = 0.0756$

8. A vessel contains 2 kg of saturated liquid water and saturated water vapor mixture at a temperature of 150⁰ C. One third of the volume is saturated liquid and two third is saturated vapor. Determine the pressure, quality, and volume of the mixture. ***(IOE 2070 Magh)***

Solution:

Given, Mass of water (m) = 2 kg

Temperature of mixture (T) = 150^0C

Volume of saturated liquid in mixture $(V_l) = \frac{V}{3}$

Volume of saturated vapor in mixture $(V_g) = \frac{2V}{3}$

Referring the Table A2.2, P_{sat} (150^0 C) = 475.72 kPa, v_l (150^0 C) = 0.001090 m^3/kg, v_g (150^0 C) = 0.3929 m^3 /kg

$\therefore$ Mass of saturated liquid $(m_l) = \frac{V_l}{v_l} = \frac{V}{3 \times 0.001090}$

And, Mass of saturated vapor $(m_g) = \frac{V_g}{v_g} = \frac{2V}{3 \times 0.3929}$

Then, total mass of water is given by

$m = m_l + m_g$

or, $2 = \frac{V}{3 \times 0.001090} + \frac{2V}{3 \times 0.3929}$

or, $2 = \frac{1}{3} \frac{(0.3929V + 2 \times 0.001090V)}{0.001090 \times 0.3929}$

$\therefore$ $V = 0.006504\ m^3 = 6.504 \times 10^{-3}\ m^3$

$\therefore$ Quality of the mixture $(x) = \frac{m_g}{m} = \frac{2 \times V}{3 \times 0.3929 \times 2} = \frac{2 \times 0.006504}{3 \times 0.3929 \times 2}$

$= 5.5179 \times 10^{-3}$

Referring to Table A2.2, h_l (150^0 C) = 632.32 kJ/kg, h_{lg} (150^0 C) = 2114.1 kJ/kg, u_l (150^0 C) = 631.80 kJ/kg, u_{lg} (150^0 C) = 1927.7 kJ/kg. therefore, specific enthalpy and specific internal energy are given as,

$h = h_l + xh_{lg} = 632.32 + 0.0055179 \times 2114.1 = 643.985$ kJ/kg

$\therefore$ Enthalpy of the mixture (H) = mh = 2 × 643.985 = 1287.970 kJ

$u = u_l + xu_{lg} = 631.80 + 0.0055179 \times 1927.7 = 642.437$ kJ/kg

$\therefore$ Internal energy of the mixture (U) = mu = 2 × 642.437 = 1284.874 kJ

9. 2 kg of water is contained in a rigid vessel of volume 0.5 m³. S Heat is added until the temperature is 150° C. Determine:

(a) The final pressure,

(b) The mass of vapor at the final state, and

(c) The volume of the vapor at the final state.

Solution:

Given, Mass of water (m) = 2 kg

Volume of vessel (V) = 0.5 m^3

Final state: $T_2 = 150^0$ C

Process: constant volume heating

Specific volume of water (ν) $= \frac{V}{m} = \frac{0.5}{2} = 0.25$ m^3/kg

Referring to Table A2.2, $P_{sat}(150^0 C) = 475.72$ kPa, $\nu_l(150^0C) = 0.001090$ m^3/kg, $\nu_g(150^0C) = 0.3929$ m^3/kg, $\nu_{lg}(150^0C) = 0.3918$ m^3/kg, Here, $\nu_l < \nu < \nu_g$, hence it is a two phase mixture.

Quality of mixture (x) $= \frac{\nu - \nu_l}{\nu_{lg}} = \frac{0.25 - 0.001090}{0.3918} = 0.6353$

a) The final pressure is given by

$P_2 = P_{sat}(150^0 C) = 475.72$ kPa

b) Mass of vapor at final state (m_g) = x × m = 0.6353 × 2 = 1.2706 kg

c) Volume of vapor at final state $(V_g)_2 = \nu_g . m_g = 0.3929 \times 1.2706 = 0.49922$ m^3

10. Saturated water vapor at 200 kPa is in a freely moving piston cylinder device. At this state piston is 0.1 m from the bottom. Determine the height of the piston when the temperature is

(a) 250° C

(b) 150° C

(c) 100° C

Solution:

Given, Initial pressure (P) = 200 kPa

Displacement of piston (x) = 0.1 m

Referring to Table A2.1, T_{sat} (200 kPa) = 120.24^0 C, v_g (200 kPa) = 0.8859 m^3/kg

Considering unit mass of water, initial volume of water is

$(V_g)_1$= 0.8859 m^3

$\therefore$ Area of piston $(A_p) = \frac{V_g}{x} = \frac{0.8859}{0.1} = 8.859$ m^2

a) T = 250^0 C

Here, T > T_{sat}, hence it is a superheated steam. Referring to Table A2.4, v_g = 1.1988m^3/kg

Then, final volume of water $(V_g)_2 = v_g \times m_g = 1.1983 \times 1 = 1.1988$ m^3

$\therefore$ Height of the piston $= \frac{(V_g)_2}{A_p} = \frac{1.1988}{8.859} = 0.1353$ m

b) 150^0 C

Here, T > T_{sat}, hence it is superheated steam. Referring to Table A2.4, v_g = 0.9597 m^3/kg

Then, final volume of water $(V_g)_2 = v_g \times m_g = 0.9597 \times 1 = 0.9597$m^3

$\therefore$ Height of the piston $= \frac{(V_g)_2}{A_p} = \frac{0.9597}{8.859} = 0.1083$ m

c) T = 100^0 C

Here, T < T_{sat}, hence it is a compressed liquid. Referring to Table A2.2 (Since 200 kPa is not available in Table A2. 3)

v_l (100^0 C)= 0.001043 m^3/kg

Then, final volume of water $(V_l) = \frac{v_l}{m_l} = \frac{0.001043}{1} = 0.001043$ m^3

$\therefore$ Height of the piston $= \frac{(V_l)_2}{A_p} = \frac{0.001043}{8.859} = 0.000117733$ mm

11. Water in a piston cylinder device evaporates at a temperature of 120^0 C. If the diameter of the piston is 0.15 m and the local atmospheric pressure is 101 kPa, what is the mass of the piston? [Take g = 9.8 m^2/s]

Solution:

Temperature at which water evaporates (T) = 120^0 C

Diameter of piston (d_p) = 0.15 m

Mass of the piston (m_p) = ?

Referring o Table A2.2, P_{sat} (120^0 C) = 198.48 kPa = P

Referring to free body diagram of the piston we can write equation for the pressure inside the cylinder as,

$$P_{abs} = P_{atm} + \frac{W}{A_p}$$

$$\text{or, } P - P_{atm} = \frac{m_p g}{A_P}$$

$$\text{or, } (198.48 - 101) \times 10^3 = \frac{m \times 9.81}{\frac{\pi}{4}(0.15)^2}$$

$$\therefore \; m = 175.598 \text{ kg}$$

12. A piston cylinder device containing water has a piston mass of 50 kg and a cross sectional area of 0.01 m². If the atmospheric pressure is 101 kPa, determine the temperature at which the water start boiling. [Take $g = 9.8$ m²/s]

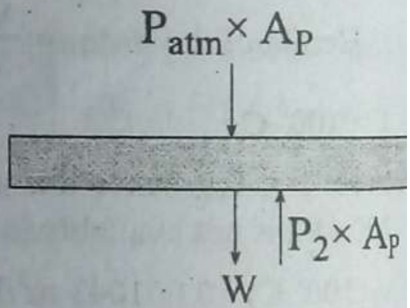
Solution:

Given, Mass of piston (m_p) = 50 kg

Area of piston (A_p) = 0.01m²

Atmospheric pressure (P_{atm})= 101 kPa

Referring to free body diagram of the piston we can write equation for the pressure inside the cylinder as,



$$P_{abs} = P_{atm} + \frac{W}{A_p}$$

$$\therefore \; P = 101 + \frac{50 \times 9.81}{0.01 \times 10^3} = 150.05 \text{ kpa} \cong 150 \text{ kPa}$$

Referring to the Table A2.2, T_{sat} (150 kPa) = 111.38°C

∴ Temperature at which water will start boiling (T) = T_{sat} (150 kPa) = 111.38°C

13. 5 kg of H_2O is contained in a closed rigid container with an initial pressure and quality of 1000 kPa and 40% respectively. Heat is added to the system until the container holds only saturated vapor. Sketch the process on P-v diagram and T-v diagrams and determine:

(a) The volume of the container, and

(b) The final pressure.

Solution:

Given, Mass of H_2O (m) = 5 kg

State 1: P_1 = 1000 kPa, x_1 = 0.4

Process: Constant volume heating

$\therefore$ Mass of saturated vapor $(m_g) = x_1 \times m = 0.4 \times 5 = 2$ kg

Referring to Table A2.1, v_l (1000 kPa) = 0.001127 m^3/kg

v_g (1000 kPa) = 0.1944 m^3/kg and v_{lg} (1000 kPa) = 0.1933 m^3/kg

$\therefore\ v_1 = v_l + x_1 v_{lg} = 0.001127 + 0.4 \times 0.1933 = 0.07845$ m^3/kg

a) The volume of the container (V) = m.v_1 = 5 × 0.7845 = 0.3922 m^3

Since, volume is constant specific volume at state 2 is given as

$v_2 = 0.07845$ m^3/kg

Referring the Table A2.1, specific volumes of saturated vapor which includes the specific volume 0.07845 m^3/kg and corresponding pressure are listed as:

P, kPa	v_g, m^3/kg	
2500	0.07995	(a)
2750	0.07272	(b)

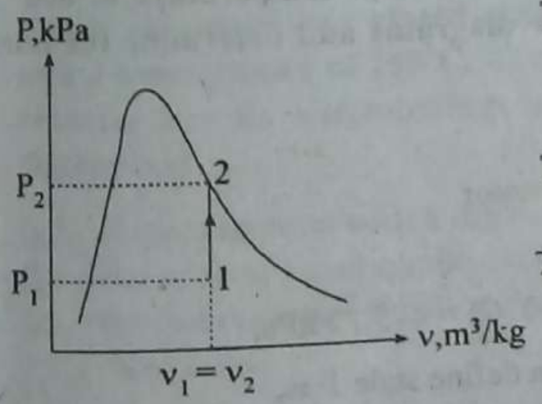
Then, applying linear interpolation for pressure

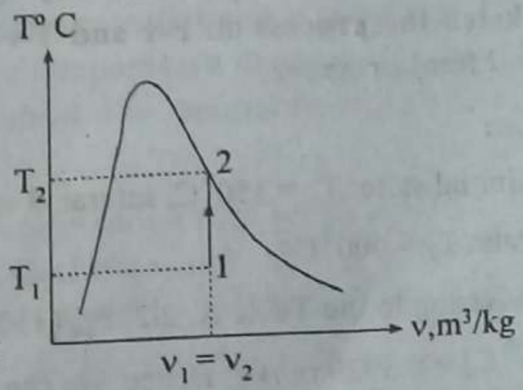
$$P_2 - P_a = \frac{P_b - P_a}{(v_g)_b - (v_g)_a}[v - (v_g)_a]$$

$$\therefore\ P_2 = P_a + \frac{P_b - P_a}{(v_g)_b - (v_g)_a}[v - (v_g)_a]$$

$$= 2500 + \frac{2750 - 2500}{0.07272 - 0.07995}(0.07845 - 07995) = 2551.87 \text{ kPa}$$

b) The final pressure, P_2 = 2551.87 kPa





14. A rigid vessel with volume of 0.4 m^3 contains 2 kg of water in the form of saturated liquid and saturated vapor mixture. Heat is supplied t the system from an external source. Determine the temperature at which the water in the vessel is completely vaporized.

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Solution:

Given, Volume of vessel (V) = 0.4 m^3

Mass of water (m) = 2 kg

Specific volume at initial state (ν) = $\frac{V}{m} = \frac{0.4}{2} = 0.2\ m^3/kg$

Since, volume is constant specific volume at state 2 is given as

$\nu_2 = 0.2\ m^3/kg$

Referring to Table A2.2 specific volume of saturated vapor which includes specific volume 0.2 m^3/kg and corresponding temperature are listed as

T, ^{0}C	ν_g, m^3/kg	
175	0.2168	(a)
180	0.1940	(b)

Then, applying linear interpolation for temperature

$$T_2 - T_a = \frac{T_b - T_a}{(\nu_g)_b - (\nu_g)_a}[\nu - (\nu_g)_a]$$

$$\therefore\ T_2 = T_a + \frac{P_b - P_a}{(\nu_g)_b - (\nu_g)_a}[\nu - (\nu_g)_a]$$

$$= 175 + \frac{180 - 175}{0.1940 - 0.2168}(0.2 - 0.2168) = 178.684^0 C$$

Therefore, the temperature at which water in the vessel is completely vaporized (T_2) = 178.684^{0}C

15. Water initially at saturated vapor state is heated in a closed rigid vessel from an initial temperature 150^0 C to a final temperature of 600^0 C. Sketch the process on P-v and T-v diagrams and determine the initial and final pressure.

Solution:

Given, Initial state: $T_1 = 150^0$ C, saturated vapor

Final state: $T_2 = 600^0$ C

Then, referring to the Table A. 2.2, P_{sat} (150^0 C) = 475.72 kPa,

ν_g (150^0 C) = 0.3929 m^3/kg. Hence, we can define state 1 as,

State 1: P_1 = 475.72 kPa, ν_1 = 0.3929 m^3/kg

Since, water initially at saturated vapor state is further heated in a closed rigid vessel, it becomes superheated steam and specific volume at state 2 is given as,

$\nu_2 = 0.3929\ m^3/kg$

Referring to the Table A2.4, specific volume of superheated steam which includes the specific volume 0.3929 m^3/kg and corresponding pressure for 600°C is listed as

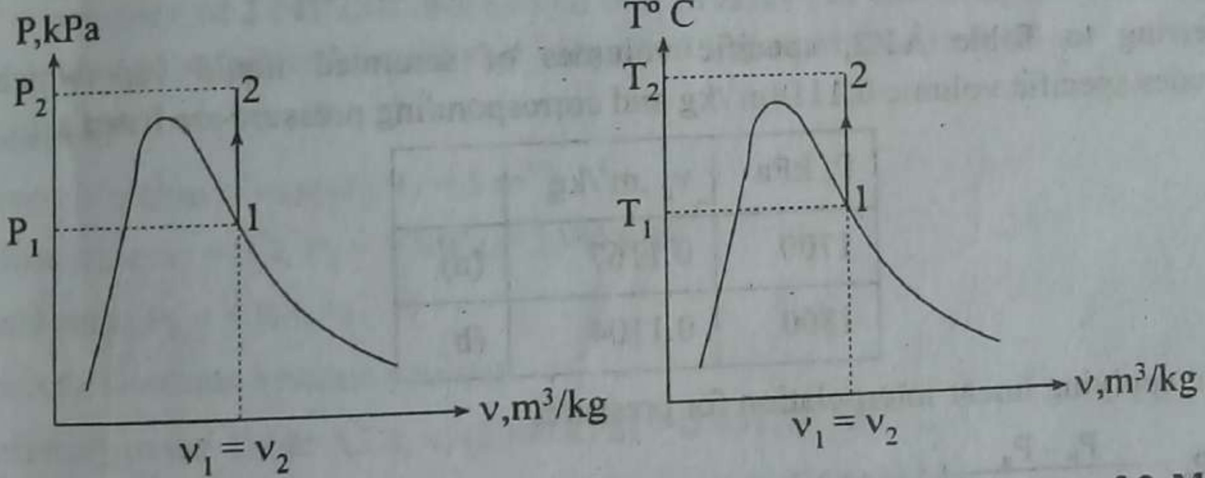
P, kPa	ν_g, m^3/kg	
1000	0.4011	(a)
1500	0.2668	(b)

Then, applying linear interpolation for pressure

$$P - P_a = \frac{P_b - P_a}{(\nu_g)_b - (\nu_g)_a}[\nu_2 - (\nu_g)_a]$$

$$\therefore\ P = P_a + \frac{P_b - P_a}{(\nu_g)_b - (\nu_g)_a}[\nu_2 - (\nu_g)_a]$$

$$= 1000 + \frac{1500 - 1000}{0.2668 - 0.4011}(0.3929 - 0.4011) = 1030.529\ kPa$$



16. Steam contained in a closed container initially at a pressure of 2 MPa and a temperature of 250°C. The temperature drops as a result of heat transfer to the surroundings until the temperature reaches 80°C. Determine:

(a) the pressure at which the condensation first occurs,

(b) the pressure and quality at final state, and

(c) the percentage of volume occupied by the saturated liquid at the final state.

Solution:

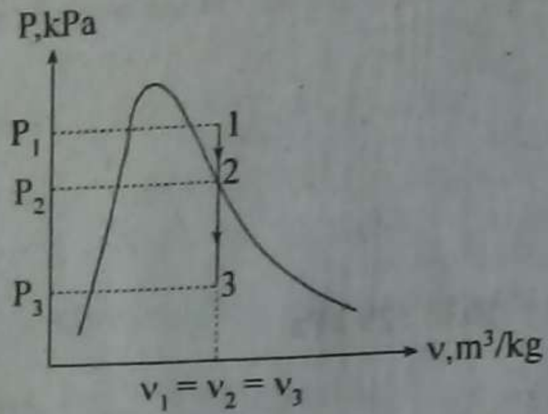
Given, Initial state: $P_1 = 2$ $MP_a = 2000$ kPa, $T_1 = 250^0$ C

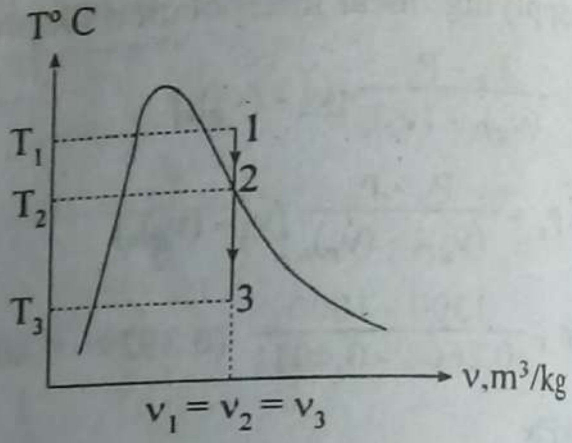
Final state: $T_{final} = 80^0$ C

Now, referring to the Table A2.1, T_{sat} (2000 kPa) = 212.42^0 C. Here, $T_1 >$ T... hence it is a superheated steam. Now, referring to Table A2.4, $v_1 = 0.1114$ m^3/kg

From P - v and T - v diagram, the condensation first occurs at P_2 kPa. Since volume is constant specific volume at state 2 is given as

$v_2 = 0.1114$ m^3/kg





Referring to Table A1.2, specific volumes of saturated liquid vaporwhich includes specific volume 0.1114m^3/kg and corresponding pressure are listed as:

P, kPa	v_g ,m^3/kg	
1700	0.1167	(a)
1800	0.1104	(b)

Then, applying linear interpolation for pressure,

$$P_2 - P_a = \frac{P_b - P_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$$

$$\therefore\ P_2 = P_a + \frac{P_b - P_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$$

$$= 1700 + \frac{1800 - 1700}{0.1104 - 0.1167}(0.1114 - 0.1167) = 1784.127 \text{ kPa}$$

a) The pressure at which the condensation first occurs, P_2= 1784.127 kPa

Specific volume of water at final state (v_3) = v_2 = 0.1114m^3/kg

Referring to table A2.2, v_l (80^0 C) = 0.001029 m^3/kg, v_{lg} (80^0 C) = 3.4078 m^3/kg, v_g (80^0 C) = 3.4088m^3/kg, P_{sat} (80^{0}C) = 47.373 kPa. Here, $v_l < v < v_g$ hence it is a two phase mixture.

b) Quality of mixture at state 3,

$$x_3 = \frac{v_3 - v_l}{v_{lg}} = \frac{0.1114 - 0.001029}{3.4078} = 0.0324$$

and, pressure at final state is given as

$P_3 = P_{sat}(80^0 C) = 47.373$ kPa

Then, mass fraction of liquid $\left(\frac{m_l}{m}\right) = 1 - x = 1 - 0.0324 = 0.9675$

And, volume of container (V) = $v_3 \times m$

$= 0.1114 \times 1 = 0.1114$ (considering m = 1 kg)

Then, volume occupied by saturated liquid $(V_l) = v_l \times m_l$

$= 0.001029 \times 0.9675 = 0.0009956$

c) Percentage of volume occupied by the saturated liquid at final state =

$$\frac{V_l}{V} = \frac{0.0009956}{0.1114} \times 100\ \% = 0.894\ \%$$

17. Water is contained in a rigid vessel of 5 m³ at a quality of 0.8 and a pressure of 2 MPa. If it is cooled to a pressure of 400 kPa, determine the mass of saturated liquid and saturated vapor at the final state.

Solution:

Given, Volume of vessel (V) = 5 m^3

Initial state: $x_1 = 0.8$, $P_1 = 2$ MPa = 2000 kPa

Final state: $P_2 = 400$ kPa

Process: Constant volume cooling

Referring to the Table A2.1, v_l (2000 kPa) = 0.001177 m^3/kg,

v_g (2000 kPa) = 0.09959 m^3/kg, v_{lg} (2000 kPa) = 0.09841 m^3/kg

Then, specific volume at state 1 is given as

$v_1 = v_l + x_1 v_{lg} = 0.00177 + 0.8 \times 0.09841 = 0.079905\ m^3/kg$

Since, volume is constant specific volume at state 2 is given as

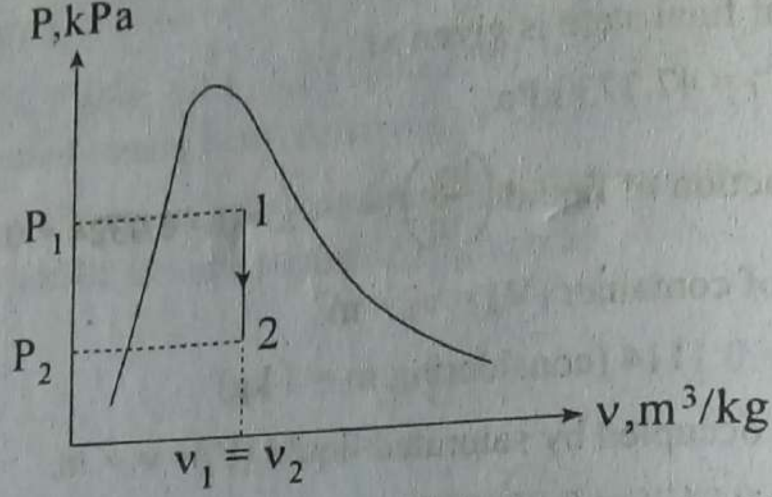
$v_2 = 0.079905\ m^3/kg$

$$\therefore \text{Mass of water (m)} = \frac{V}{v_1} = \frac{5}{0.079905} = 62.5743 \text{ kg}$$

Referring to Table A2.1, v_l(400 kPa) = 0.001084 m^3/kg

v_{lg} (400 kPa) = 0.4614 m^3/kg and v_g (400 kPa) = 0.4625 m^3/kg. Here, $v_l < v < v_g$, hence it is a two phase mixture. Quality of mixture at state 2 is given as

$$\therefore\ x = \frac{v_2 - v_l}{v_{lg}} = \frac{0.079905 - 0.001084}{0.4614} = 0.17083$$



Then, mass of saturated vapor at final state $(m_g)_2 = x.\ m = 0.17083 \times 62.5743$
$= 10.6896$ kg

And, mass of saturated liquid at final state $(m_l)_2 = m - m_g = 62.5743 - 10.6896$
$= 51.8847$ kg

18. A rigid container with a volume of 0.170 m3 is initially filled with steam at 200 kPa, 3000C. It is cooled to 900C.

(a) At what temperature does a phase change start to occur?

(b) What is the final pressure?

(c) What mass fraction of the water is liquid in the final state?

Solution: (IOE 2067 Mangsir)

Given, Volume of vessel (V) = 0.170 m^3

Initial state: $P_1 = 200$ kPa, $T_1 = 300^0$ C

Final state: $T_{final} = 90^0$ C

Process : Constant volume cooling

Referring to Table A 2.1, T_{sat} (200 kPa) = 120.24^0 C. Here, $T > T_{sat}$, hence it is superheated steam. Referring to table A2.4, v_g (300^0 C) = 1.3162 m^3/kg

Since, volume is constant specific volume at state 2 is given as

$v_2 = 1.3162$ m^3/kg

Since, constant volume cooling process

$\therefore\ v_1 = v_2 = v_3 = v_g = 1.3162$ m^3/kg

Referring to Table A2.2, specific volume of saturated vapor which includes specific volume 1.3162 m³/kg and its corresponding temperature are listed as

T, ⁰C	v_g ,(m³/kg)	
105	1.4200	(a)
110	1.2106	(b)

Then, applying linear interpolation for temperature

$$T_2 - T_a = \frac{T_b - T_a}{(v_g)_b - (v_g)_a}\,[v_2 - (v_g)_a]$$

$$\therefore\ T_2 = T_a + \frac{T_b - T_a}{(v_g)_b - (v_g)_a}\,[v_2 - (v_g)_a]$$

$$= 105 + \frac{110 - 105}{1.2106 - 1.4200}\,(1.3162 - 1.4200) = 107.48^0\,C$$

a) The temperature at which phase change start to occur (T_2) = 107.48⁰ C

Referring to Table A2.2, P_{sat} (90⁰C) = 70.117 kPa, v_l (90⁰ C) = 0.001036 m³/kg, v_{lg} (90⁰ C) = 2.3607 m³/kg

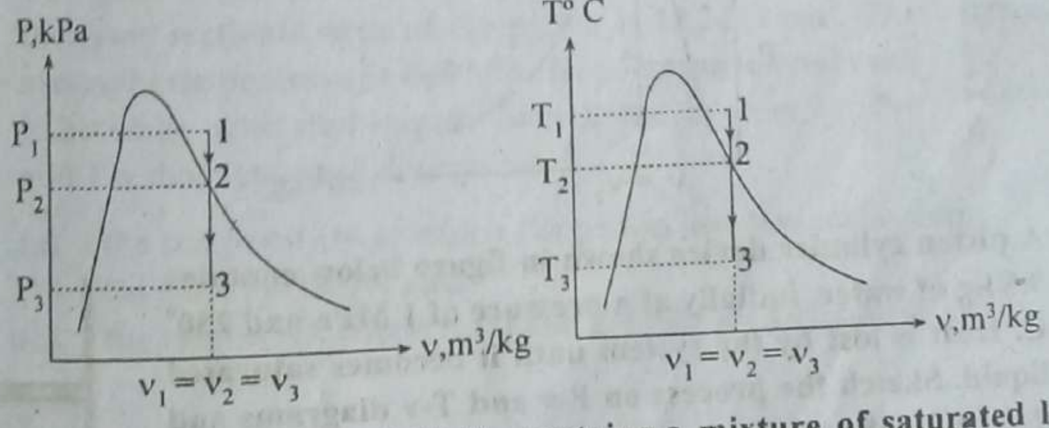
$$\therefore\ x_3 = \frac{v_3 - v_l}{v_{lg}} = \frac{1.3162 - 0.001036}{2.3607} = 0.5571$$

b) The final pressure is given as

$P_2 = P_{sat}$ (90⁰ C) = 70.117 kPa

c) Mass fraction of saturated liquid is given as

$$\frac{m_l}{m} = 1 - x = = 1 - 0.5571 = 0.4429 = 44.29\%$$



19. A 0.05 m³ rigid vessel initially contains a mixture of saturated liquid and saturated vapor at 100 kPa. The water is now heated until it reaches the critical state. Determine the mass and volume of the saturated liquid water at initial state.

Solution:

Given, Volume of rigid vessel (V) = 0.05 m^3

Initial state: P_1 = 100 kPa, Two phase mixture

Final state: critical point

Process: constant volume heating

Specific volume at final state, $v_2 = v_{cr} = 0.00311\ m^3/kg$

Since, volume is constant specific volume at state 2 is given as

$v_1 = v_{cr} = 0.00311\ m^3/kg$

Then, mass of water (m) $= \frac{V}{v_1} = \frac{0.05}{0.00311} = 16.077$ kg

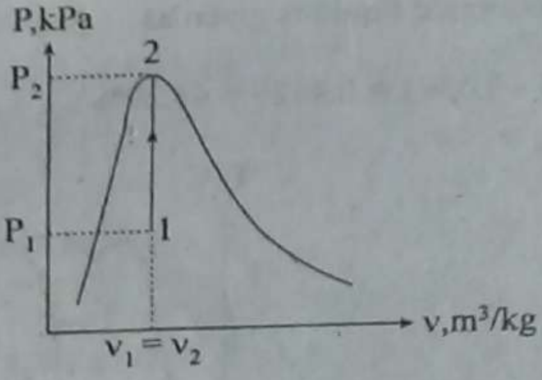
Referring to Table A 2.1, v_l (100 kPa) = 0.001043 m^3/kg, v_{lg} (100 kPa) = 1.6933 m^3/kg. Quality of mixture at state 1 is given as

$$x_1 = \frac{v_1 - v_l}{v_{lg}} = \frac{0.00311 - 0.001043}{1.6933} = 0.00122$$

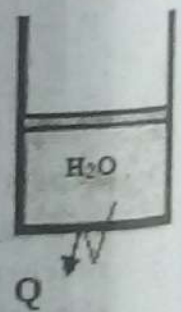
Then, mass fraction of liquid $\left(\frac{m_l}{m}\right) = 1 - x_1 = 1 - 0.00122 = 0.99878$

∴ Mass of saturated liquid water at state 1, $(m_l)_1 = x_1 \times m = 0.99878 \times 16.077 =$ 16.0574 kg

And, volume of saturated liquid water at initial state $(V_l)_1 = (m_l)_1 \times v_1 = 16.0574 \times 0.001043 = 0.01675\ m^3$



20. A piston cylinder device shown in figure below contains 0.5 kg of water initially at a pressure of 1 MPa and 250° C. Heat is lost by the system until it becomes saturated liquid. Sketch the process on P-v and T-v diagrams and determine the work transfer.



Solution:

Given, Mass of water (m) = 0.5 kg

Initial state: P_1 = 1 MPa = 1000 kPa, $T_1 = 250^0$ C

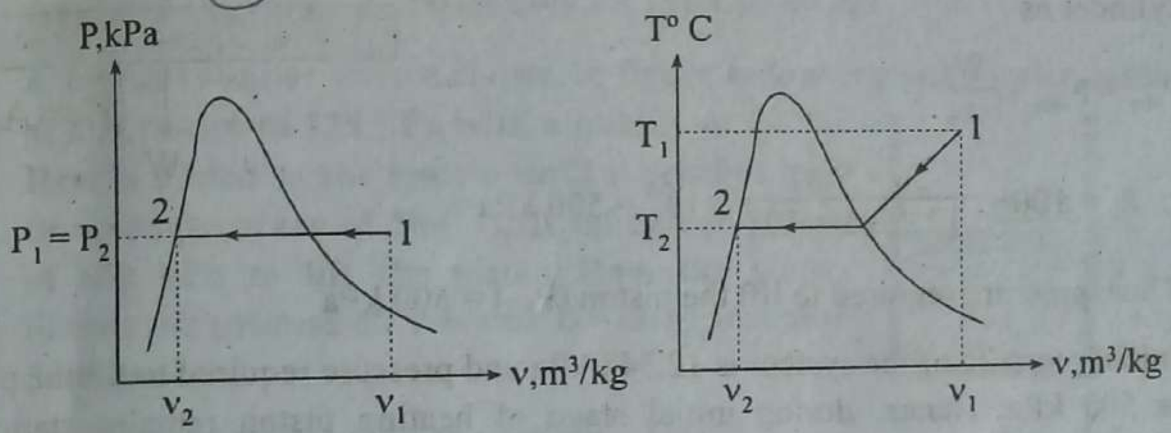
Final state: saturated liquid

Referring to Table A2.1, T_{sat} (1000 kPa) = 179.92°C, T > T_{sat}, hence it is a superheated vapor. Referring to Table A2.4, v_f (1000kPa) = 001172 m³/kg, v_g = 0.2326m³/kg. Therefore, specific volume of water at state 1 is given as

$v_1 = v_g = 0.2326 m^3/kg$

Since the heat is lost by the system until it becomes saturated liquid, the process occurs at constant pressure of 1000 kPa (Process 1-2), we can define state 2 as

State 2: v_2 = 001172 m³/kg



∴ Work transfer for the process is given as

$W = W_{12} = P_1 (V_1 - V_2) = mP_1 (v_1 - v_2)$

= 0.5 × 1000 (0.001172 - 0.2326) = - 115.714 kJ

21. A piston cylinder device shown in figure below contains 0.2 kg of a mixture of saturated liquid water and saturated water vapor at a temperature of 50°C and a volume of 0.03 m³. The mass of the piston resting on the stops is 50 kg and the cross sectional area of the piston is 12.2625 cm². The atmospheric pressure is 100 kPa. Heat is transferred until it becomes saturated vapor. Sketch the process on P-v and T-v diagrams and determine:

(a) the temperature at which the piston just leaves the stops,

(b) the final pressure, and

(c) the total work transfer. [Take g = 9.81 m/²](*IOE 2069 chaitra*)

Solution:

Given, mass of water (m) = 0.2 kg

Initial state: $T_1 = 50^0C$, $V_1 = 0.03$ m³, Two phase mixture

Mass of piston (m_p) = 50 kg

Area of piston (A_p) = 12.2625 cm^2 = 12.2625×10^{-4} m^2

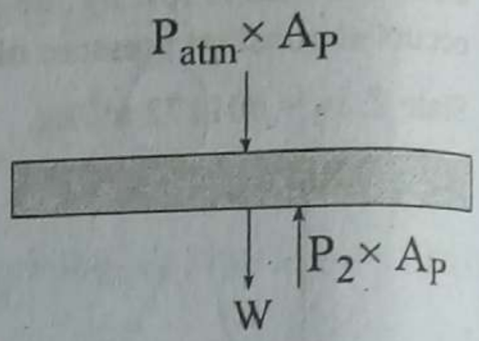
Specific volume at initial state (v_1) = $\frac{V_1}{m} = \frac{0.03}{0.2} = 0.15$ m^3/kg

Referring to Table A2.2, $P_1 = P_{sat}$ (50^0 C) = 12.344 kPa, v_l (50^0 C) = 0.00101 m^3/kg, v_g (50^0 C) = 12.037 m^3/kg, v_{lg} (50^0 C) = 12.036 m^3/kg

∴ Pressure at state 1 is given as

$P_1 = P_{sat}$ (50^0 C) = 12.344 kPa

Referring to the free body diagram of the piston we can write equation for the pressure inside the cylinder as



$$P_{abs} = P_{atm} + \frac{W}{A_P}$$

$$\therefore\ P_2 = 100 + \frac{50 \times 9.81}{12.2625 \times 10^{-4}} \times 10^{-3} = 500 \text{ kPa}$$

Thus, pressure required to lift the piston (P_{lift}) = 500 kPa

Initial pressure of the system is 12.344 kPa and pressure required to lift the piston is 500 kPa. Hence, during initial stage of heating piston remains stationary although heat is supplied to the system, so process is constant volume heating (Process 1 -2). During constant volume heating, pressure increases from 12.344 kPa to 500 kPa. Hence, we can define state 2 as,

State 2: P_2 = 500 kPa, V_2 = 0.03 m^3, v_2 = 0.15 m^3/kg

Referring to Table A2.1, T_{sat} (500kPa) = 151.87^{0}C ,v_l (500 kPa) = 0.001093 m^3/kg, v_{lg} (500 kPa) = 0.3738 m^3/kg, v_g (500 kPa) = 0.3749 m^3/kg. Here, $v_l < v < v_g$, hence it is a two phase mixture.

But the required final state is saturated vapor state, hence it should be further heated until it becomes completely saturated vapor and the process occurs at constant pressure of 500 kPa (Process 2-3). Hence, we can define state 3 as,

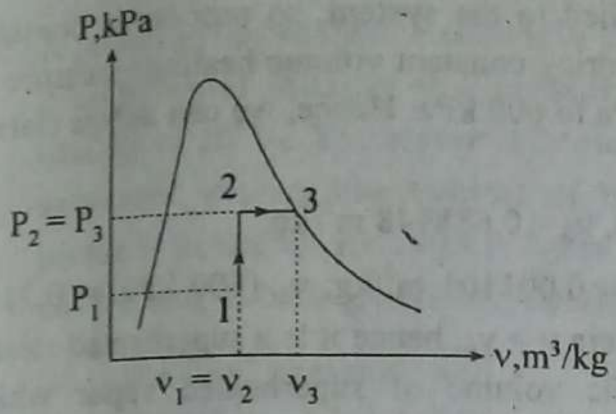
State 3: P_3 = 500 kPa, v_3 = 0.3749 m^3/kg,

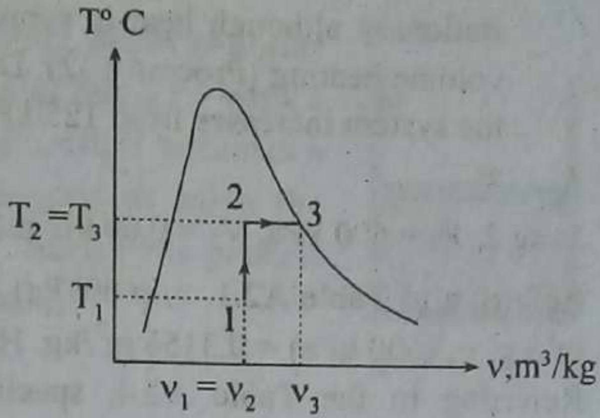
a) The temperature at which piston just leaves the stops is given as

$T_2 = T_{sat}$ (500kPa) = 151.87^{0}C

b) The final pressure is given as

P_3 = 500 kPa

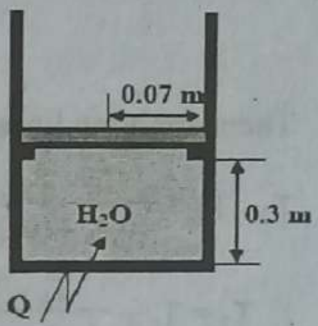




c) Total work transfer, $W = W_{12} + W_{23} = 0 + P_2 (V_3 - V_2)$

$= mP_2 (v_3 - v_2) = 0.2 \times 500 (0.3749 - 0.15) = 22.49$ kJ

22. A piston cylinder device shown in figure below contains water initially at a pressure of 125 kPa with a quality of 50 %. Heat is added to the system until it reaches to a final temperature of 800⁰ C. It takes a pressure of 600 kPa to lift the piston from the stops. Sketch the process on P-v and T-v diagrams and determine:



(a) the mass of H_2O in the system, and

(b) the total work transfer.

Solution:

Given, Initial state: $P_1 = 125$ kPa, $x_1 = 50\%$

Final state: $T_{final} = 800^0$C

Pressure required to lift the piston (P_{lift}) = 600 kPa

Radius of the piston (r_p) = 0.07 m

Volume of water at initial state $(V_1) = A_p \times 0.3 = \pi(r_p)^2 \times 0.3$

$= \pi (0.07)^2 \times 0.3 = 0.00462$ m^3

Referring to Table A2.1, T_{sat} (125 kPa) = 105.99^0C,v_l (125 kPa) = 0.001048 m^3/kg, v_{lg} (125 kPa) = 1.3742 m^3/kg, v_g (125 kPa) = 1.3752 m^3/kg. Quality of mixture at state 1 is given as

$$v_1 = v_l + x_1 v_{lg} = 0.001048 + 0.5 \times 1.3742 = 0688148 \text{ m}^3/\text{kg}$$

a) The mass of H_2O in the system $(m) = \dfrac{V_1}{v_1} = \dfrac{0.00462}{0.688148}$

$= 0.006714$ kg

Initial pressure of the system is 125 kPa and pressure required to lift the piston is 600 kPa. Hence, during initial stage of heating piston remains stationary although heat is supplied to the system, so process is consta volume heating (Process 1 -2). During constant volume heating, pressure the system increases from 125 kPa to 600 kPa. Hence, we can define state as

State 2: $P_2 = 600$ kPa, $V_2 = 0.00462$ m^3, $\nu_2 = 0.688148$ m^3/kg.

Referring to Table A2.1, ν_l (600kPa) = 0.001101 m^3/kg, ν_{lg} (600 kPa) = 0.31 m^3/kg, ν_g (600 kPa) = 0.3156 m^3/kg. Here $\nu > \nu_g$, hence it is a superheated stea Referring to the Table A2.4, specific volume of superheated vapor whi includes the specific volume 0.688148 m^3/kg and corresponding temperature a listed as

T,^{0}C	ν_g, m^3/kg	
600	0.6697	(a)
650	0.7085	(b)

Then, applying linear interpolation for temperature

$$T_2 - T_a = \frac{T_b - T_a}{(\nu_g)_b - (\nu_g)_a} [\nu_2 - (\nu_g)_a]$$

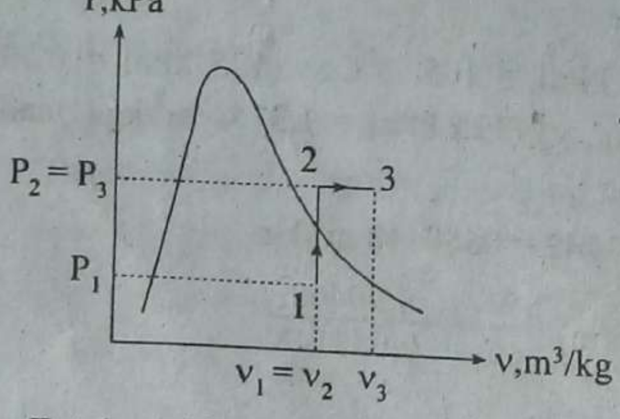
$$\therefore\ T_2 = T_a + \frac{T_b - T_a}{(\nu_g)_b - (\nu_g)_a} [\nu_2 - (\nu_g)_a]$$

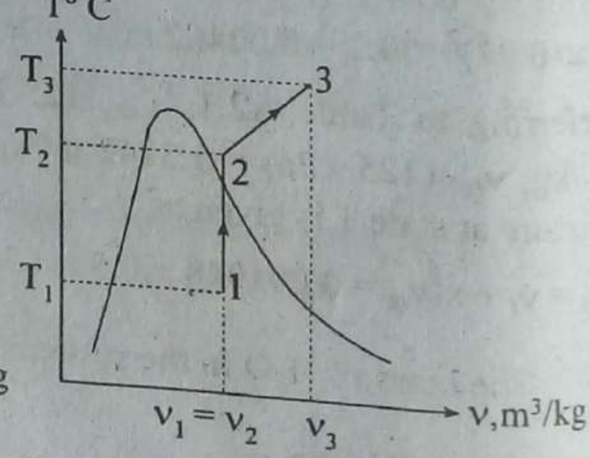
$$= 600 + \frac{650 - 600}{0.7085 - 0.6697} (0.688148 - 0.6697) = 623.773^0\ C$$

But the required final temperature is 800^0 C, hence it should be further heated increase the temperature from 623.773^0 C to 800^0 C and the process occurs constant pressure of 600 kPa (Process 2-3). Hence, we can define state 3 as,

State 3: $T_3 = 800^0$ C, $P_3 = 600$ kPa, superheated steam

Referring to the Table A2.4, $\nu_3 = \nu_g = 0.8246$ m^3/kg





b) Total work transfer, $W = W_{12} + W_{23} = 0 + P_2 (V_3 - V_2)$

$= mP_2 (\nu_3 - \nu_2) = 0.006714 \times 600 (0.8246 - 0.688148) = 0.5497$ kJ

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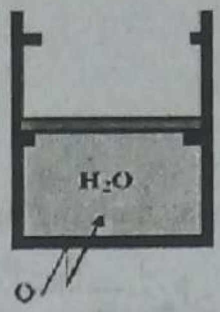
23. **A piston cylinder device shown in figure below contains 2 kg of water initially at a pressure of 500 kPa with a quality of 20 %. The water is heated until it becomes a saturated vapor. The volume of the system when the piston is at the upper stops is 0.4 m³. Sketch the process on P-v and T-v diagrams and determine:**



(a) the final pressure, and

(b) the total work transfer

Solution:

Given, mass of water (m) = 2 kg

Initial state: P_1 = 500 kPa, x_1 = 20%

Final state: V_{final} = 0.4 m³

Referring to Table A2.1, T_{sat} (500 kPa) = 151.87°C, ν_f (500 kPa) = 0.001093m³/kg, ν_{fg} (500 kPa) = 0.3738 m³/kg, ν_g (500 kPa) = 0.3749 m³/kg

∴ Specific volume of water at state 1 is given as

$\nu_1 = \nu_f + x_1\nu_{fg} = 0.001093 + 0.2 \times 0.3738 = 0.07583$ m³/kg

Thus, initial volume of water (V_1) = $\nu_1 \times m = 0.07853 \times 2 = 0.15171$ m³

∴ Specific volume at final state $(\nu_3) = \frac{V_3}{m} = \frac{0.4}{2} = 0.2$ m³/kg

Initial pressure of the system is 500 kPa with quality of 20 % and it is heated until it becomes saturated vapor. The specific volume of saturated vapor is 0.3749 m³/kg but the specific volume of system when the piston is at upper stops is 0.2 m³/kg. Therefore, it should be further heated to increase specific volume from 0.07583 m³/kg to 0.2 m³/kg and the process occurs at constant pressure of 500 kPa (Process 1-2). It is further heated at constant volume until it becomes saturated vapor (Process 2-3). Hence, we can define state 2 and state 3 as,

State 2: P_2 = 500 kPa, ν_2 = 0.2 m³/kg

State 3: ν_3 = 0.2 m³/kg

Referring to Table A2.1 specific volume of saturated vapor which includes the specific volume 0.2 m³/kg and corresponding pressure are listed as

P, kPa	v_g, m^3/kg	
950	0.2041	(a)
1000	0.1944	(b)

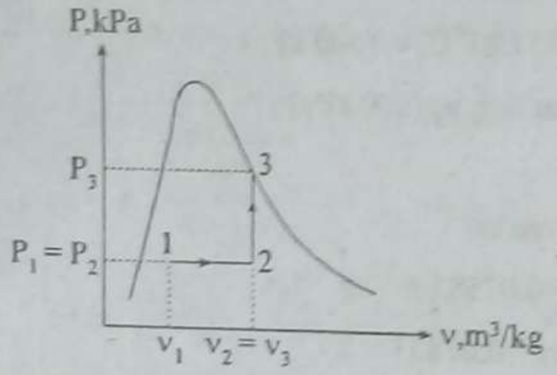
Then, applying linear interpolation for pressure,

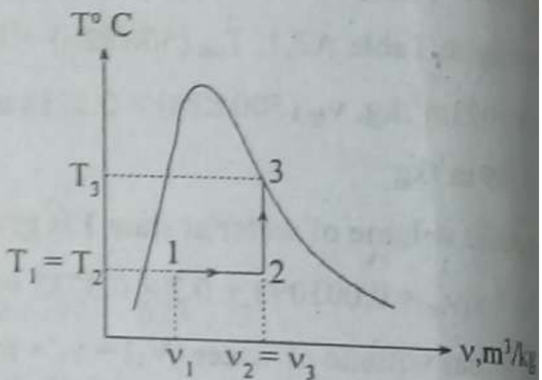
$$P_3 - P_a = \frac{P_b - P_a}{(v_g)_a - (v_g)_b}\,[v_3 - (v_g)_a]$$

$$\therefore\ P_3 = P_a + \frac{P_b - P_a}{(v_g)_a - (v_g)_b}\,[v_3 - (v_g)_a]$$

$$= 950 + \frac{1000 - 950}{0.1944 - 0.2041}\,(0.2 - 0.2041) = 971.134 \text{ kPa}$$

a) The final pressure, $P_3 = 971.134$ kPa



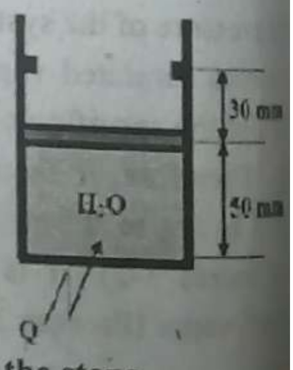


b) Total work transfer is given as

$W = W_{12} + W_{23} = 0 + P_2\,(V_2 - V_1) = mP_2\,(v_2 - v_1)$

$= 2 \times 500\,(0.02 - 0.075853) = 124.147$ kJ

24. The frictionless piston shown in figure below has a mass of 20 kg and a cross sectional area of 78.48 cm^2. Heat is added until the temperature reaches 400°C. If the quality of the H_2O at the initial state is 0.2, determine:



(a) the initial pressure,

(b) the mass of H_2O,

(c) the quality of the system when the piston hits the stops,

(d) the final pressure, and

(e) the total work transfer.

[Take p_{atm} = 100 kPa and g = 9.81 m/s^2]

Solution:

Given, mass of piston (m_p) = 20 kg

Area of piston (A_p) = 78.48 cm^2 = 78.48 × 10^{-4} m^2

Initial state: $x_1 = 0.2$

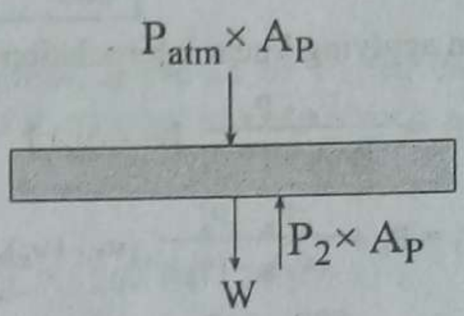
Final state: $T_{final} = 400^0$ C

Atmospheric pressure (P_{atm}) = 100 kPa

Initial volume of the system (V_1) = A_p × 0.05 = 78.48 × 10^{-4} × 0.05 = 0.0003924 m^3

Final volume of the system (V_{final}) = A_p × 0.08 = 78.48 × 10^{-4} × 0.08 = 0.00062784 m^3

Referring to the free body diagram of the piston we can write equation for the pressure inside the cylinder as



$$P_{abs} = P_{atm} + \frac{W}{A_p} = P_{atm} + \frac{m_p g}{A_p}$$

$$\therefore\ P_1 = 100 + \frac{20 \times 9.81 \times 10^{-3}}{78.48 \times 10^{-4}} = 125 \text{ kPa}$$

Now, referring to Table A 2.1, T_{sat} (125 kPa) = 105.99^0C, v_l (125kPa) = 0.001048 m^3/kg, v_g (125kPa) = 1.3752, v_{lg} (125 kPa) = 1.3742 m^3/kg,

∴ Specific volume at state 1 (v_1) = $v_l + x_1 v_{lg}$ = 0.001088 + 0.2 × 1.3742 = 0.27589 m^3/kg

$$\text{Then, mass of } H_2O\ (m) = \frac{V_1}{v_1} = \frac{0.0003924}{0.27589} = 0.001422 \text{ kg}$$

$$\therefore \text{ Specific volume at final state } (v_{final}) = \frac{V_{final}}{m} = \frac{0.00062784}{0.001422} = 0.44142 \text{ m}^3/\text{kg}$$

Initial specific volume of system is 0.27589 m^3/kg but the final specific volume is 0.44142 m^3/kg. Therefore, the system is heated until the specific volume becomes 0.44142 m^3/kg and the process occurs at constant pressure of 125 kPa (Process 1-2). Hence, we can define state 2 as,

State 2: P_2 = 125 kPa, v_2 = 0.44142 m^3/kg

Here, $v_l < v_2 < v_g$, hence, it is a two phase mixture. Quality of steam at state 2 is given as

$$x_2 = \frac{v_2 - v_l}{v_{lg}} = \frac{0.44142 - 0.001048}{1.3742} = 0.321$$

and, temperature at state 2 is given as

$T_2 = T_{sat}$ (125 kPa) = 105.99° C

But the required final temperature is 400° C, hence it should be further heated to increase the temperature from 105.99° C to 400° C and the process occurs at constant volume (Process 2-3). Hence, we can define state 3 as,

State 3: $v_3 = 0.44142$ m³/kg, $T_3 = 400°$ C

Referring to Table A2.2, $T_{cr} = 373.98°$ C. Here, $T_3 > T_{cr}$, hence it is a superheated vapor. Then, referring to Table A2.4, specific volumes of superheated vapor which includes the specific volume 0.3715m³/kg and corresponding pressure for 400°C are listed as

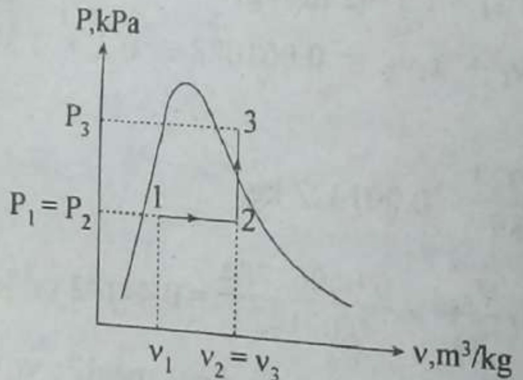
P, kPa	v_g, m³/kg	
600	0.5137	(a)
800	0.3843	(b)

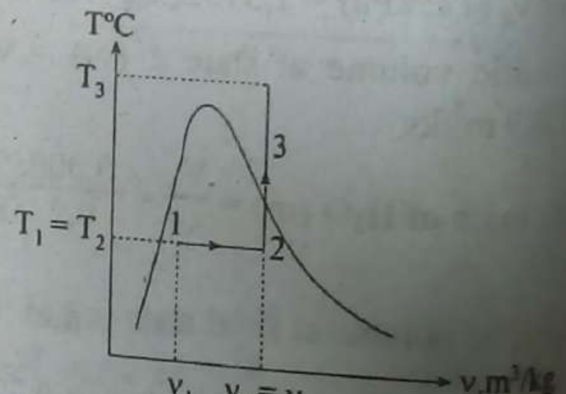
Then applying linear interpolation for pressure,

$$P_3 - P_a = \frac{P_b - P_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$\therefore\ P_3 = P_a + \frac{P_b - P_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$= 600 + \frac{800 - 600}{0.3843 - 0.5137}(0.44142 - 0.5137) = 711.72 \text{ kPa}$$





Now, total work transfer is given as

$W = W_{12} + W_{23} = P_2(V_2 - V_1) + 0 = 125\,(0.00062784 - 0.0003924) = 0.02943$ kJ

25. A piston cylinder arrangement shown in figure below contains 0.2 kg of water initially at a pressure of 150 kPa with a quality of 40 %. The system is heated to a position where the piston is locked, and then cooled till it becomes a saturated vapor at a temperature of 60°C. Sketch the process on P-v and T-v diagrams and determine the total work transfer.

Solution:

Given, mass of water (m) = 0.2 kg

Initial state: P_1 = 150 kPa, x_1 = 40% = 0.4

Final state: T_{final} = 60° C

Referring to Table A2.1, T_{sat} (150 kPa) = 111.38°C , v_f (150 kPa) = 0.001053m³/kg, v_{fg} (150 kPa) = 1.1584 m³/kg,v_g (150 kPa) = 1.1595 m3/kg,

∴ Specific volume at state 1 (v_1) = $v_f + x_1 v_{fg}$

$= 0.001053 + 0.4 \times 1.1584 = 0.464413$ m³/kg

The system is heated until the piston reaches to a position where it is locked and the process occurs at constant pressure of 150 kPa (Process 1-2). Hence, we can define state 2 as,

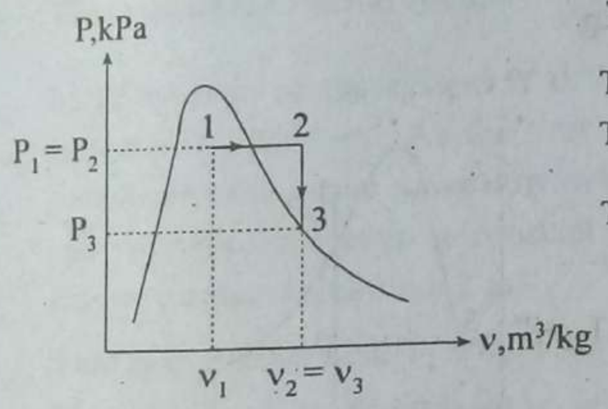
State 2: P_2 = 150 kPa, superheated steam

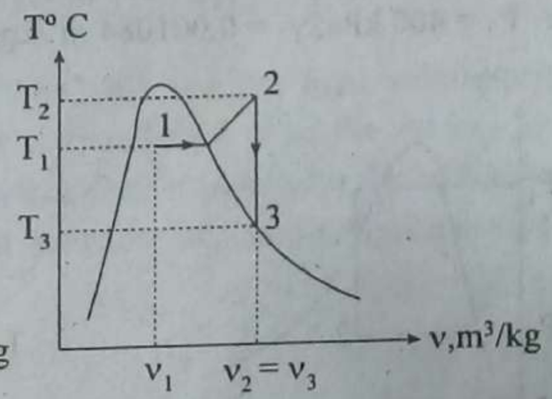
But the required final state is saturated vapor hence, it should be cooled until temperature reaches 60° C and the process occurs at constant volume (Process 2-3). Hence, we can define state 3 as,

State 3: T_3 = 60° C

Referring to Table A2.2, v_g (60°C) = 7.6743 m³/kg, P_{sat} (60°C) = 19.932 kPa

Hence, v_3 = 7.6743 m³/kg, P_3 = 19.932 kPa





Then, total work transfer is given as,

$W = W_{12} + W_{23} = 0 + P_2 (V_2 - V_1) = mP_2 (v_2 - v_1)$

$= 0.2 \times 150 (7.6743 - 0.464413) = 216.297$ kJ

26. A piston cylinder arrangement shown in figure below contains 1 kg of water initially at a pressure of 1 MPa and a temperature of 500°C. The water is cooled until it is completely converted into the saturated liquid. It requires a pressure of 400 kPa to support the piston. Sketch the process on P-v and T-v diagrams and determine the total work transfer.

Solution:

Given, mass of water (m) = 1 kg

Initial state: P_1 = 1MPa = 1000 kPa, $T_1 = 500^0$ C

Pressure required to support the piston ($P_{support}$) = 400 kPa

Referring to Table A2.1, T_{sat} (1000 kPa) = 179.92^{0}C. Here, T > T_{sat}, hence it is a superheated vapor. Referring to Table A2. 4,v_g (500^0 C) = 0.3541 m^3/kg

∴ Specific volume at state 1 (v_1) = 0.3541 m^3/kg

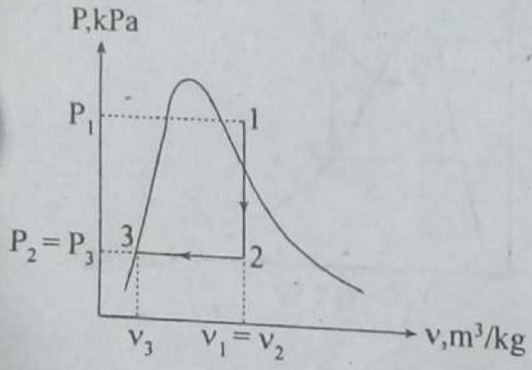
Initial pressure of the system is 1000 kPa and pressure required to support the piston is 400 kPa. Hence, during initial stage of cooling piston remain stationary although heat is removed from the system, so process is constant volume cooling (Process 1-2). During constant volume cooling, pressure of the system decreases from 1000 kPa to 400 kPa. Hence, we can define state 2 as

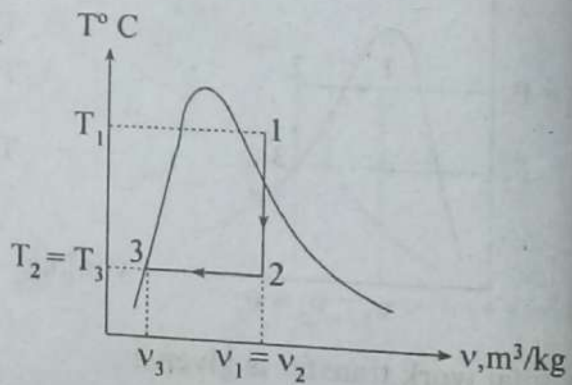
State 2: P_2 = 400 kPa, v_2 = 0.3541 m^3/kg

Referring to Table A2.1,v_l (400 kPa) = 0.001084 m^3/kg , v_{lg} (400 kPa) = 0.4612 m^3/kg, v_g (400 kPa) = 0.4625 m^3/kg. Here, $v_l < v < v_g$, hence it is a two phase mixture.

But the required final state is saturated liquid hence it should be further cooled until it becomes saturated liquid and the process occurs at constant pressure of 400 kPa (Process 2-3). Hence, we can define state 3 as

State 3: P_3 = 400 kPa, v_3 = 0.001084 m^3/kg

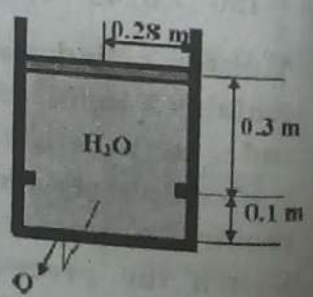




Hence, total work transfer is given as

$W = W_{12} + W_{23} = 0 + P_2 (V_3 - V_2) = mP_2 (v_3 - v_2)$

$= 1 \times 400 (0.001084 - 0.3541) = -141.21$ kJ



27. A piston cylinder arrangement shown in figure below contains water initially at a pressure of 1 MPa and a temperature of 4000C. Heat is transferred from the system to the surroundings until its pressure drops to 100 kPa. Sketch the process on P-v and T-v diagrams and determine:

(a) the mass of H2O in the system, and

(b) the total work transfer.

Solution:

Given, Initial state: $P_1 = 1\ MP_a = 1000\ kPa$, $T_1 = 500^0 C$

Final state: $P_2 = 100\ kPa$

Radius of the piston $(r_p) = 0.28\ m$

$\therefore$ Area of the piston $(A_p) = \pi(r_p)^2 = \pi\ (0.28)^2$

$= 0.2463\ m^2$

Volume of system at initial state $(V_1) = A_p \times 0.4 = 0.2463 \times 0.4$

$= 0.09852\ m^3$

Volume of system at final state $(V_{final}) = A_p \times 0.1 = \pi \times (0.28)^2 \times 0.1 = 0.02463$ m^3

Referring to Table A2.1, T_{sat} (1000 kPa) = 174 .92^{0}C.Here, $T_1 > T_{sat}$, hence it is a superheated steam. Then, referring to Table A 2.4, $v_g = 0.3066\ m^3/kg$

$\therefore$ Specific volume at state 1 $(v_1) = 0.3066\ m^3/kg$

a) The mass of H_2O in the system $= \dfrac{V_1}{v_1} = \dfrac{0.09852}{0.3066} = 0.3213$ kg

Initial volume of the system is 0.09852 m^3 and the final volume of the system is 0.02463 m^3. As the heat is transferred from the system to the surrounding the piston move downwards until it reaches to the bottom stops and the process occurs at constant pressure of 1000 kPa (Process 1-2). Hence, we can define state 2 as

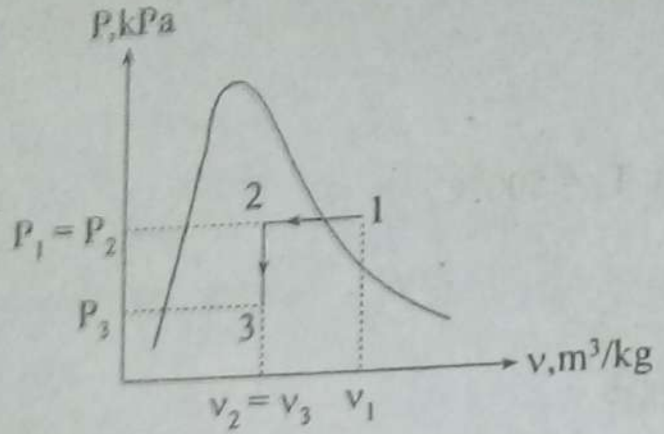
State 2: $P_2 = 1000\ kPa$, $V_2 = 0.02463\ m^3$

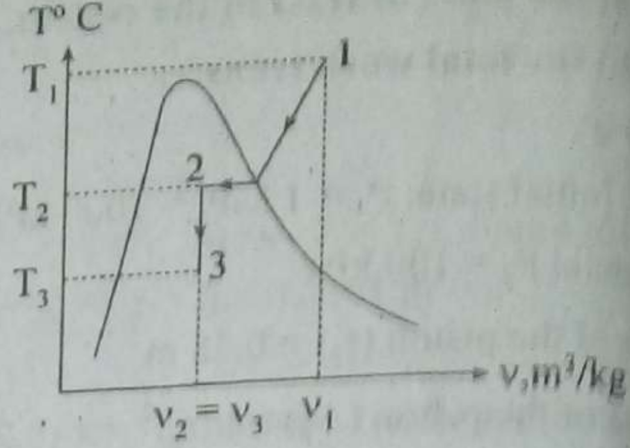
Specific volume at state 2 is given as

$$v_2 = \frac{V_2}{m} = \frac{0.02463}{0.3213} = 0.07666\ m^3/kg$$

But the required final pressure is 100 kPa and it should be further cooled to decrease pressure from 1000 kPa to 100 kPa the process occurs at constant volume (Process 2-3). Hence, we can define state 3 as

State 3: $P_3 = 100\ kPa$, $v_3 = 0.07666\ m^3/kg$



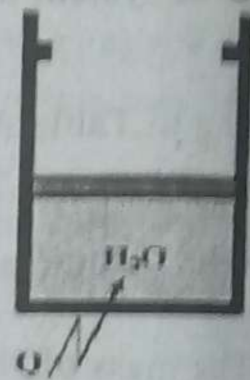


b) Total work transfer is given as

$W = W_{12} + W_{23} = P_2 (V_2 - V_1) + 0 = 1000\ (0.02463 - 0.09852)$

$= -73.89$ kJ

28. A piston cylinder arrangement shown in figure below contains 2 kg of water initially at a pressure of 200 kPa and a temperature of 50°C. Heat is added until the piston reaches the upper stops where the total volume is 1.5 m³. It takes a pressure of 600 kPa to lift the piston. Sketch the process on P-v and T-v diagrams and determine the final temperature and the work transfer.



Solution:

Given, Mass of water (m) = 2 kg

Initial state: $P_1 = 200$ kPa, $T_1 = 50^0$ C

Final state: $V_{final} = 1.5\ m^3$

Pressure required to lift the piston, $P_{lift} = 600$ kPa

Referring to Table A2.1, T_{sat} (200 kPa) = 120.24°C. Here, $T_1 < T_{sat}$, hence it is a compressed liquid. Then, referring to Table A2.2 (since 200 kPa is not available in Table A2.3), $v_f\ (50^0C) = 0.001012\ m^3/kg$

∴ Specific volume at state 1 $(v_1) = 0.001012\ m^3/kg$

And, specific volume at final state $(v_{final}) = \frac{V_{final}}{m} = \frac{1.5}{2} = 0.75\ m^3/kg$

Initial pressure of the system is 200kPa and pressure required to lift the piston is 600 kPa. Hence, during initial state of heating piston remain stationary although heat is supplied to the system so process is constant volume process (Process 1-2). During constant volume heating, pressure of the system increases from 200 kPa to 600 kPa. Hence, we can define state 2as

State 2: $P_2 = 600$ kPa, $v_2 = 0.001012$ m^3/kg

Referring to the Table A2.1, v_f (600 kPa) = 0.001101 m^3/kg. Here $v_f > v$, hence it is a compressed liquid.

But the final specific volume is 0.75 m^3/kg hence, it should be further heated to increase specific volume from 0.001012 m^3/kg to 0.75 m^3/kg and process occurs at constant pressure of 600 kPa (Process 2-3). Hence, we can define state 3 as,

State 3: $P_3 = 600$kPa, $V_3 = 1.5$ m^3, $v_3 = 0.75$ m^3/kg

Referring to Table A. 2.1, v_g (600kPa) = 0.3180 m^3/kg. Here, $v_3 > v_g$, hence it is a superheated vapor. Then, referring to Table A2.4, specific volumes of super heated steam which includes the specific volume 0.75 m^3/kg and corresponding temperature are listed as

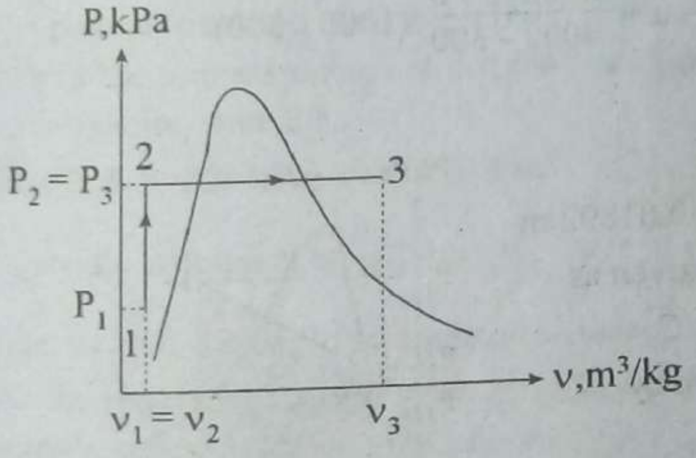
T, ^{0}C	v_g, m^3/kg	
700	0.7472	(a)
750	0.7859	(b)

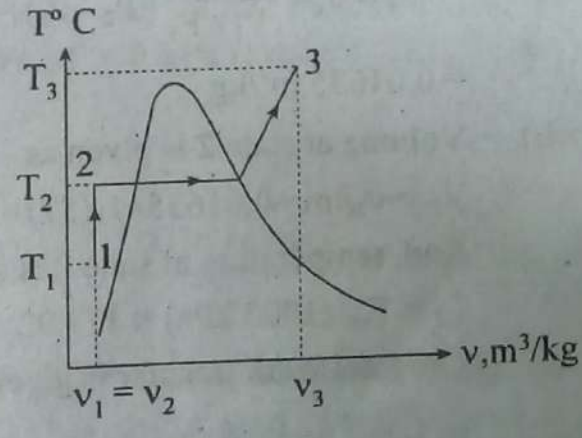
Then applying linear interpolation for temperature,

$$T_3 - T_a = \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$\therefore\ T_3 = T_a + \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$= \frac{750 - 700}{0.7859 - 0.7472}(0.75 - 0.7472) = 703.65^0\ C$$



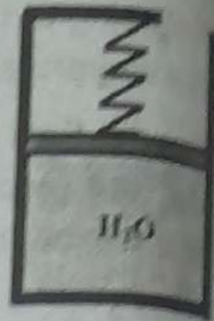


Then, total work transfer is given as

$W = W_{12} + W_{23} = P_2(V_3 - V_2) = mP_2(v_3 - v_2)$

$= 2 \times 600 \times (0.75 - 0.001012) = 898.786$ kJ

29. A piston cylinder device with a linear spring initially contains water at a pressure of 4 MPa and 500°C with the initial volume being 0.1 m³, as shown in figure below. If the piston is at the bottom, the system pressure is 300 kPa. The system now cools until the pressure reaches 1000 kPa. Sketch the process on P-v diagram and determine.



(a) the mass of H_2O

(b) the final temperature and volume, and

(c) total work transfer. *(IOE 2070 Bhadra) (IOE 21070 Ashad)*

Solution:

Given, Initial state: $P_1 = 4$ MPa $= 4000$ kPa, $V_1 = 0.1$ m³, $T_1 = 500^0$ C

Final state $P_{final} = 1000$kPa

Referring to Table A 2.1, T_{sat} (4000 kPa) = 250.39°C. Here, $T_1 > T_{sat}$, hence it is superheated steam. Then referring to Table A2.4, $v_g = 0.08642$ m³/kg

∴ Specific volume at state 1 (v_1) = 0.08642 m³/kg

a) Mass of $H_2O = \frac{V_1}{v_1} = \frac{0.1}{0.08642} = 1.1571$ kg

When the piston is at the bottom, we can define state 3 as,

State 3: $V_3 = 0$ m³, $v_3 = 0$ m³/kg, $P_3 = 300$ kPa

Using linear relationship,

$$v_2 - v_3 = \frac{v_1 - v_3}{P_1 - P_3}(P_2 - P_3)$$

$$\therefore\ v_2 = v_3 + \frac{v_1 - v_3}{P_1 - P_3}(P_2 - P_3) = 0 + \frac{0.08642 - 0}{4000 - 300}(1000 - 300)$$

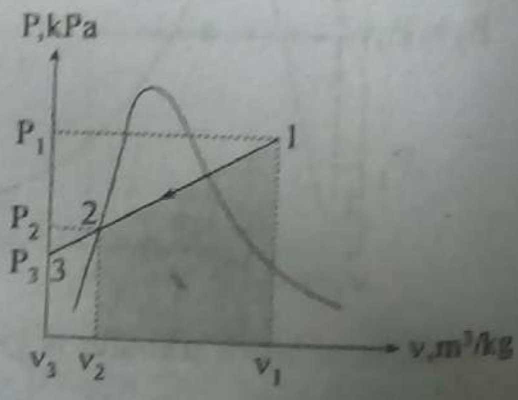
$= 0.01635$ m³/kg

b) Volume at state 2 is given as

$V_2 = v_2 \times m = 0.01635 \times 1.1571 = 0.01892$ m³

And, temperature at state 2 is given as

$T_2 = T_{sat}$ (1000 kPa) $= 179.92^0$ C



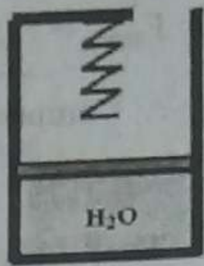
c) The total work transfer is given by

$$W = W_{12} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1)$$

$$= \frac{1}{2} \times (4000+1000)(0.01892 - 0.1)$$

$= -202.695$KJ

30. **A piston cylinder arrangement shown in figure below contains water initially at $P_1 = 100$ kPa, $x_1 = 0.8$ and $V_1 = 0.01\ m^3$. When the system is heated, it encounters a linear spring (k = 100 kN/m). At this state volume is 0.015 m^3. The heating continues till its pressure is 200 kPa. If the diameter of the piston is 0.15 m, determine**



(a) the final temperature, and

(b) the total work transfer.

Also sketch the process on P-v diagram. *(IOE 2068 Magh)*

Solution:

Given, *State 1:* $P_1 = 100$ kPa, $x_1 = 0.8$, $V_1 = 0.001\ m^3$

State 2: $V_2 = 0.015\ m^3$

State 3: $P_3 = 200$ kPa

Spring constant (k) = 100 kN/m

Diameter of the piston (d_P) = 0.15 m

$\therefore$ Area of the piston $(A_P) = \frac{\pi(d_p)^2}{4} = \frac{\pi}{4} \times 0.15^2 = 0.01767\ m^2$

Referring to Table A2.1. T_{sat} (100 kPa) = 99.632°C, v_f (100 kPa) = 0.001043 m^3/kg, v_{fg} (100 kPa) = 1.6933 m^3/kg

Specific volume at state 1 $(v_1) = v_f + x_1 v_{fg} = 0.001043 + 0.8 \times 1.6933 = 1.35568$ m^3/kg

$\therefore$ Mass of H_2O (m) $= \frac{V_1}{v_1} = \frac{0.01}{1.35568} = 0.007376$ kg

The system is heated until it encounters the spring and volume increases to 0.015 m^3 and the process occurs at constant pressure of 100 kPa (Process 1-2). Hence, we can define state 2 as,

State 2: $P_2 = 100$ kPa, $V_2 = 0.015\ m^3$

$\therefore$ Specific volume at state 2 $(v_2) = \frac{V_2}{m} = \frac{0.015}{0.007376} = 2.0336\ m^3/kg$

Here, $v_2 > v_g$, hence, it is superheated steam.

But the required final pressure is 200 kPa hence it should be further heated to increase pressure from 100 kPa to 200 kPa and the spring gets compressed. Hence, we can define state 3 as,

State 3: $P_3 = 200$ kPa

But, the pressure due to spring $(P_{spring}) = P_3 - P_2 = 200 - 100$
$= 100$ kPa

Then, spring force is given by

$F_{spring} = P_{spring} \times A_p = kx$

$\therefore$ Compression of spring $(x) = \frac{P_{Spring} \times A_p}{k} = \frac{100 \times 0.01767}{100}$

$= 0.01767$ m

Thus, volume at state 3 is given as

$V_3 = V_2 + A_p x = 0.015 + 0.01767 \times 0.01767 = 0.015312\ m^3$

And, specific volume at state 3 is given as

$v_3 = \frac{V_3}{m} = \frac{0.015312}{0.007376} = 2.076\ m^3/kg.$

Referring to Table A2.1, v_g (200 kPa) = 0.08859 m^3/kg. Here, $v_3 > v_g$, hence it is a superheated vapor. Then, referring to Table A2.4, specific volume of superheated vapor which includes the specific volume 2.075 m^3/kg and corresponding temperatures are listed as

T, ^{0}C	v_g, m^3/kg	
600	2.0130	(a)
650	2.1287	(b)

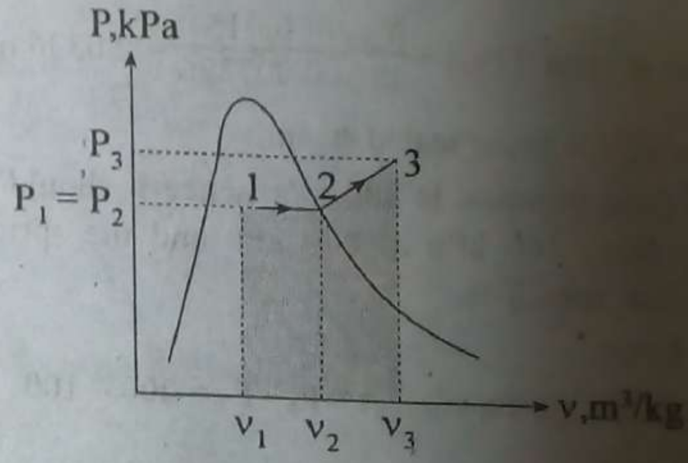
Then applying linear interpolation for temperature,

$$T_3 - T_a = \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$\therefore\ T_3 = T_a + \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$= 600 + \frac{650 - 600}{2.1287 - 2.0130}(2.076 - 2.0.130) = 627.226°C$$

a) The final temperature, $T_3 = 627.226^0$ C



b) Total work transfer is given as

$$W = W_{12} + W_{23} = P_1 (V_2 - V_1) + \frac{1}{2} (P_2 + P_3) (V_3 - V_2)$$

$$= 100 (0.015 - 0.01) + \frac{1}{2} (100 + 200) (0.015312 - 0.015) = 0.5468 \text{ kJ}$$

Chapter 4

First Law of Thermodynamics

4.1 Numerical Problems

1. **A control mass containing 0.5 kg of a gas undergoes a process in which there is a heat transfer of 120 kJ from the system to the surroundings. Work done in the system is 60 kJ. If the initial specific internal energy of the system is 4000 kJ/kg, determine its final specific internal energy.**

Solution:

Given, Mass of gas (m) = 0.5 kg

Total heat transfer (Q) = -120 kJ

Work done on the system (W) = - 60 kJ

Initial specific internal energy of the system (u_1) = 400 kJ/kg

Final specific internal energy (u_2) = ?

Total heat transfer is given as

$Q = \Delta U + W = (U_2 - U_1) + W = m(u_2 - u_1) + W$

or, $-120 = 0.5 (u_2 - 400) - 60$

or, $u_2 - 400 = -120$

$\therefore u_2 = 280$ kJ/kg

2. **A gas contained in a piston cylinder device undergoes a polytropic process for which pressure volume relationship is given by $PV^{2.5}$ = constant. The initial pressure is 400 kPa, the initial volume is 0.2 m^3 and the final volume is 0.4 m^3. The internal energy of the gas decreases by 20 kJ during the process. Determine the work transfer and heat transfer for the process.**

Solution:

Given, Initial state: $P_1 = 400$ kPa, $V_1 = 0.2$ m^3

Final state: $V_2 = 0.4$ m^3

Process relation: $PV^{2.5}$ = constant

Change in internal energy (ΔU) = -20 kJ

$\therefore$ Pressure of gas at final state $(P_2) = P_1\left(\frac{V_1}{V_2}\right)^{2.5} = 400\left(\frac{0.2}{0.4}\right)^{2.5}$

= 70.71 kPa

Work transfer during the polytropic process is given as

$$W = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{70.71 \times 0.4 - 400 \times 0.2}{1-2.5} = 34.48 \text{ kJ}$$

$\therefore$ Heat transfer during the process, $Q = \Delta U + W$

= - 20 + 34.48= 14.48 kJ

3. **A piston cylinder arrangement contains 0.01 m^3 air at 150 kPa 27°C. The air is now compressed in a process for which pressure volume is given by PV$^{1.25}$ = constant to a final pressure 600 Determine the work transfer and heat transfer for the process.**

Solution:

Given, Initial state: $V_1 = 0.01$ m^3, $P_1 = 150$ kPa, $T_1 = 27^0C = 27 + 273 = 300$

Final state: $P_2 = 600$ kPa

Process relation: $PV^{1.25}$ = constant

$\therefore$ Volume of the gas at final state, $V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.25}} \times V_1$

$$= \left(\frac{150}{600}\right)^{\frac{1}{1.25}} \times 0.01 = 0.003299 \text{ m}^3$$

$\therefore$ Work transfer during the process is given as

$$W = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{600 \times 0.003299 - 150 \times 0.01}{1-1.25} = -1.9176\text{kJ}$$

Also, mass of air$(m) = \frac{P_1V_1}{RT_1} = \frac{150 \times 10^3 \times 0.01}{287 \times 300} = 0.0174$ kg

Temperature at final state, $T_2 = \frac{P_2V_2}{mR} = \frac{600 \times 10^3 \times 0.003299}{0.0174 \times 287} = 396.37$ K

$\therefore$ Change in internal energy for the process is given by

$\Delta U = mc_v(T_2 - T_1) = 0.0174 \times 718\,(396.37 - 300) = 1203.97\text{J} = 1.20397$ kJ

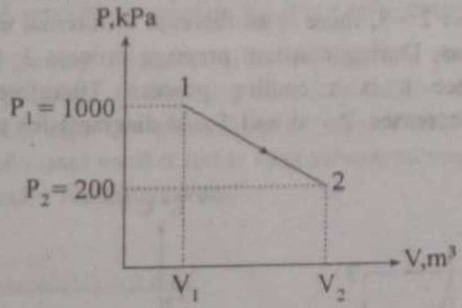
Total heat transfer for the process is given as

$Q = \Delta U + W = 1.20397 - 1.9176 = -0.71363$ kJ

4. **A closed system undergoes a process A from state 1 to state 2 as shown in figure below, which requires a heat input of Q_A = 65 kJ. The system returns adiabatically from state 2 to state 1 through process B. Determine the work transfer for process B.**

Solution:

Given, $Q_A = 65$ kJ



Initial State: $P_1 = 1000$ kPa, $V_1 = 0.05$ m^3

Final state: $P_2 = 200$kPa, $V_2 = 0.2$ m^3

Then, work transfer during the process is given as

$$W_{12} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2} \times (1000 + 200) \times (0.2 - 0.05) = 90\text{kJ}$$

Then, for process A, change in internal energy is given by

$\Delta U = Q_A - W_{12} = 65 - 90 = -25$ kJ

Now, process B (Process 2 -1) is adiabatic i.e. $Q_B = 0$

$\therefore Q_B = \Delta U + W_{21}$

or, $0 = \Delta U + W_{21}$

$\therefore W_{21} = -\Delta U = -(-25) = 25$ kJ

5. **A gas undergoes a thermodynamic cycle consisting of the following three processes:**

Process 1-2: **expansion with PV = constant, P_1 = 800 kPa, $U_2 = U_2$**

Process 2-3: **constant volume with $V_2 = V_3$ = 2 m^3, $U_3 - U_2$ = 300 kJ**

Process 3-1: **constant pressure, W_{32} = - 1200 kJ**

(a) **Sketch the process on P-V and T-V diagrams.**

(b) **Calculate the net work for the cycle.**

(c) **Calculate the net heat for the cycle.**

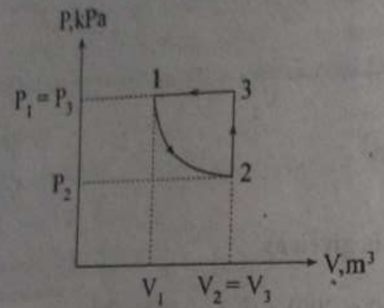
(d) **Calculate the heat transfer for process 1-2.**

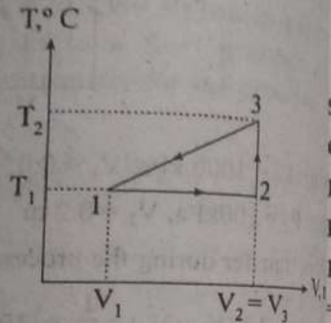
(e) **Calculate the heat transfer for process 3-1.**

(f) **Is this power cycle or a refrigeration cycle?** ***(IOE 2068 Ma...***

Solution:

a) During expansion (PV = constant) process 1-2, temperature ... remains constant, volume increases and pressure decreases. During ... volume process 2 - 3, there is an increase in internal energy, hen... heating process, During constant pressure process 3 -1, work ... negative, hence it is a cooling process. Therefore, its vol... temperature decreases. P - V and T - V diagrams for the cycle ... below.





a) Net work for the cycle is given by

$\Sigma W = W_{12} + W_{23} + W_{31}$

Work transfer during the process 3 -1 is given as

$W_{31} = P_1 (V_1 - V_3)$

$\therefore V_1 = \frac{W_{31}}{P_1} + V_3 = \frac{-1200}{800} + 2 = 0.5 \text{ m}^3$

Also, work transfer during process 1-2 is given as

$W_{12} = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) = 800 \times 0.5 \ln\left(\frac{2}{0.5}\right) = 554.518 \text{ kJ}$

$\therefore \Sigma W = 554.518 + 0 + (-1200) = -645.4823 \text{ kJ}$

b) For a cycle, net work transfer is equal to net heat transfer, therefore,

$\Sigma Q = \Sigma W = -645.4823 \text{ kJ}$

c) Heat transfer for the process 1 - 2 is given as

$Q_{12} = (\Delta U)_{12} + W_{12} = 0 + 554.518 = 554.518 \text{ kJ}$

d) For a complete cycle, (ΔU) cycle $= 0$

or, $(\Delta U)_{12} + (\Delta U)_{23} + (\Delta U)_{31} = 0$

or, $0 + 300 + (\Delta U)_{31} = 0$

$\therefore \Delta U_{31} = -300 \text{ kJ}$

Then, heat transfer for the process 3 - 1 is given as

$Q_{31} = (\Delta U)_{31} + W_{31} = -300 - 1200 = -1500 \text{ kJ}$

Since, net work transfer is negative. so, it is a refrigeration cycle.

6. **A rigid vessel having a volume of 0.4 m³ initially contains a two-phase mixture at a pressure of 100 kPa with 2 % of its volume occupied by saturated liquid and the remaining by the saturated vapor. Heat is supplied to the vessel until it holds only saturated vapor. Determine the total heat transfer for the process.**

Solution:

Given, Volume of vessel (V) = 0.4 m³

Initial state: $P_1 = 100$ kPa

Final state: Saturated vapor

Initial volume occupied by saturated liquid, $(V_l)_1 = 2\%$ of V $= 0.02 \times 0.4 = 0.008 \text{ m}^3$

Initial volume occupied by saturated vapor, $(V_g)_1 = V - (V_l)_1 = 0.4 - 0.008 = 0.392 \text{ m}^3$

Referring to Table A . 2.1 v_l (100kPa) = 0.001043 m³/kg, v_{lg} (100 kPa) = 1.6933 m³/kg, v_g (100kPa) = 1.6943 m³/kg

$\therefore$ Mass of saturated liquid $(m_l) = \frac{(V_l)_1}{v_l} = \frac{0.008}{0.001043} = 7.6702 \text{ kg}$

Mass of saturated vapor $(m_g) = \frac{(V_g)_1}{v_g} = \frac{0.392}{1.6943} = 0.2314 \text{ kg}$

$\therefore$ Total mass of H_2O (m) $= m_l + m_g = 7.6702 + 0.2314 = 7.9016 \text{ kg}$

Quality of the steam $(x) = \frac{m_g}{m_l + m_g} = \frac{0.2314}{7.9016} = 0.0293$

Referring to Table A 2. 1, u_l (100kPa) = 417.41 kJ/kg, u_{lg} (100 kPa) = 2088.3 kJ/kg

$\therefore$ Specific internal energy at state 1 is given as

$u_1 = u_l + x\, u_{lg} = 417.41 + 0.0293 \times 2088.3 = 478.5972 \text{ kJ/kg}$

Heat is supplied to the vessel until it holds only saturated vapor and the vessel is rigid hence it is constant volume heating process.

$\therefore v_2 = v_1 = v_f + x\, v_{fg} = 0.001043 + 0.0293 \times 1.6933 = 0.05066 m^3/kg$

Referring to Table A2.1, specific volume of saturated vapor which includes the specific volume 0.05066 m^3/kg and corresponding specific internal energy are listed as:

v_g, m^3/kg	u_g, kJ/kg	
0.05318	2602.3	(a)
0.04977	2601.5	(b)

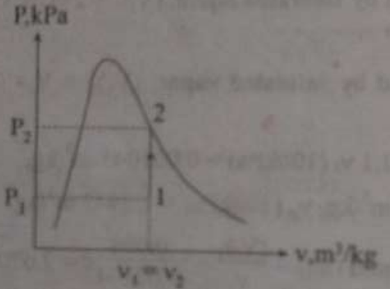
Applying linear interpolation for specific internal energy,

$$u_2 - (u_g)_a = \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$$

$$\therefore u_2 = (u_g)_a + \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$$

$$= 2602.3 + \frac{2601.5 - 2602.3}{0.04977 - 0.05318}[0.05066 - 0.05318]$$

$= 2601.7088$ kJ/kg



Change in total internal energy is given by

$\Delta U = m(u_2 - u_1) = 7.9016\,(2601.7088 - 478.5912)$

$= 16775.98$ kJ

Work transfer during the process, $W = W_{12} = 0$

Total heat transfer, $Q = \Delta U + W = 16775.98 + 0 = 16775.98$ kJ

7. **A rigid vessel with a volume of 0.1 m^3 contains water initially at 500 kPa with a quality of 60 %. A heater is turned on heating the water at a rate of 2 kW. Determine the time required to vaporize all the liquid.**

Solution:

Given, Volume of vessel (V) = 0.1 m^3

Initial state: $P_1 = 500$ kPa, $x_1 = 60\% = 0.6$

Power supply ($\dot{Q}$) = 2 kW

Final state: saturated vapor

Process: constant volume heating

Referring to Table A2.1, v_f(500 kPa) = 0.001093 m^3/kg, v_{fg} (500 kPa) = 0.3738 m^3/kg

$\therefore$ Specific volume at state 1,

$v_1 = v_f + x_1 v_{fg} = 0.001093 + 0.6 \times 0.3738 = 0.225373 m^3/kg$

$$\text{Mass of } H_2O\ (m) = \frac{V}{v_1} = \frac{0.1}{0.225373} = 0.4437 \text{ kg}$$

Referring to Table A2.1, u_f (500 kPa) = 639.84 kJ/kg

u_{fg} (500 kPa) = 1921.4 kJ/kg

$\therefore$ specific volume at state 1

$u_1 = u_f + x_1 u_{fg} = 639.84 + 0.6 \times 1921.4 = 1792.68$ kJ/kg

Since the process is constant volume heating process

$\therefore v_2 = v_1 = 0.225373 m^3/kg$

Referring to Table A2.1, specific volume of saturated vapor which includes the specific volume 0.225373 m^3/kg and corresponding specific internal energy are listed as

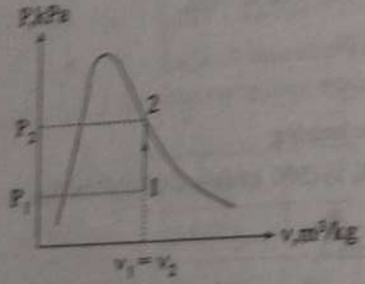
v_g, m^3/kg	u_g, kJ/kg	
0.2269	2578.5	(a)
0.2149	2580.2	(b)

Applying linear interpolation for specific internal energy,

$$u_2 - (u_g)_a = \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$$

$$\therefore u_2 = (u_g)_a + \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$$

$$= 2578.5 + \frac{2580.2 - 2578.5}{0.2149 - 0.2269}(0.225373 - 0.2269) = 2579.098 \text{ kJ/kg}$$



Change in total internal energy is given as

$\Delta U = m(u_2 - u_1) = 0.4437\,(2579.098 - 1792.68) = 348.9337$ kJ

Work transfer during the process, $W = W_{12} = 0$

∴ Total heat transfer is given by

$Q = \Delta U + W = \Delta U + W = 348.9337 + 0 = 348.9337$ kJ

Hence, time required to vaporize all the liquid is given by

$$t = \frac{Q}{\dot{Q}} = \frac{348.9337}{2} = 174.467 \text{ sec}$$

8. A closed rigid tank contains 2 kg of saturated water vapor initially at ~~500 kPa~~ 160° C. heat transfer occurs from the system and the pressure drops to 150 kPa. Determine the amount of heat lost by the system.

Solution:

Given, Mass of saturated water vapor $(m_g)_1 = 2$ kg

Initial state: $T_1 = 160^0$ C

Final pressure: $P_2 = 150$ kPa

Referring to Table A 2.2, $v_g(160^0C) = 0.3071$ m³/kg, $u_g(160^0C) = 2568.3$ kJ/kg

Initially the vessel contains only saturated vapor, therefore,

Mass of H_2O (m) = $(m_g)_1 = 2$ kg

And, specific volume at state 1, $v_1 = v_g(160^0C) = 0.3071$ m³/kg

Specific internal energy at state 1, $u_1 = u_g(160^0C) = 2568.3$ kJ/kg

Since the process is constant volume cooling process,

∴ $v_1 = v_2 = 0.3071$ m³/kg

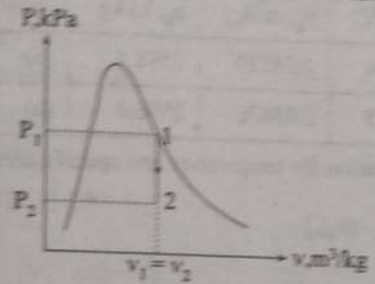
Referring to Table A2.1, v_f (150 kPa) = 0.001053 m³/kg, v_{fg} (150 kPa) = 1.1584 m³/kg, v_g (150 kPa) = 1.1595 m³/kg, u_f (150 kPa) = 467.02 kJ/kg, u_{fg} (150 kPa) = 2052.4 kJ/kg. Here, $v_f < v < v_g$, hence it is a two phase mixture.

∴ Quality of steam at state 2 is given as

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.3071 - 0.001053}{1.1584} = 0.2642$$

Specific internal energy at state 2 is given by

$u_2 = u_f + x_2 u_{fg} = 467.02 + 0.2642 \times 2052.4 = 1009.2641$ kJ/kg



Change in total internal energy is given by

$\Delta U = m(u_2 - u_1) = 2\,(1009.2641 - 2568.3)$

$= -3118.072$ kJ

Total work transfer for the process, $W = W_{12} = 0$

∴ The amount of heat last by the system is given by

$Q = \Delta U + W = -3118.072 + 0 = -3118.072$ kJ

9. A rigid vessel initially contains 4 kg of a saturated liquid water vapor. It is cooled to a final state where the temperature is 150° C and quality is 0.1. Determine the initial temperature and the heat transfer from the system.

Solution:

Given, Mass of saturated water vapor $(m_g)_1 = 4$ kg

Mass of H_2O (m) = $(m_g)_1 = 4$ kg

Final state: $T_2 = 150^0$ C, $x_2 = 0.1$

Process: constant volume cooling process.

Referring to Table A2.2, $v_f(150^0C) = 0.001090$ m³/kg

$v_{fg}(150^0C) = 0.3918$ m³/kg, $u_f(150^0C) = 631.80$ kJ/kg, $u_{fg}(150^0C) = 1927.7$ kJ/kg

∴ specific volume at state 2 is given as

$v_2 = v_f + x_2 v_{fg} = 0.001090 + 0.1 \times 0.3918 = 0.04027$ m³/kg

Since the process is constant volume cooling process,

$v_1 = v_2 = 0.04027$ m³/kg

Specific internal energy at state 2 is given by

$u_2 = u_f + x_2\, u_{fg} = 631.80 + 0.1 \times 1927.7 = 824.57$ kJ/kg

Referring to Table A2.2, the specific volume of saturated vapor which include the specific volume 0.04027 m^3/kg and corresponding temperatures and specific internal energy are listed as:

T,°C	v_g, m^3/kg	u_g, kJ/kg	
260	0.04219	2598.4	(a)
265	0.03876	2596.0	(b)

Applying linear interpolation for temperature and specific internal energy,

$$T_1 - T_a = \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$$

$$\therefore T_1 = T_a + \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$$

$$= 260 + \frac{265 - 260}{0.03876 - 0.04219}(0.04027 - 0.04219) = 262.799^\circ C$$

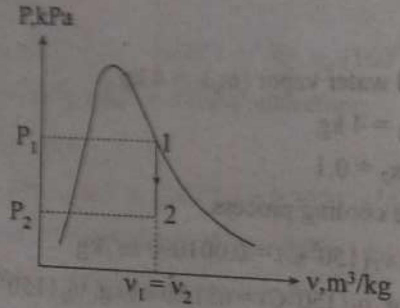
Specific internal energy at state 1 is given by

$$u_1 = (u_g)_a + \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_1 - (v_g)_a]$$

$$= 2598.4 + \frac{2596.0 - 2598.4}{0.03876 - 0.04219}(0.04027 - 0.04219) = 2597.057 \text{ kJ/kg}$$

Change in total internal energy is given by

$\Delta U = m(u_2 - u_1) = 4(824.57 - 2597.057) = -7089.948$ kJ



Work transfer during the process, $W = W_{12} = 0$

$\therefore$ The heat transfer from the system is given by

$Q = \Delta U + W = -7089.948 + 0 = -7089.948$ kJ

10. A rigid vessel of volume 0.2 L contains water at its critical state. It is cooled down to room temperature of 25° C. Determine the heat loss from the water.

Solution:

Given, Volume of vessel (V) = 0.2L = 0.2×10^{-3} m^3

Initial state: Critical state

Final state: $T_2 = 25^\circ C$

Process: constant volume cooling

specific volume at state 1,

$v_1 = v_{cr} = 0.00311$ m^3/kg

Specific internal energy at state 1,

$u_1 = u_{cr} = 2017$ kJ/kg

$\therefore$ Mass of H_2O (m) $= \frac{V}{v_1} = \frac{0.2 \times 10^{-3}}{0.00311} = 0.06431$ kg

Since the process is constant volume cooling process,

Specific volume at state 2 is given as

$v_2 = v_1 = 0.003$ m^3/kg

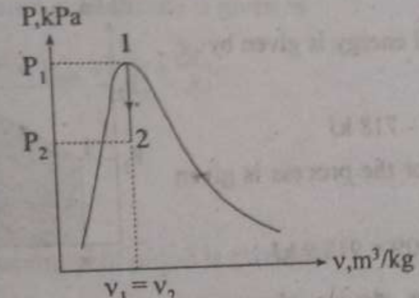
Referring to Table A2.1, v_f (25° C) = 0.001003 m^3/kg, v_{fg} (25° C) = 43.356 m^3/kg, v_g (25° C) = 43.356 m^3/kg, u_f (25° C) = 104.75 kJ/kg, u_{fg} (25° C) = 2304.1 kJ/kg.

Here, $v_f < v < v_g$, hence it is a two phase mixture. Quality at state 2 is given as

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.00311 - 0.001003}{43.356} = 0.000049$$

Specific internal energy at state 2 is

$u_2 = u_f + x_2\, u_{fg} = 104.75 + 0.000049 \times 2304.1 = 104.8629$ kJ/kg



Change in total internal energy is given by

$\Delta U = m(u_2 - u_1) = 0.06431(104.8629 - 2017) = -122.969$ kJ

Work transfer during the process, $W = W_{12} = 0$

∴ Total heat loss from the water is given by

$Q = \Delta U + W = -122.969 + 0 = -122.969$ kJ

11. A piston cylinder device shown in figure below restrained by a linear spring contains 2 kg of air initially at 150 kPa and 27° C. It is now heated until its volume doubles at which time temperature reaches 527° C. Sketch the process on P-v and determine the total work and heat transfer in the process. [Take R = 287 J/kgK and c_v = 718 J/kgK.]

Solution:

Given, Mass of air (m) = 2 kg

Initial state: $P_1 = 150$ kPa, $T_1 = 27^0 C = 27 + 273 = 300$ K

Final state: $V_{final} = 2V_1$, $T_{final} = 527^0 C = 527 + 273 = 800$K

Volume of air at state 1 is given by

$$V_1 = \frac{mRT_1}{P_1} = \frac{2 \times 0.287 \times 300}{150} = 1.148 \text{ m}^3$$

Volume of air at final state is given as

$V_2 = V_{final} = 2V_1 = 2 \times 1.148 = 2.296$ m³

Pressure at state 2 is given as

$$P_2 = \frac{mRT_2}{V_2} = \frac{2 \times 0.287 \times 800}{2.296} = 200 \text{ kPa}$$

Total work for the process is given by

$$W = W_{12} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2}(150 + 200)(2.296 - 1.148)$$

$= 200.9$ kJ

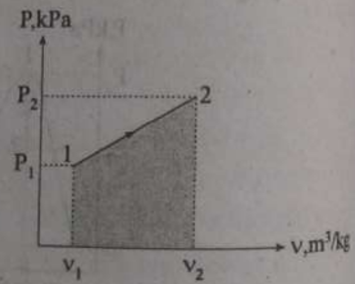
Total change in internal energy is given by

$\Delta U = mc_v (T_2 - T_1)$

$= 2 \times 0.718 (800 - 300) - 718$ kJ

∴ Total heat transfer for the process is given as

$Q = \Delta U + W = 718 + 2009 = 918.9$ kJ



12. A piston cylinder device shown in figure below loaded with a linear spring (k = 20 kN/m) contains 0.5 kg of H_2O initially at a pressure of 200 kPa and a volume of 0.4 m³. Heat is transferred to the H_2O until a final pressure of 400 kPa is reached. If the cross sectional area of the piston is 0.05 m², determine the final temperature and the heat transfer for the process.

Solution:

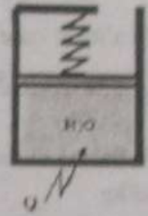
Given, Mass of H_2O (m) = 0.5 kg

Initial state: $P_1 = 200$ kPa, $V_1 = 0.4$ m³

Final state: $P_2 = 400$ kPa

Cross-sectional area of the piston (A_p) = 0.05 m²

Spring constant (k) = 20 kN/m



Specific volume at state 1 is given as

$$v_1 = \frac{V_1}{m} = \frac{0.4}{0.5} = 0.8 \text{ m}^3/\text{kg}$$

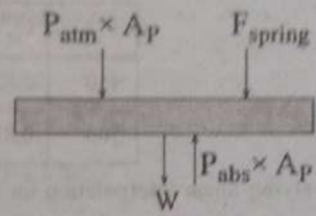
Referring to Table A2.1, v_f (200 kPa) = 0.00106 m³/kg, v_{fg} (200kPa) = 0.8848m³/kg, v_g (200kPa) = 0.8859 m³/kg. Here, $v_f < v < v_g$, hence it is a two phase mixture.

Temperature at state 1 is given as

$T_1 = T_{sat}(200^0 C) = 120.24^0 C$

Referring to the free body diagram of the piston, we can write equation for the pressure inside the cylinder as

$$P = P_{abs} = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p}$$



Initially, $F_{spring} = 0$

$$\therefore P_1 = P_{atm} + \frac{W}{A_p} = 200 \text{ .....(i)}$$

Let x_2 be the amount of spring being compressed then pressure at final state is given as

$$P_2 = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p} = P_{atm} + \frac{W}{A_p} + \frac{kx}{A_p}$$

$$\text{or, } 400 = 200 + \frac{20x_2}{0.05}$$

$\therefore x_2 = 0.5$ m

At initial stage, displacement of piston is given as

$$x_1 = \frac{V_1}{A_p} = \frac{0.4}{0.05} = 8 \text{ m}$$

Volume at final state is given by

$V_2 = A_p(x_1 + x_2) = 0.05 (8 + 0.5) = 0.425$ m³

Specific volume at state 2 is given by

$$v_2 = \frac{V_2}{m} = \frac{0.425}{0.5} = 0.85\ m^3/kg$$

Quality at state 1is given by

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.8 - 0.00106}{0.8848} = 0.90296$$

Referring to the Table A2.1, u_f (200kPa) = 5049.59 kJ/kg, u_{fg} (200 kPa) = 2024.8 kJ/kg

Specific internal energy at state 1 is given by

$$u_1 = u_f + x_1 u_{fg} = 504.59 + 0.90296 \times 2024.8 = 2332.9034\ kJ/kg$$

Referring to the Table A2.1, v_f (400 kPa) = 0.001084 m^3/kg

v_g (400kPa) = 0.4625 m^3/kg. Here, $v > v_g$,hence it is a superheated vapor. Now referring to the Table A 2.4, specific volume of the superheated vapor which includes the specific volume 0.85m^3/kg and corresponding temperature specific internal energy are listed as:

T,°C	v_g, m^3/kg	u_g, kJ/kg	
450	0.8311	3046.0	(a)
500	0.8894	3129.3	(b)

Applying linear interpolation for temperature and internal energy,

$$T_2 - T_a = \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$$

$$\therefore T_2 = T_a + \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$$

$$= 450 + \frac{500 - 450}{0.8894 - 0.8311}(0.85 - 0.8311) = 466.21^\circ C$$

Similarly, $u_2 = (u_g)_a + \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_2 - (v_g)_a]$

$$= 3046 + \frac{3129 - 3046.0}{0.8894 - 0.8311}(0.85 - 0.8311) = 3072.91\ kJ/kg$$

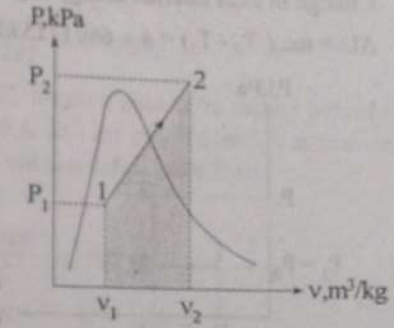
Change in total internal energy is given as

$$\Delta U = m(u_2 - u_1) = 0.5(3072.91 - 2332.9034) = 370\ kJ/kg$$

Work transfer during the process is given as

$$W = W_{12} = \frac{1}{2}\ m(P_1 + P_2)(v_2 - v_1)$$

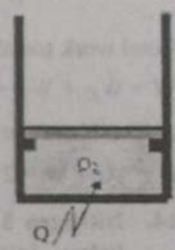
$$= \frac{1}{2} \times 0.5(200 + 400)(0.85 - 0.8) = 7.5\ kJ$$



$\therefore$ Total heat transfer for the process is given by

$$Q = \Delta U + W = 3707 + 7 = 377\ kJ/kg$$

13. Oxygen (4 kg) is contained in a piston cylinder device shown in figure below initially at a pressure of 1000 kPa and a temperature of 77° C. There is a heat transfer to the system until the piston reaches the upper stops, at which time volume inside the cylinder is 0.6 m^3. The oxygen is further heated until the pressure reaches to 2000 kPa. Sketch the process on P-V and T-V diagrams and determine the total work and heat transfer in the process. [Take R = 260 J/kgK and c_v = 660 J/kgK]



Solution:

Given, Mass of oxygen (m) = 4 kg

Initial state: P_1 = 1000 kPa, $T_1 = 77^0$ C = 77 + 273 = 350 K

Final state: V_{final} = 0.6 m^3, P_{final} = 2000 kPa

Volume at state 1 is given by

$$V_1 = \frac{mRT_1}{P_1} = \frac{4 \times 260 \times 350}{1000 \times 10^3} = 0.364\ m^3$$

The cylinder is heated until the piston reaches the upper stops and the process is constant pressure heating process Of 1000 kPa (Process 1-2). Hence, we can define state 2 as

State 2: $P_2 = P_1$ = 1000 kPa, V_2 = 0.6 m^3

Temperature at state 2, $T_2 = \frac{P_2V_2}{mR} = \frac{1000 \times 0.6}{4 \times 0.260} = 576.92\ K$

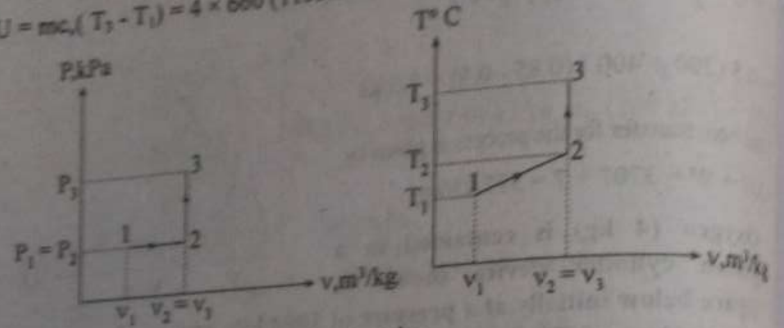
It is further heated till the pressure reaches to 2000 kPa. Hence, the process is constant volume heating process (process 2-3) and we can define state 3 as

State 3: $V_3 = V_2$ = 0.6 m^3, P_3 = 2000 kPa

Temperature at state 3, $T_3 = \frac{P_3V_3}{mR} = \frac{2000 \times 0.6}{4 \times 0.260} = 1153.84$ K

Change in total internal energy is given as

$\Delta U = mc_v(T_3 - T_1) = 4 \times 660\,(1153.84 - 350) = 2122.1376$ kJ



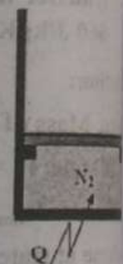
Total work transfer in the process is given by

$W = W_{12} + W_{23} = P_1(V_2 - V_1) + 0 = 1000\,(0.6 - 0.364) = 236$ kJ

$\therefore$ Total heat transfer in the process is given by

$Q = \Delta U + W = 2122.1376 + 236 = 2358.1376$ kJ

14. Nitrogen 5 kg is contained in a piston cylinder device shown in figure below initially at a pressure of 800 kPa and a temperature of 127° C. There is a heat transfer to the system until the temperature reaches to 527° C. It takes a pressure of 1500 kPa to lift the piston. Sketch the process on P-V and T-V diagrams and determine the total work and heat transfer in the process. [Take R = 297 J/kgK and c_v = 743 J/kgK].



Solution:

Given, Mass of N_2 (m) = 5 kg

Initial state: $P_1 = 800$ kPa, $T_1 = 127^0 C = 127 + 273 = 400$ K

Final state: $T_{final} = 527^0 C = 527 + 273 = 800$ K

Pressure required to lift the piston (P_{lift}) = 1500 kPa

Volume of N_2 at state 1 is given as

$V_1 = \frac{mRT_1}{P_1} = \frac{5 \times 297 \times 400}{800 \times 10^3} = 0.7425\ m^3$

Initial pressure of the system is 800 kPa and pressure required to lift the piston is 1500 kPa. Hence, during initial stage of heating piston remains stationary although heat is supplied to the system so process is constant volume heating (Process 1 - 2). During constant volume heating, pressure of the system increases from 800 kPa to 1500 kPa. Hence, we can define state 2 as,

State 2: $P_2 = 1500$ kPa, $V_2 = V_1 = 0.7425\ m^3$

Temperature at state 2, $T_2 = \frac{P_2V_2}{mR} = \frac{1500 \times 0.7425}{5 \times 297} = 750$ K

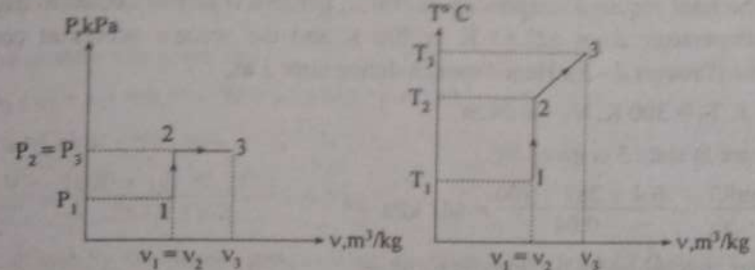
But, the required final temperature is 800 K, hence it should be further heated to increase the temperature from 750 K to 800 K and the process occurs at constant pressure of 1500 kPa (Process 2-3). Hence, we can define state 3 as,

State 3: $P_3 = 1500$ kPa, $T_3 = 800$ K

Volume at state 3, $V_3 = \frac{mRT_3}{P_3} = \frac{5 \times 297 \times 800}{1500 \times 10^3} = 0.792\ m^3$

Change in total internal energy is given by

$\Delta U = mc_v(T_3 - T_1) = 5 \times 743 \times (800 - 400) = 1486$ kJ



Total work transfer in the process is given by

$W = W_{12} + W_{23} = 0 + P_2(V_3 - V_1) = 1500\,(0.792 - 0.7425) = 74.25$ kJ

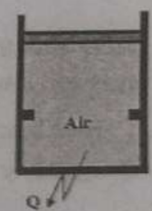
Total heat transfer in the process is given by

$Q = \Delta U + W = 1486 + 74.25 = 1560.25$ kJ

15. Air (0.4 kg) is contained in a piston cylinder device shown in figure below initially at a pressure of 1500 kPa and 800 K. The cylinder has stops suchthat the minimum volume of the system is 0.04 m³. The air in the cylinder is cooled to 300 K. Sketch the process on P-V and T-V diagrams and determine

(a) the final volume and pressure of the air, and

(b) the total work and heat transfer in the process. [Take R=287 J/kgK and c_v = 718 J/kgK].



Solution:

Given, Mass of air (m) = 0.4 kg

Initial state: $P_1 = 1500$ kPa, $T_1 = 800$ K

Final state: $T_{final} = 300$ K

Minimum volume of the system (V_{min}) = 0.04 m³

Volume at initial state is given by

$$V_1 = \frac{mRT_1}{P_1} = \frac{0.4 \times 287 \times 800}{1500 \times 10^3} = 0.06123 \text{ m}^3$$

If heat is lost by the system, piston drops downward and process (Process 1 - 2) occurs at constant pressure of 1500 kPa and volume decreases to 0.04 m³ as the piston reaches the stops. Hence, we can define state 2 as,

State 2: $P_2 = 1500$ kPa, $V_2 = 0.04$ m³

Temperature at state 2 is given by

$$T_2 = \frac{P_2V_2}{mR} = \frac{1500 \times 10^3 \times 0.04}{0.4 \times 287} = 522.65 \text{ K}$$

But, the final required temperature is 300 K, hence it is further cooled to decrease the temperature from 522.65 K to 300 K and the process occurs at constant volume (Process 2 - 3). Hence we can define state 3 as,

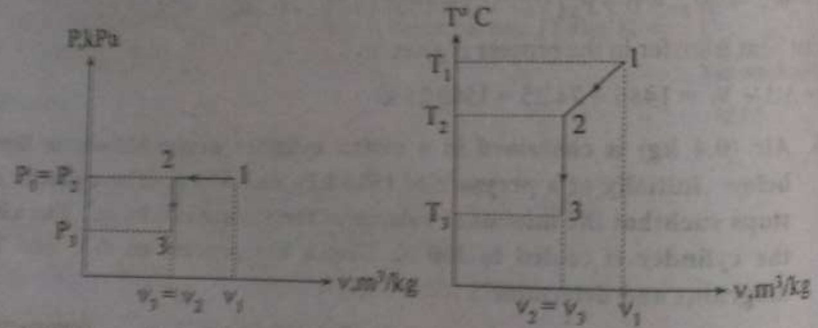
State 3: $T_3 = 300$ K, $V_3 = 0.04$ m³

Pressure at state 3 is given by

$$P_3 = \frac{mRT_3}{V_3} = \frac{0.4 \times 287 \times 300}{0.04} = 861 \text{ kPa}$$

Change in total internal energy is given as

$$\Delta U = mc_v (T_3 - T_1) = 0.4 \times 718 \times (300 - 500) = -143.6 \text{ kJ}$$



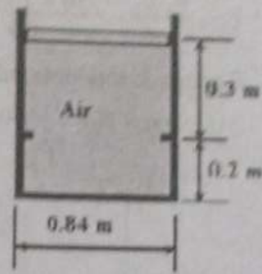
Total work transfer in the process is given by

$$W = W_{12} + W_{23} = P_1 (V_2 - V_1) + 0 = 1500 (0.04 - 0.06123) = -31.845 \text{ kJ}$$

∴ Total heat transfer in the process is given by

$$Q = \Delta U + W = -143.6 - 31.845 = 175.445 \text{ kJ}$$

16. Air is contained in a piston cylinder device shown in figure below initially at a pressure and temperature of 1000 kPa and 800°C. Heat is lost by the system until its pressure drops to 750 kPa. Sketch the process on P - V and T - V diagrams and determine the total work and heat transfer. [Take R = 287 J/kgK and c_V = 718 J/kgK]



Solution:

Given, Initial state $P_1 = 1000$ kPa, $T_1 = 800^0$ C = 800 + 273 = 1073 K

Final state: $P_{final} = 750$ kPa

Diameter of the piston (A_p) 0.84 m

$$\text{Area of piston } (A_p) = \frac{\pi (D_p)^2}{4} = \frac{\pi (0.84)^2}{4} = 0.554177 \text{ m}^2$$

Volume at state 1 is given as

$$V_1 = A_p (x_1 + x_2) = 0.554177 \times (0.2 + 0.3) = 0.2771 \text{ m}^3$$

∴ Mass of air is given by

$$m = \frac{P_1V_1}{RT_1} = \frac{1000 \times 10^3 \times 0.2771}{287 \times 1073} = 0.9 \text{ kg}$$

If heat is lost by the piston, piston drops downward and process (Process 1 - 2) occurs at constant pressure of 1000 kPa and volume decreases to V_2 as the piston reaches the stops.

Hence, we can define state 2 as

State 2: $P_2 = 1000$ kPa

Volume at state 2 is given as

$$V_2 = A_p \times x_1 = 0.554177 \times 0.2 = 0.11084 \text{ m}^3$$

And temperature at state 2 is given by

$$T_2 = \frac{P_2V_2}{mR} = \frac{1000 \times 10^3 \times 0.11084}{0.9 \times 287} = 429.1 \text{ K}$$

But the final required pressure is 750 kPa, hence it is further cooled to decrease the pressure from 1000 kPa to 750 kPa and the process occurs at constant volume (Process 2 - 3). Hence we can define state 3 as,

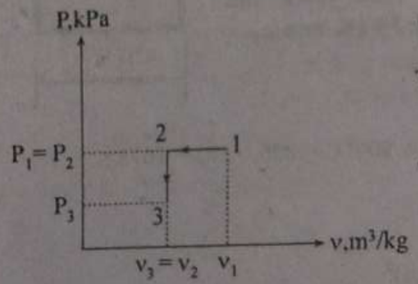
State 3: $P_3 = 750$ kPa, $V_3 = 0.11084$ m³

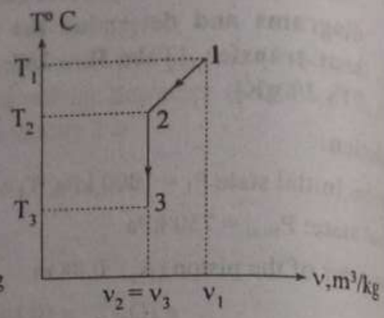
Temperature at state 3 is given by

$$T_3 = \frac{P_3 V_3}{mR} = \frac{750 \times 10^3 \times 0.11084}{0.9 \times 287} = 321.84 \text{ K}$$

Change in total internal energy is given as

$$\Delta U = mc_v (T_1 - T_3) = 0.9 \times 718\,(1073 - 321.84) = -4531 \text{ kJ}$$





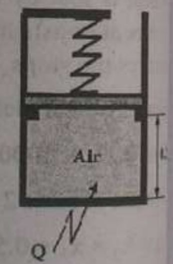
The total work transfer in the process is given by

$$W = W_{12} + W_{23} = P_1 (V_2 - V_1) + 0 = 1000\ (0.11084 - 0.2771) = -166.26 \text{ kJ}$$

∴ The total heat transfer during the process is given by

$$Q = \Delta U + W = -453.1 - 166.26 = -651.66 \text{ kJ}$$

17. Air (0.1 kg) is contained in piston/cylinder assembly as shown in figure below. Initially, the piston rests on the stops and is in contact with the spring, which is in its unstretched position. The spring constant is 100 kN/m. The piston weighs 30 kN and atmospheric pressure is 101 kPa. The air is initially at 300 K and 200 kPa. Heat transfer occurs until the air temperature reaches the surrounding temperature of 700 K.



(a) Find the final pressure and volume.

(b) Find the process work.

(c) Find the heat transfer.

(d) Draw the P-V diagram of the process. [Take R=287 J/kgK and c_V=718 J/kgK].

Solution:

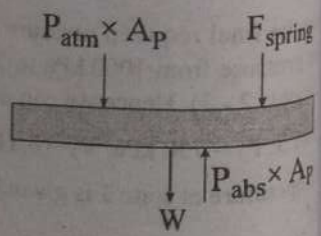
Given, Mass of air (m) = 0.1 kg

Spring constant (k) = 100kN/m

Weight of piston (W) = 30 kN

Atmospheric pressure (P_{atm}) = 101 kPa

Initial state: T_1 = 300 K, P_1 = 200 kPa



Final state: T_{final} = 700 K

a) Volume at state 1 is given by

$$V_1 = \frac{mRT_1}{P_1} = \frac{0.1 \times 287 \times 300}{200 \times 10^3} = 0.04305 \text{ m}^3$$

∴ Area of the piston is given by

$$A_p = \frac{V_1}{x_1} = \frac{0.04305}{0.2} = 0.21525 \text{ m}^2$$

Referring to the free body diagram of the piston, we can write equation for the pressure inside the cylinder as

$$P = P_{abs} = P_{atm} + \frac{W}{A_p} + \frac{F_{spring}}{A_p}$$

Initially, the spring touches the piston but exerts no force. So, $F_{spring} = 0$. Pressure required to lift the piston is given as

$$P = P_{lift} = 101 + \frac{30}{0.21525} + 0 = 240.37 \text{ kPa}$$

Initial pressure of the system is 200 kPa but pressure required to lift the piston is 240.37 kPa. Hence, during initial stage of heating piston remains stationary although heat is supplied to the system, so process is constant volume heating (Process 1 - 2).During constant volume heating pressure of the system increases from 200 kPa to 240.37 kPa. Hence, we can define state 2 as.

State 2 : P_2 = 240.37 kPa, V_2 = 0.04305 m³

Temperature at state 2 is given

$$T_2 = \frac{P_2 V_2}{mR} = \frac{240.37 \times 10^3 \times 0.04305}{0.1 \times 287} = 360.56 \text{ K}$$

But, the final required temperature is 700 K, hence it is further heated to increases temperature from 360.56 K to 700 K. Hence, we can define state T_3 as,

State 3: T_3 = 700 K

The pressure equation for any intermediate state is given as

$$P = P_{atm} + \frac{W}{Ap} + \frac{F_{spring}}{Ap}$$

∴ Pressure at state 3 is given as

$$P_3 = P_2 + \frac{kx}{A_p} = 240.37 - \frac{100x}{0.21525} \quad \text{......(i)}$$

Let x be the displacement of piston above the stops.

Then, volume at state 3 is given as

$V_3 = A_p\,(0.2 + x) = 0.2525\,(0.2 + x)$ ....(ii)

Also, pressure at state 3 is given by

$$P_3 = \frac{mRT_3}{V_3}$$

$$\text{or, } \left(240.37 + \frac{100x}{0.21525}\right) \times 10^3 = \frac{0.1 \times 287 \times 700}{0.21525\,(0.2 + x)}$$

$$\text{or, } 51.74 + 100x = \frac{20.09}{0.2 + x}$$

or, $(51.74 + 100x)(0.2 + x) = 20.09$

or, $10.378 + 51.74x + 20x + 100x^2 = 20.09$

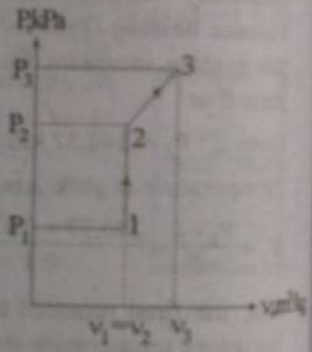
or, $100x^2 + 71.74x - 9.742 = 0$

$\therefore\ x = 0.1168$ m

Hence, substituting value of x in equations (i) and equation (ii), volume and pressure at final state are:

$V_3 = 0.21525\,(0.2 + 0.1168) = 0.0682\ m^3$

$$P_3 = 240.37 + \frac{100 \times 0.1168}{0.21525} = 294.63\ \text{kPa}$$



b) The process work is given as

$W = W_{12} + W_{23}$

$$= 0 + \frac{1}{2}(P_2 + P_3)(V_3 - V_2)$$

$$= \frac{1}{2}(240.37 + 294.63)(0.0682 - 0.04305) = 6.73\ \text{kJ}$$

c) Change in total internal energy is given as

$\Delta U = mc_v(T_3 - T_1) = 0.1 \times 718 \times (700 - 300) = 28.72$ kJ

$\therefore$ Total heat transfer during the process is given by

$Q = \Delta U + W = 28.72 + 6.73 = 35.45$ kJ

18. Water (1.5 kg) is contained in a piston cylinder device shown in figure below initially at a pressure of 400 kPa with a quality of 50 %. There is a heat transfer to the system until it reaches a final temperature of 500°C. It takes a pressure of 800 kPa to lift the piston. Sketch the

process on P-v and T-v diagrams and determine the total work and heat transfer.

Solution:

Given, Mass of H_2O (m) = 1.5 kg

Initial state: $P_1 = 400$ kPa, $x_1 = 50\% = 0.5$

Final state: $T_{final} = 500^0$C

Pressure required to lift the piston (P_{lift}) = 800 kPa

Referring to the Table A2.1, v_f (400 kPa) = 0.001084 m^3/kg, v_{fg} (400 kPa) = 0.4614 m^3/kg, u_f (400kPa) = 604.47 kJ/kg,

u_{fg} (400 kPa) = 1949.0 kJ/kg, T_{sat} (400 kPa) = 143.64^{0}C

Specific volume at state 1 is given by

$v_1 = v_f + x_1 v_{fg} = 0.001084 + 0.5 \times 0.4614 = 0.231784\ m^3/kg$

Specific internal energy at state 1 is given as

$u_1 = u_f + x_1 u_{fg} = 604.47 + 0.5 \times 1749.0 = 1578.97$ kJ/kg

Initial pressure of the system is 400 kPa and pressure

required to lift the piston is 800 kPa. Hence, during initial stage of heating piston remains stationary although heat is supplied to the system, so process is constant volume process (Process 1-2). During constant volume heating, pressure of the system increases from 400 kPa to 800 kPa. Hence, we can define state 2 as,

State 2: $P_2 = 800$kPa, $v_2 = 0.231784\ m^3/kg$

Referring to the Table A2.1, v_g (800 kPa) = 0.2404 m^3/kg, v_{fg} (800 kPa) = 0.2393 m^3/kg, v_f (800 kPa) = 0.001115m^3/kg. Here, $v_f < v < v_g$, hence it is a two phase mixture.

Temperature at state 2 is given as

$T_2 = T_{sat}$ (800kPa) = 170.44^{0}C

Quality at state 2 is given as

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.23178 - 0.001115}{0.2393} = 0.96392$$

But the required final temperature is 500^{0}C, hence it should be further heated to increases the temperature from 170.44^0 C to 500^0 C and the process occurs at constant pressure of 800 kPa. (Process 2-3). Hence, we can define state 3 as,

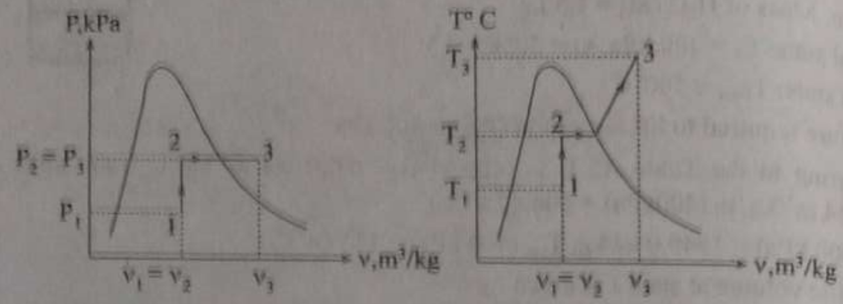
State 3: $P_3 = 800$kPa, $T_3 = 500^0$C

Referring to Table A2.1, T_{sat} (800 kPa) = 170.44^{0}C, Here $T > T_{sat}$, hence it is a superheated steam. Referring to the Table A2.4, $v_g = 0.4433\ m^3/kg$, $u_g = 3126.1$ kJ/kg

$\therefore v_3 = 0.4433 m^3$, $u_3 = 3126.1$ kJ/kg

Change in total internal energy is given by

$\Delta U = m(u_3 - u_1) = 1.5\,(3126.1 - 1578.97) = 2320.695$ kJ/kg



Total work transfer is given by

$W = W_{12} + w_{23} = 0 + P_3(V_3 - V_2) = mP_2(v_3 - v_2)$

$= 1.5 \times 800 \times (0.4433 - 0.231784) = 253.8192$ kJ

$\therefore$ Total heat transfer during the process is given by

$Q = \Delta U + W = 2320.695 + 253.8192 = 2574.51$ kJ

19. Water (0.5 kg) is contained in a piston cylinder device shown in figure below initially at a pressure of 200 kPa with a quality of 80 %. Mass of the piston is such that a pressure of 300 kPa is required to lift it. Heat transferred to the system until its volume doubles. Sketch the process on P-v and T-v diagrams and determine:

(a) the final temperature

(b) the total work transfer, and

(c) the total work transfer.

Solution:

Given, Mass of H_2O (m) = 0.5 kg

Initial state: $P_1 = 200$ kPa, $x_1 = 80\% = 0.8$

Final state: $V_{final} = 2V_1$

Pressure required to lift the piston (P_{lift}) = 300 kPa

a) Referring to the Table A2.1, v_f (200kPa) = 0.001060 m³/kg, v_{fg} (200 kPa) = 0.8848 m³/kg, u_f (200kPa) = 504.59 kJ/kg, u_{fg} (200 kPa) = 2024.8 kJ/kg, T_{sat} (200 kPa) = 120.24° C

Specific volume at state 1 is given as

$v_1 = v_f + x_1 v_{fg} = 0.001060 + 0.8 \times 0.8848 = 0.7089$ m³/kg

Specific volume at final state, $v_{final} = 2v_1 = 2 \times 6.7089 = 1.4178$ m³/kg

$\therefore$ Volume at state 1 is given as

$V_1 = v_1 \times m = 0.7089 \times 0.5 = 0.35445$ m³

Temperature at state 1 is given as

$T_1 = T_{sat}$ (200 kPa) = 120.24° C

Specific internal energy at state 1 is given by

$u_1 = u_f + x_1 u_{fg} = 504.59 + 0.8 \times 2024.8 = 2124.43$ kJ/kg

Initial pressure of the system is 200 kPa and pressure required to lift the piston is 300 kPa. Hence, during initial stage of heating piston remains stationary although heat is supplied to the system so process is constant volume heating (Process 1-2). During constant volume heating, pressure of the system increases from 200 kPa to 300 kPa. Hence, we can define state 2 as,

State 2: $P_2 = 300$kPa, $v_2 = 0.7089$ m³/kg

Referring to Table A2.1, v_g (300 kPa) = 0.6059 m³/kg. Here, $v > v_g$, hence it is superheated vapor.

But the required final specific volume is 1.4178 m³/kg, hence is should be heated further to increases the specific volume from 0.7089 m³/kg to 1.4178 m³/kg and the process occurs at constant pressure of 300 kPa (Process 2 - 3) Hence, we can define state 3 as,

State 3: $P_3 = 300$ kPa, $v_3 = 1.4178$m³/kg

Referring to Table A2.4, the specific volume of superheated vapor which includes the specific volume 1.4178 m²/kg at 300 kPa and corresponding temperature and specific initial energy are listed as:

T°C	v_g, m³/kg	u_g, kJ/kg	
600	1.3414	3301.1	(a)
650	1.4186	3389.1	(b)

Applying linear interpolation for temperature and specific internal energy,

$$T_3 - T_a = \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$\therefore T_3 = T_a + \frac{T_b - T_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$= 600 + \frac{650 - 600}{1.4186 - 1.3414}[1.4178 - 1.3414]$$

$= 649.482^\circ C$

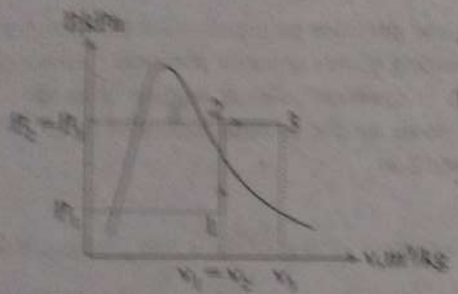
Similarly,

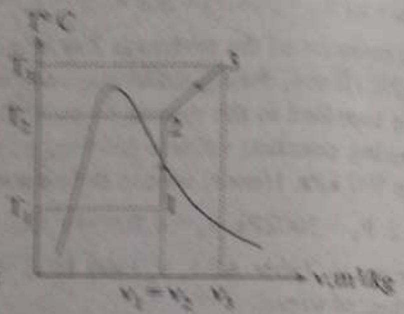
$u_3 = (u_g)_a + \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$

$= 3301.1 + \frac{3398.1 - 3301.1}{1.4186 - 1.3414}(1.4178 - 1.3414) = 3388.19 \text{ kJ/kg}$

Change in total internal energy is given as

$\Delta U = m(u_3 - u_1) = 0.5(3388.19 - 2124.43) = 631.88 \text{ kJ}$





b) Total work transfer for the process is given as

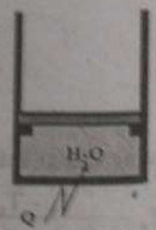
$W = W_{1-2} + W_{2-3} = 0 + P_2(V_3 - V_2) = mP_2(v_3 - v_2)$

$= 0.5 \times 300(1.4178 - 0.7089) = 106.335 \text{ kJ}$

c) Total heat transfer for the process is given as

$Q = \Delta U + W = 631.88 + 106.335 = 738.215 \text{ kJ}$

20. Water (4 kg) is contained in a piston cylinder device shown in figure below initially at a pressure of 100 kPa with a quality of 10 %. The piston has a mass of 100 kg and a cross sectional area of 24.525 cm². Heat is now added until H2O reaches a saturated vapor state. Sketch the process on P - v and T - v diagrams and determine



(a) the initial volume

(b) the final pressure

(c) the total work transfer, and

(d) the total work transfer.

Solution:

Given, Mass of H_2O (m) = 4 kg

Initial state: $P_1 = 100$ kPa, $x_1 = 10\% = 0.1$

Mass of piston (m) = 100 kg

Cross sectional area of piston $(A_p) = 24.525 \text{ m}^2$

Final state: saturated vapor state

Referring to the Table A2.1, $v_f(100 \text{ kPa}) = 0.001043 \text{m}^3/\text{kg}$, $v_{fg}(100\text{kPa}) = 1.6933 \text{m}^3/\text{kg}$, $u_f(100 \text{ kPa}) = 417.41 \text{ kJ/kg}$, $u_{fg}(100 \text{ kPa}) = 2088.3 \text{ kJ/kg}$, $T_{sat}(100 \text{ kPa}) = 99.632^\circ C$

Specific volume at state 1 is given as

$v_1 = v_f + x_1 v_{fg} = 0.001043 + 0.1 \times 1.6933 = 0.170373 \text{ m}^3/\text{kg}$

a) Volume at state 1 is given by

$V_1 = v_1 \times m = 0.170373 \times 4 = 0.681492 \text{ m}^3$

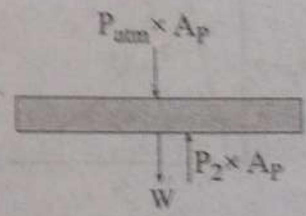
Temperature at state 1 is given as

$T_1 = T_{sat}(100 \text{ kPa}) = 99.632^\circ C$

Specific enthalpy at state 1 is given by

$u_1 = u_f + x_1 u_{fg} = 417.41 + 0.1 \times 2088.3 = 62624 \text{ kJ/kg}$

Referring to the free body diagram of the piston, we can write the equation for the pressure inside the cylinder as



$P_{abs} = P = P_{atm} + \frac{W}{A_p} = P_{atm} + \frac{m_p g}{A_p}$

$\therefore P = 100 + \frac{100 \times 9.81}{24.525 \times 10^{-4} \times 10^3} = 500 \text{ kPa}$

Hence, Pressure required to left the piston, $P_{lift} = 500$ kPa.

Initial pressure of the system is 100 kPa and pressure required to lift the piston is 500 kPa. Hence, during initial stage of heating piston remains stationary although heat is supplied to the system so pressure is constant volume heating (Process 1-2). During constant volume heating, pressure of the system increases from 100kPa to 500kPa. Hence, we can define state 2 as,

State 2: $P_2 = 500$ kPa, $v_2 = 0.170373 \text{ m}^3/\text{kg}$

Referring to Table A2.1, $v_f(500 \text{ kPa}) = 0.001093 \text{ m}^3/\text{kg}$, $v_g(500 \text{ kPa}) = 0.3749 \text{ m}^3/\text{kg}$, $v_{fg}(500 \text{ kPa}) = 6.3738 \text{ m}^3/\text{kg}$

$T_{sat}(500^\circ C) = 151.87^\circ C$. Here, $v_f < v < v_g$, hence it is a two phase mixture.

Temperature at state 2 is given as

$T_2 = T_{sat}(500\text{kPa}) = 151.87^\circ C$

But the required final state is saturated vapor state, hence is should be heated further to increase specific volume from 0.170373 m^3/kg to v_g (500 kPa) = 0.3749 m^3/kg and the process occurs at constant pressure of 500 kPa and constant temperature of 151.87° C (process 2-3). Hence, we can define state 3 as

State 3: $P_3 = 500$ kPa, $v_3 = 0.3749\ m^3/kg$, $T_3 = 151.87^0 C$

b) Final pressure, $P_{final} = P_3 = 500$ kPa

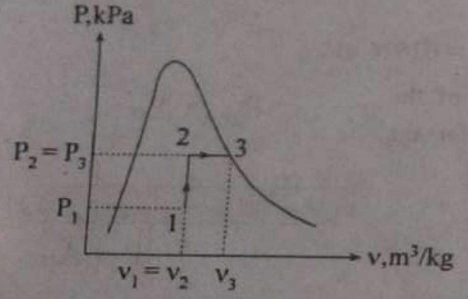
Referring to Table A2.1, u_g (500 kPa) = 2561.2 kJ/kg

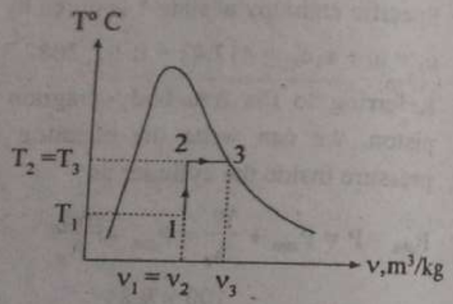
∴ Specific volume at state 3 is given as

$u_3 = u_g (500\ kPa) = 2561.2$ kJ/kg

Change in total internal energy is given as

$\Delta U = m\,(u_3 - u_1) = 4 \times (2561.2 - 626.24) = 7739.8$ kJ





c) Total work transfer for the process is given as

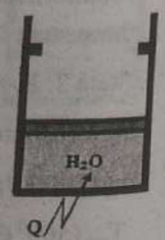
$W = W_{12} + W_{23} = 0 + P_2 (V_3 - V_2) = mP_2 (v_3 - v_2)$

$= 4 \times 500\ (0.3749 - 0.170373) = 409.054$ kJ

d) Total heat transfer for the process is given by

$Q = \Delta U + W = 7739.84 + 409.054 = 8148.894$ kJ

21. A piston cylinder device shown in figure below contains 4 kg of water initially at saturated liquid state at 5 MPa. There is a heat transfer to the system until it hits the stops at which time its volume is 0.08 m^3. There is further heat transfer to the device until water is completely vaporized. Sketch the process on P-v and T-v diagrams and determine the total work and heat transfer.



Solution:

Given, Mass of H_2O (m) = 4 kg

Initial state: $P_1 = 5$ MPa = 5000 kPa, saturated liquid

State 2: $V_2 = 0.08\ m^3$

Final state: saturated vapor

Referring to Table A2.1, v_f (5000 kPa) = 0.001286 m^3/kg, v_g (5000 kPa) = 0.03944m^3/kg, u_f (5000 kPa) = 1147.8 kJ/kg, u_g (5000 kPa) = 2596.5 kJ/kg, T_{sat} (5000 kPa) = 263.98° C

State 1 is saturated liquid state, hence, we can define state 1 as

$v_1 = 0.001286\ m^3/kg$, $u_1 = 1147.8$ kJ/kg

Specific volume at state 2 is given by

$$v_2 = \frac{V_2}{m} = \frac{0.08}{4} = 0.02\ m^3/kg$$

The system is heated until the piston hits the stops and the process occurs at constant pressure of 5000 kPa (Process 1-2).

Hence, we can define state 2 as

State 2: $v_2 = 0.02\ m^3/kg$, $P_2 = 5000$ kPa

Here, $v_f < v < v_g$, hence it is a two phase mixture.

But the required final state is saturated vapor hence it should be further heated and the process occurs at constant volume (Process 2-3) as piston hits the stops. Hence, we can define state 3 as,

State 3: $v_3 = 0.02\ m^3/kg$

Referring to the Table A2.1, specific volume of saturated vapor which includes the specific volume 0.02 m^3/kg and corresponding specific internal energy is listed as:

v_g, m^3/kg	u_g, kJ/kg	
0.02048	2557.6	(a)
0.01803	2544.2	(b)

Applying linear interpolation for specific internal energy

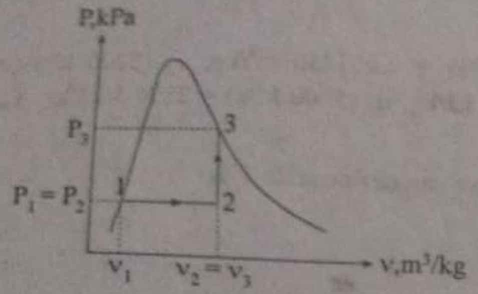
$$u_3 + (u_g)_a = \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

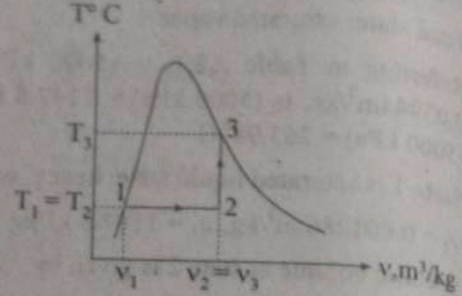
$$\therefore u_3 = (u_g)_a + \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$= 2557.6 + \frac{2544.2 - 2557.6}{0.01803 - 0.02048}(0.02 - 0.02048) = 2554.975 \text{ kJ/kg}$$

Change in total internal energy is given as

$$\Delta U = m(u_3 - u_2) = 4 \times (2554.975 - 1147.8) = 5628.7 \text{ kJ}$$





∴ Total work transfer in the process is given as

$$W = W_{12} + W_{23} = P_1(V_2 - V_1) + 0 = mP_1(v_2 - v_1)$$

$$= 4 \times 5000 (0.02 - 0.001286) = 374.28 \text{ kJ}$$

And, total heat transfer during the process is given by

$$Q = \Delta U + W = 5628.7 + 374.28 = 6002.98 \text{ kJ}$$

22. A piston cylinder device shown in figure below contains water initially at P_1=1 MPa and T_1=500° C. A pressure of 400 kPa is required to support the piston. There is a heat transfer from the device until its temperature drops to 30° C. Sketch the process on P-v and T-v diagrams and determine the total work and heat transfer.

Solution:

Given, Initial state: $P_1 = 1$ MPa = 1000 kPa, $T_1 = 500°C$

Pressure required to support the piston ($P_{support}$) = 400 kPa

Final state = $T_{final} = 30°C$

Referring to the Table A2.1, T_{sat} (1000kPa) = 179.92° C

Here, $T > T_{sat}$ (1000 kPa), hence, it is a superheated vapor.

Referring to the Table A2.4, v_g (500°C) = 0.3541 m³/kg,

u_g (500°C) = 3124.5 m³/kg,

∴ $v_1 = v_g(500°C) = 0.3541$ m³/kg

And, $u_1 = u_g(500°C) = 3124.5$m³/kg

Initial pressure of the system is 1000 kPa and pressure required to support the piston is 400 kPa. Hence, during initial stage of cooling piston remains stationary although heat is rejected from the system, so process is constant volume cooling (Process 1-2). During constant volume cooling, pressure of the system decreases from 100 kPa to 400kPa. Hence we can define state 2 as,

State 2: $P_2 = 400$ kPa, $v_2 = 0.3541$ m³/kg

Referring to Table A2.1, T_{sat} (400 kPa) = 143.64° C, v_f (400 kPa) = 0.001084 m³/kg, v_{fg} (400 kPa) = 0.4614 m³/kg, v_g (400 kPa) = 0.4625 m³/kg. Here, $v_f < v < v_g$, hence it is a two phase mixture.

Temperature at state 2, $T_2 = T_{sat}$ (400 kPa) = 143.64° C

But the final temperature is 30°C, hence it should be cooled further to decrease the temperature from 143.64° C to 30° C and the process occurs at constant of 400 kPa (Process 2-3). Hence, we can define state 3 as,

State 3: $T_3 = 30°$ C, $P_3 = 400$ kPa

$T < T_{sat}$ (400 kPa), hence it is a compressed liquid.

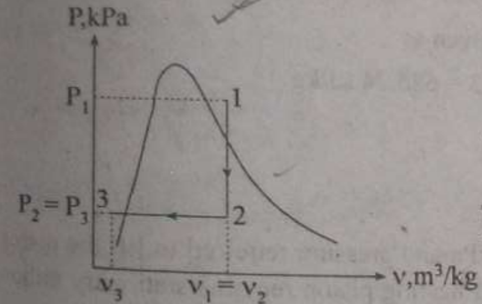
Referring to Table A2.2 (since 400 kPa is not available in Table A2.3), v_f (30° C) = 0.001004m³/kg

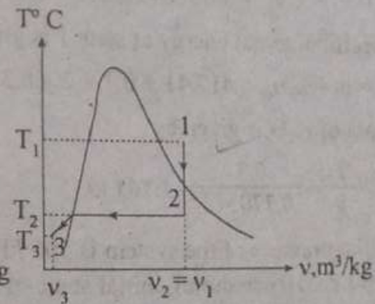
Specific volume at state 3 is given as

$v_3 = v_f(30°C) = 0.001004$m³/kg, $u_3 = 125.67$ kJ/kg

Change in total internal energy is given as

$$\Delta u = (u_3 - u_2) = 125.67 - 3124.5 = -2998.83 \text{ kJ/kg}$$





Total work transfer in the process is given as

$$w = w_{12} + w_{23} = 0 + P_2(v_3 - v_2) = 400(0.001004 - 0.3541) = -141.238 \text{ kJ/kg}$$

And, total heat transfer during the process is given by

$$q = \Delta u + w = -2998.83 - 141.238 = -3140.1 \text{ kJ/kg}$$

23. A piston is free to move between two sets of stops in a piston cylinder device shown in figure below. When the piston rests on the bottom stops, the volume is 0.3 m^3 and when the piston reaches the upper stops the volume is 0.6 m^3. The system initially contains water at 100 kPa with a quality 10 %. Heat is supplied to the system until it contains only saturated vapor. The mass of the piston is such that a pressure of 250 kPa is required to lift the piston. Sketch the process on P-v and T-v diagrams and determine

(a) the final pressure

(b) the total work transfer, and

(c) the total work transfer.

Solution:

Given, Initial state: $P_1 = 100$ kPa, $x_1 = 10\% = 0.1$, $V_1 = 0.3$ m^3

Final state: saturated vapor

Pressure required to lift the piston (P_{lift}) = 250 kPa

Referring to the Table A2.1, v_f(100 kPa) = 0.001043 m^3/kg,

v_{fg} (100 kPa) = 1.6933 m^3/kg, u_f (100 kPa) = 417.41 kJ/kg, u_{fg} (100kPa) = 2088.3 kJ/kg, T_{sat} (100 kPa) = 99.632°C

Specific volume at state 1 is given as

$$v_1 = v_f + x_1 v_{fg} = 0.001043 + 0.1 \times 1.6933 = 0.170373 \text{ m}^3/\text{kg}$$

Specific internal energy at state 1 is given as

$$u_1 = u_f + x_1 u_{fg} = 417.41 + 0.1 \times 2088.3 = 626.24 \text{ kJ/kg}$$

Mass of H_2O is given by

$$m = \frac{V_1}{v_1} = \frac{0.3}{0.170373} = 1.761 \text{ kg}$$

Initial pressure of the system is 100 kPa and pressure required to lift the piston is 250 kPa. Hence during initial stage of heating piston remains stationary although heat is supplied to the system, so process is constant volume heating (Process 1-2). During constant volume heating, pressure of the system increases from 100kpa to 250 kPa. Hence, we can define state 2 as,

State 2: $P_2 = 250$ kPa, $v_2 = 0.17037$ m^3/kg

Referring to the Table A2.1, v_g (250 kPa)= 0.7188m^3/kg, v_f (250 kPa) = 0.001067 m^3/kg. Here $v_f < v < v_g$, hence it is a two phase mixture.

The maximum volume of the cylinder is 0.6 m^3

∴ Specific volume at state 3 is given as

$$v_3 = \frac{V_3}{m} = \frac{0.6}{1.761} = 0.340716 \text{ m}^3/\text{kg}$$

When the piston reaches the upper stops, its specific volume becomes 0.340716 m^3/kg. Hence it is further heated to increase the specific volume from 0.170373 m^3/kg to 0.340716 m^3/kg and the process occurs at constant pressure of 250 kPa (Process 2-3). Hence, we can define state 3 as,

State 3: $P_3 = 250$ kPa, $v_3 = 0.340716$ m^3/kg

Referring to the Table A2.1, $v_f < v < v_g$, hence it is a two phase mixture.

But the final state is saturated vapor, hence it should be heated further until it contains only saturated vapor and the process occurs at constant volume (Process 3-4). Hence we can define state 4 as,

State 4: $v_4 = 0.340716$ m^3/kg. $V_4 = 0.6$ m^3, saturated vapor Referring to the Table A2.1, the specifc volume of saturated vapor which includes the specific volume 0.340716 m^3/kg and corresponding pressure and specific internal energy are listed as:

v_g, m^3/kg,	P, kPa	u_g, kJ/kg	
0.3426	550	2564.4	(a)
0.31560	600	2567.3	(b)

Applying linear interpolation for pressure and specific internal energy,

$$P_4 - P_a = \frac{P_b - P_a}{(v_g)_b - (v_g)_a}[v_4 - (v_g)_a]$$

$$\therefore P_4 = P_a + \frac{P_b - P_a}{(v_g)_b - (v_g)_a}[v_4 - (v_g)_a]$$

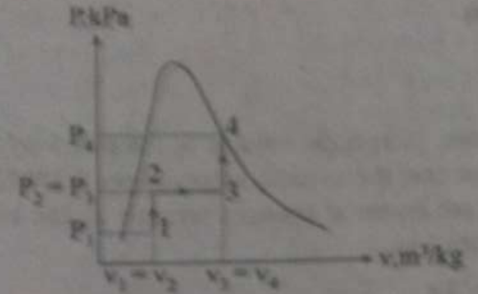
$$= 550 + \frac{500 - 550}{0.31560 - 0.3426}(0.340716 - 0.3426)$$

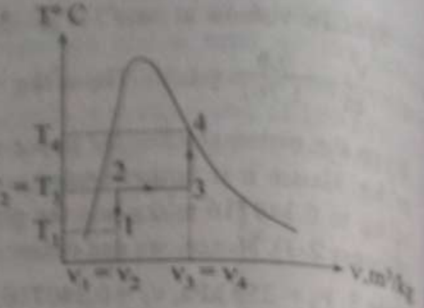
$$= 553.49 \text{ kPa}$$

$$u_4 = (u_g)_a + \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$= 2564.4 + \frac{2567.3 - 2564.4}{0.3156 - 0.3426}(0.340716 - 0.3426)$$

$$= 2564.6024 \text{ kJ/kg}$$





Total work transfer for the process is given by

$W = W_{12} + W_{23} + W_{34} = 0 + P_2 (V_3 - V_2) + 0 = P_2 (V_4 - V_1)$

$= 250 (0.6 - 0.3) = 75$ kJ

Change in total internal energy is given by

$\Delta U = m (u_4 - u_1) = 1.761 \times (2564.6024 - 626.24) = 3413.546$ kJ

$\therefore$ Total heat transfer during the process is given by

$Q = \Delta U + W = 3413.456 + 75 = 3488.456$ kJ

24. Steam enters through a turbine operating under steady state condition at a rate of 5 kg/s. The properties of the steam at turbine inlet are $P_1 = 1$ MPa and $T_1 = 500^0$ C and at the turbine exit $P_2 = 50$ kPa and $x_2 = 0.9$. If the heat loss from the turbine surface occurs at a rate of 500 kW, determine the power output from the turbine.

Solution:

Given, Mass flow rate of steam ($\dot{m}$) = 5 kg/s

Properties of stream at inlet: $P_1 = 1$ MPa = 1000kPa, $T_1 = 5000^0$ C = 500 + 273 = 7785 K

Properties of steam at outlet: $P_2 = 50$ kPa , $x_2 = 0.9$

Heat loss from the turbine surface ($\dot{Q}_{CV}$) = -500 kW

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (1000 kPa) = 179.92^{0}C. Here T > T_{sat}, hence it is a superheated vapor. Now, referring to the Table A2.4, specific enthalpy of steam at inlet, $h_1 = 3478.6$ kJ/kg

For other properties of steam at outlet, referring to the Table A2.1, h_f (50 kPa) = 340.54 kJ/kg, h_{fg} (50 kPa) = 2304.8 kJ/kg

Therefore, specific enthalpy of steam at outlet is given by

$h_2 = h_f + x_2 h_{fg} = 340.54 + 0.9 \times 2304.8 = 2414.86$ kJ/kg

Now, applying steady state energy equation,

$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m} [(h_2 - h_1) + \frac{1}{2} (\overline{V_2^2} - \overline{V_1^2}) + g (z_2 - z_1)]$

$\therefore \dot{W}_{CV} = \dot{Q}_{CV} - \dot{m} [(h_2 - h_1) + \frac{1}{2} (\overline{V_2^2} - \overline{V_1^2}) + g (z_2 - z_1)]$

$= - 500 - 5 [(2414.86 - 3478.6) + 0 + 0] = 4818.7$ kW

Hence, power output from the turbine ($\dot{W}_{CV}$) = 4818.7 kW

25. Steam enters the turbine operating at steady state with a mass flow rate of 1.2 kg/s. Properties of the steam at the inlet are $P_1 = 5$ MPa, $T_1 = 450^0$ C, $\overline{V_1} = 10$ m/s and at the exit are $P_2 = 100$ kPa, $x_2 = 80$ % $\overline{V_2} = 50$ m/s. If the power output of the turbine is 1200 kW, determine the rate of heat transfer from the turbine.

Solution:

Given, Mass flow rate of steam ($\dot{m}$) = 1.2 kg/s

Properties of the steam at intel: $P_1 = 5$ MPa = 5000 kPa, $T_1 = 450^0$ C, $\overline{V_1} = 10$ m/s

Properties of the steam at outlet: $P_2 = 100$ kPa, $x_2 = 80\% = 0.8$, $\overline{V_2} = 50$ m/s

Power output of the turbine ($\dot{W}_{CV}$) = 1200 kW

Rate of heat transfer from the turbine ($\dot{Q}_{CV}$) = ?

For other properties of steam of the inlet of turbine, referring to the Table A2.1, T_{sat} (5000 kPa) = 263.98^0 C. Here, T > T_{sat}, hence, it is a super heated vapor. Now, referring to the Table A2.4, $h_g = 3316.3$ kJ/kg

Specific enthalpy of steam at inlet of turbine, $h_1 = 3316.3$ kJ/kg

For other properties of steam at the exit of turbine, referring to the Table A2.1, h_f (100kPa) = 417 kJ/kg, h_{fg} (100kPa) = 2257.6 kJ/kg. Therefore, specific enthalpy of steam at exit of turbine is given by

$h_2 = h_f + x_2 h_{fg} = 417.51 + 0.8 \times 2257.6 = 2223.59$ kJ/kg

Now applying steady state energy equation,

$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m} [(h_2 - h_1) + \frac{1}{2} (\overline{V_2^2} - \overline{V_1^2}) + g (z_2 - z_1)]$

$\therefore \dot{Q}_{CV} = \dot{W}_{CV} + \dot{m} [(h_2 - h_1) + \frac{1}{2} (\overline{V_2^2} - \overline{V_1^2}) + g (z_2 - z_1)]$

$= 1200 + 1.2 [(2223.59 - 3316.3) + \frac{1}{2000} (50^2 - 10^2) + 0] = - 109.812$ kW

26. Air enters a compressor operating at steady state at 100 kPa, 300 K and leaves at 1000 kPa, 400 K, with a volumetric flow rate of 1.5 m³/min. The work consumed by the compressor is 250 kJ per kg of air. Neglecting the effects of potential and kinetic energy, determine the heat transfer rate, in kW.

Solution:

Given, Volumetric flow rate at exist ($\dot{V}_2$) = 1.5 m³/min = $\frac{1.5}{60} = 0.025\ m^3/s$

Properties of air at inlet: $P_1 = 100$ kPa, $T_1 = 300$ K

Properties of air at exit: $P_2 = 1000$ kPa, $T_2 = 400$ K

Work consumed per unit mass of air by the compressor (w_{cv}) = -250 kJ/kg

Mass flow rate is given by

$$\dot{m} = \frac{P_2 V_2}{RT_2} = \frac{1000 \times 10^3 \times 0.025}{287 \times 400} = 0.218\ kg/s$$

∴ Work consumed rate ($\dot{W}_{CV}$) = $\dot{m}\, w_{cv} = 0.218 \times (-250) = -54.5$ kW

Now, applying steady state energy equation for compressor,

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\,[(h_2 - h_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2) + g(z_2 - z_1)]$$

For an ideal gas using $h_1 - h_2 = c_p (T_1 - T_2)$ and neglecting kinetic energy and potential energy,

$$\dot{Q}_{CV} = \dot{W}_{CV} + \dot{m}\,[c_p (T_2 - T_1) + 0 + 0]$$

$$= -54.5 + 0.218\,[1.005\,(300 - 400)] = -32.591\ kW$$

27. Air expands through an adiabatic turbine from 1000 kPa, 1000 K to 100 kPa, 400 K. the inlet velocity is 10 m/s whereas exit velocity is 100 m/s the power output of the turbine is 3600 kW. Determine the mass flow rate of air and the inlet and exit areas. [Take R = 287 J/kgK and c_p = 1005 J/kgK]

Solution:

Given, Properties of air at inlet: $P_1 = 1000$ kPa, $T_1 = 1000$ K, $\overline{V_1} = 10$ m/s

Properties of air at exit: $P_2 = 1000$ kPa, $T_2 = 400$K, $\overline{V_2} = 100$ m/s

Power output of the turbine ($\dot{W}_{CV}$) = 3600 kW

Applying steady state energy equation for an adiabatic turbine,

$$\dot{W}_{CV} = \dot{m}\,[(h_1 - h_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + g(z_1 - z_2)]$$

For an ideal gas using $h_1 - h_2 = cp\,(T_1 - T_2)$ and neglecting potential energy,

$$\dot{W}_{CV} = \dot{m}\,[c_p (T_1 - T_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + 0]$$

$$\therefore \dot{m} = \frac{\dot{W}_{CV}}{c_p (T_1 - T_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2)}$$

$$= \frac{3600 \times 10^3}{1005\,(1000 - 400) + \frac{1}{2}(10^2 - 100^2)} = 6.02\ kg/s$$

Specific volumes of air at the inlet and outlet are given by

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 1000}{1000 \times 10^3} = 0.287\ m^3/kg$$

$$v_2 = \frac{RT_2}{P_2} = \frac{287 \times 400}{100 \times 10^3} = 1.148\ m^3/kg$$

Inlet area and exit area are given by

$$A_1 = \frac{\dot{m}\,v_1}{\overline{V_1}} = \frac{6.02 \times 0.287}{10} = 0.172774\ m^2$$

$$A_2 = \frac{\dot{m}\,v_2}{\overline{V_2}} = \frac{6.02 \times 1.148}{100} = 0.06911\ m^2$$

28. Air enters the turbine at 1 MPa and 327°C with a velocity of 100 m/s and exits at 100 kPa and 27°C and with a low velocity. Heat transfer loss from the turbine surface is 1200 kJ/min and the power output of the turbine is 240 kW. Determine the mass flow rate of air through the turbine. [Take R = 287 J/kgK and c_p = 1005 J/kgK]

Solution:

Given, Properties of air at inlet : P_1 = 1 MPa = 1000 kPa, T_1 = 327°C = 327 + 273 = 600 K, $\overline{V_1}$ = 100 m/s

Properties of air at exits: P_2 = 100 kPa, T_2 = 27°C = 27 + 273 = 300 K

Heat transfer rate from the turbine surface ($\dot{Q}_{CV}$) = -1200 kJ/min

$$= -\frac{1200}{60} = -20\ kJ/s = -20\ kW$$

Power output of the turbine ($\dot{W}_{CV}$) = 240 kW

Applying steady state energy equation for turbine,

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\,[(h_2 - h_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2) + g\,(z_2 - z_1)]$$

For ideal gas using $h_2 - h_1 = c_p\,(T_2 - T_1)$ and neglecting potential energy,

$$\dot{m} = \frac{\dot{Q}_{CV} - \dot{W}_{CV}}{c_p\,(T_2 - T_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2) + 0}$$

$$= \frac{-20 - 240}{1.005\,(300 - 600) + \frac{1}{2000}(0 - 10^2)} = 0.8622 \text{ kg/s}$$

29. Air flows steadily through an adiabatic compressor entering at 150 kPa, 150°C and with a velocity of 200 m/s and leaving at 1000 kPa, 500°C and with a velocity of 100 m/s. The exit area of the compressor is 100 cm². Determine

(a) the mass flow rate of air through the compressor, and

(b) the power required to drive the compressor.

[Take R=287 J/kgK and c_p=1005 J/kgK *(IOE 2067 Chaitra)*

Solution:

Given, Properties of air at inlet: P_1 = 150 kPa, T_1 = 150° C = 150 + 273=423 K, $\overline{V_1}$ = 200 m/s

Properties of air at state 2: P_2 = 1000 kPa, T_2 = 500° C = 500 + 273= 773 K, $\overline{V_2}$ = 100 m/s

$A_2 = 100\text{ cm}^2 = 100 \times 10^{-4}\text{ m}^2$

Specific volume of air at exit is given by

$$v_2 = \frac{RT_2}{P_2} = \frac{287 \times 773}{100 \times 10^3} = 2.2185\text{ m}^3/\text{kg}$$

Mass flow rate of air through the compressor is given as

$$\dot{m} = \frac{A_2\,\overline{V_2}}{v_2} = \frac{1000 \times 10^{-4} \times 100}{2.2185} = 4.51\text{ kg/s}$$

Now, applying steady state energy equation for adiabatic compressor,

$$\dot{W}_{CV} = \dot{m}\,[(h_1 - h_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + g\,(z_1 - z_2)]$$

For an ideal gas ideal using $h_1 - h_2 = c_p\,(T_1 - T_2)$ and neglecting potential energy.

$$\dot{W}_{CV} = \dot{m}\,[c_p\,(T_1 - T_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + 0]$$

$$= 4.51\,[1005\,(423 - 773) + \frac{1}{2}(200^2 - 100^2)] = -1518.743\text{ kW}$$

30. An adiabatic turbine operating under steady state condition develops 12 MW of power output for a steam mass flow rate of 15 kg/s. the steam enters at 4 MPa with a velocity of 20 m/s and exits at 60 kPa with a quality of 85% and velocity of 100 m/s. determine the inlet temperature of steam.

Solution:

Given, mass flow rate of steam ($\dot{m}$) = 15 kg/s

Properties of steam at inlet: P_1 = 4 MPa = 4000 kPa, $\overline{V_1}$ = 20 m/s

Properties of steam at exit: P_2 = 60 kPa, x_2 = 85% = 0.85, $\overline{V_2}$ = 100 m/s

Power output ($\dot{W}_{CV}$) = 12 MW = 12 × 10³ kW

For other properties of steam as exit, referring to Table A2.1,

h_f (50 kPa) = 359.9 kJ/kg, h_{fg} (60kPa) = 2293.1 kJ/kg,

Therefore, specific enthalpy of steam at exit of turbine is given by

$h_2 = h_f + x_2\,h_{fg}$ = 359.9 + 0.85 × 2293.1 = 2309.035 kJ/kg

Now, applying steady state energy equation for adiabatic turbine,

$$\dot{W}_{CV} = \dot{m}\,[(h_1 - h_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + g\,(z_1 - z_2)]$$

$$\text{or, } 12 \times 10^3 = 15\,[(h_1 - 2309.035) + \frac{1}{200}(20^2 - 100^2) + 0]$$

$\therefore\ h_1$ = 3113.835 kJ/kg

Referring to the Table A2.1, h_g (4000 kPa) = 2800.6kJ/kg. Here, h > h_g, hence it is a superheated vapor, Now referring to the Table A 2.4, specific enthalpy of the steam which includes the specific enthalpy 3113.835 kJ/kg and corresponding temperatures for 4000 kPa are listed as:

T, °C	h_g, kJ/kg	
350	3091.8	(a)
400	3213.4	(b)

Applying linear interpolation for temperature,

$$T_1 - T_a = \frac{T_b - T_a}{(h_g)_b - (h_g)_a}[h_1 - (h_g)_a]$$

$$\therefore T_1 = T_a + \frac{T_b - T_a}{(h_g)_b - (h_g)_a}[h_1 - (h_g)_a]$$

$$= 350 + \frac{400 - 350}{3213.4 - 3091.8}[3113.835 - 3091.8] = 359.1^\circ C$$

31. **Air enters an adiabatic nozzle steadily at 300 kPa, 150°C and a velocity of 20 m/s and leaves at 100 kPa and with a velocity of 200 m/s.the inlet area of the nozzle is 0.01 m². Determine**

   (a) the mass flow rate of air through the nozzle,

   (b) the exit temperature of the air, and

   (c) the exit area of the nozzle. [Take R = 287 J/kgK and c_p = 1005 J/kgK

Solution:

Given, Properties of air at inlet: $P_1 = 300$ kPa, $T_1 = 150^\circ C = 150 + 273 = 423$ K, $\overline{V_1} = 20$ m/s

Properties of air at exit: $P_2 = 100$ kPa, $\overline{V_2} = 200$ m/s

Inlet area of the nozzle$(A_1) = 0.01$ m²

Specific volume of air at inlet is given by

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 423}{300 \times 10^3} = 0.40467 \text{ m}^3/\text{kg}$$

∴ The mass flow rate of air through the nozzle is given by

$$m = \frac{A_1 \overline{V_1}}{v_1} = \frac{0.01 \times 20}{0.40467} = 0.4942 \text{ m}^2$$

Now, applying energy equation for an adiabatic nozzle,

$$(h_2 - h_1) + \frac{1}{2}(\overline{V_2^2} - \overline{V_1^2}) = 0$$

For an ideal gas using $h_2 - h_1 = c_p(T_2 - T_1)$

$$c_p(T_2 - T_1) + \frac{1}{2}(\overline{V_2^2} - \overline{V_1^2}) = 0$$

$$\therefore T_2 = \frac{1}{2}\frac{(\overline{V_1^2} - \overline{V_2^2})}{c_p} + T_1 = \frac{1}{2}\frac{(90^2 - 200^2)}{1005} + 423 = 403.299 \text{ K} = 130.299^\circ C$$

Specific volume of air at exit is given by

$$v_2 = \frac{RT_2}{P_2} = \frac{2.87 \times 403.299}{100 \times 10^3} = 1.1575 \text{ m}^3/\text{kg}$$

Therefore, the exit area of the nozzle is given by

$$A_2 = \frac{\dot{m} v_2}{\overline{V_2}} = \frac{0.4942 \times 1.1575}{200} = 0.0028602 \text{ m}^2 = 28.602 \text{ cm}^2$$

32. **Air at 100 kPa and 127° C enters an adiabatic diffuser at a rate of 1.5 kg/s and leaves at a pressure of 150 kPa. The velocity of the air is decreased from 250 m/s to 50 m/s as it passes through the diffuser. Determine:**

   (a) the exit temperature of the air, and

   (b) the exit area of the diffuser.

Solution:

Given, Mass flow rate $(\dot{m}) = 1.5$ kg/s

Properties of air at inlet: $P_1 = 100$ kPa, $T_1 = 127^\circ C = 127 + 273 = 400$ K, $\overline{V_1} = 250$ m/s

Properties of air at exit: $P_2 = 150$ kPa, $\overline{V_2} = 50$ m/s

Applying energy equation for an adiabatic diffuser,

$$(h_2 - h_1) + \frac{1}{2}(\overline{V_2^2} - \overline{V_1^2}) = 0$$

For an ideal gas using $h_2 - h_1 = c_p(T_2 - T_1)$

$$c_p(T_2 - T_1) + \frac{1}{2}(\overline{V_2^2} - \overline{V_1^2}) = 0$$

$$\therefore T_2 = \frac{1}{2}\frac{(\overline{V_2^2} - \overline{V_1^2})}{c_p} + T_1 = \frac{1}{2}\frac{(250^2 - 50^2)}{1005} + 400$$

$$= 429.85 \text{ K} = 156.85^\circ C$$

Specific volume of air at exit is given by

$$v_2 = \frac{RT_2}{P_2} = \frac{287 \times 429.85}{150 \times 10^3} = 0.82245 \text{ m}^3/\text{kg}$$

Therefore, the exit area of the diffuser is given by

$$A_2 = \frac{\dot{m} v_2}{\overline{V_2}} = \frac{1.5 \times 0.8224}{50} = 0.024672 \text{ m}^2 = 246.72 \text{ cm}^2$$

33. **Air enters a nozzle steadily at 300 kPa, 127° C and with a velocity of 40 m/s and leaves at 100 kPa and with a velocity of 300 m/s. The heat loss from the nozzle surface is 20 kJ/kg of the air. The inlet area of the nozzle is 100 cm². Determine:**

(a) the exit temperature of the air, and

(b) the exit area of the nozzle.

Solution:

Given, Properties of air at inlet: $P_1 = 300$ kPa, $T_1 = 127^0 C = 127 + 273 = 400$ K

$\overline{V_1} = 40$ m/s

Properties of air at exit: $P_2 = 100$ kPa, $\overline{V_2} = 300$ m/s

Heat loss per unit mass of air from the nozzle surface $(q_{cv}) = -20$ kJ/kg

Inlet area $(A_1) = 100\ cm^2 = 100 \times 10^{-4}\ m^2$

Specific volume of air at inlet is given by

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 400}{300 \times 10^3} = 0.38267\ m^3/kg$$

Therefore, mass flow rate of are is given by

$$\dot{m} = \frac{A_1 \overline{V_1}}{v_1} = \frac{100 \times 10^{-4} \times 40}{0.38267} = 1.0453\ kg/s$$

Heat loss rate from the nozzle surface $(\dot{Q}_{CV}) = \dot{m}\, q_{cv} = 1.0453 \times (-20) = -20.906$ kW

Applying energy equation for a nozzle,

$$\dot{Q}_{CV} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2)\right]$$

For an ideal gas using $h_2 - h_1 = c_p (T_2 - T_1)$

$$\dot{Q}_{CV} = \dot{m}\left[(c_p (T_2 - T_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2)\right]$$

or, $-20.906 \times 10^3 = 0.0453\,[1005\,(T_2 - 400) + \frac{1}{2}(300^2 - 40^2)]$

$\therefore T_2 = 336.119\ K = 63.119^0 C$

Specific volume of air at exit is given by

$$v_2 = \frac{RT_2}{P_2} = \frac{287 \times 336.119}{100 \times 10^3} = 0.96446\ m^3/kg$$

Therefore, the exit area of nozzle is given as

$$A_2 = \frac{\dot{m}\, v_2}{\overline{V_2}} = \frac{1.0453 \times 0.96466}{300} = 0.0033612\ cm^2 = 33.612\ m^2$$

34. **Steam enters an adiabatic nozzle with $P_1 = 2.5$ MPa, $T_1 = 250^0$ C and with very low velocity. The steam exits the nozzle with $P_2 = 0.8$ MPa and velocity of 65 m/s. the mass flow rate of steam is 2 kg/s. Determine the exit area of the nozzle.**

Solution:

Given, Mass flow rate of steam $(\dot{m}) = 2$ kg/s

Properties of steam at inlet: $P_1 = 2.5$ MPa = 2500 kPa, $T_1 = 250^0$ C, $\overline{V_1} = 0$ (low velocity)

Properties of steam of exit: $P_2 = 0.8$ MPa = 800 kPa, $\overline{V_2} = 65$ m/s

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (2500 kPa) = 223.99⁰ C. Here $T > T_{sat}$, hence it is a superheated vapor. Now, referring to the Table A2.4,

$v_1 = 0.08698\ m^3/kg$, $h_1 = 2879.1$ kJ/kg

Now, applying energy equation for an adiabatic nozzle,

$$h_2 + \frac{1}{2}\overline{V_2}^2 = h_1 + \frac{1}{2}\overline{V_1}^2$$

$$\therefore h_2 = h_1 + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) = 2879.1 + \frac{1}{2000}(0 - 65^2) = 2876.9875\ kJ/kg$$

Referring to the Table A2.1, h_g (800 kPa) = 2768.9 kJ/kg. Here, $h > h_g$, hence it is a superheated vapor. Hence, referring to the Table A2.4, specific enthalpy of steam which includes the specific enthalpy 2876.9875 kJ/kg and corresponding specific volume are listed as:

v_g, m³/kg	h_g, kJ/kg	
0.2607	2838.8	a
0.2931	2949.3	b

Applying linear interpolation for specific volume,

$$v_2 - (v_g)_a = \frac{(v_g)_b - (v_g)_a}{(h_g)_b - (h_g)_a}[h_2 - (h_g)_a]$$

$$\therefore v = (v_g)_a + \frac{(v_g)_b - (v_g)_a}{(h_g)_b - (h_g)_a}[h_2 - (h_g)_a]$$

$$= 0.2607 + \frac{0.2931 - 0.2607}{2949.3 - 2838.8}(2876.9875 - 2838.8) = 0.27188 \text{ m}^3/\text{kg}$$

Therefore, the exit area of the nozzle is given by

$$A_2 = \frac{\dot{m} v_2}{\overline{V_2}} = \frac{2 \times 0.27188}{65} = 0.008365 \text{ m}^2 = 8.366 \times 10^{-3} \text{ m}^2$$

35. Steam at 4 MPa, 450° C enters a nozzle operating at steady state with velocity of 50 m/s. Steam leaves the nozzle at 2 MPa and 300° C. The inlet area of the nozzle is 80 cm² and heat loss from the nozzle surface occurs at the rate of 100 kW. Determine:

(a) the mass flow rate of steam,

(b) the exit velocity of the steam, and

(c) the exit area of the nozzle. *(IOE, 2070 Magh)*

Solution:

Given, Properties of steam at inlet: $P_1 = 4$ MPa = 4000 kPa, $T_1 = 450^0$C, $\overline{V_1}$ = 50 m/s

Properties of steam at outlet: $P_2 = 2$ MPa = 2000 kPa, $T_2 = 300^0$C

Inlet area $(A_1) = 80 \text{ cm}^2 = 80 \times 10^{-4} \text{ m}^2$

Heat loss rate from the nozzle surface ($\dot{Q}_{CV}$) = - 100 kW

For other properties of steam an inlet, referring to the Table A2.1, T_{sat} (4000 kPa) = 250.39° C. Here, T > T_{sat}, hence it is a superheated vapor. Now referring to the Table A2.4,

$v_1 = 0.08002 \text{ m}^3/\text{kg}$, $h_1 = 3330.4$ kJ/kg

Therefore, mass flow rate of steam is given by

$$\dot{m} = \frac{A_1 \overline{V_1}}{v_1} = \frac{80 \times 10^{-4} \times 50}{0.08002} = 4.99875 \text{ kg}$$

For other properties of steam at exit, referring to the Table A2.1,

T_{sat} (2000 kPa) = 212.42° C. Here T > T_{sat}, hence it is a superheated vapor. Now referring to the Table A2.4,

$v_2 = 0.1254 \text{ m}^3/\text{kg}$, $h_2 = 3022.7$ kJ/kg

Applying energy equation for a nozzle,

$$\dot{Q}_{CV} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}\left(\overline{V_2^2} - \overline{V_1^2}\right)\right]$$

$$\text{or, } -100 = 4.99875\left[(3022.7 - 3330.4) + \frac{1}{2000}\left(\overline{V^2} - 50^2\right)\right]$$

$$\therefore \overline{V_2} = 760.191 \text{ m/s}$$

Therefore, the exit area of the nozzle is given by

$$A_2 = \frac{\dot{m} v_2}{\overline{V_2}} = \frac{4.99875 \times 0.1254}{760.191} = 0.00082459 \text{ m}^2 = 8.2459 \text{ cm}^2$$

36. Steam enters into a nozzle at P1 = 1000 kPa, T1 = 3000C and with a velocity of 75 m/s. Steam leaves the diffuser at P2 = 80 kPa, T2 = 2000C and with a velocity of 350 m/s. Determine the heat loss per unit mass of the steam from the nozzle surface.

Solution:

Given, Properties of steam at inlet: P_1 = 1000kPa, $T_1 = 300^0$ C, $\overline{V_1}$ = 75 m/s

Properties of steam at outlet: P_2 = 80 kPa, $T_2 = 200^0$ C, $\overline{V_2}$ = 350 m/s

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (1000kPa) = 179.92° C. Here, T > T_{sat}, hence it is a superheated vapor. Now, referring to the Table A2.4,

$v_1 = 0.2579 \text{m}^3/\text{kg}$, $h_1 = 3050.6$ kJ/kg

For other properties of steam at exit, referring to the table A2.1,

T_{sat} (80 kPa) = 93.511. Here, T > T_{sat} , hence it is a superheated vapor. Now, referring to the Table A2.4, specific enthalpy of a steam at 200° C for pressure 50 kPa and 100 kPa are listed.

P, kPa	h_g, kJ/kg	
50	2877.2	(a)
100	2874.8	(b)

Applying linear interpolation for specific enthalpy,

$$h_2 - (h_g)_a = \frac{(h_g)_b - (h_g)_a}{P_b - P_a}[P_2 - P_a]$$

$$\therefore h_2 = (h_g)_a + \frac{(h_g)_b - (h_g)_a}{P_b - P_a}[P_2 - P_a]$$

$$= 2877.2 + \frac{2874.8 - 2877.2}{100 - 50}(80 - 50)$$

$$= 2875.76 \text{ kJ/kg}$$

Now, applying energy equation for a nozzle,

$$\dot{Q}_{CV} = \dot{m}\ [(h_2 - h_1) + \frac{1}{2}(\overline{V_2^2} - \overline{V_1^2})]$$

$$\therefore \frac{\dot{Q}_{CV}}{\dot{m}} = (h_2 - h_1) + \frac{1}{2}(\overline{V_2^2} - \overline{V_1^2})$$

$$= (2875.76 - 3050.6) + \frac{1}{2000}(350^2 - 75^2)$$

$$= -116.403 \text{ kJ/kg}$$

37. A water heating arrangement operates at steady state with liquid entering at inlet 1 with P_1 = 500 kPa and T_1 = 50° C. Steam at P_2 = kPa and T_2 = 200° C enters at inlet 2. Saturated liquid water exits with pressure of P_3 = 500 kPa from the outlet 3. Determine the ratio of flow rates $\dot{m}_1/\dot{m}_2$.

Solution:

Given, Properties of water at inlets and outlet are given as

Properties at inlet 1: P_1 = 500 kPa, T_1 = 50° C

Properties at inlet 2: P_2 = 500 kPa, T_2 = 200° C

Properties at outlet 3: 500 kPa, saturated liquid

For other properties at inlet 1, referring to the Table A2. 1

T_{sat} (500 kPa) = 151.87° C. Here, T < T_{sat}, hence it is a compressed liquid. Now, referring to Table A2.2 (since 500 kPa is not available in Table A2.3), h_1 (500 kPa) = 209.33 kJ/kg

For other properties at inlet 2, referring to the Table A2.1, T_{sat} (500 kPa) = .87° C. Here T > T_{sat}, hence it is a superheated vapor. Now, referring to the Table A2.4, specific enthalpy of steam at 200° C for pressure 400 kPa and 600 kPa listed as:

P, kPa	h_g, kJ/kg	
400	2860.1	(a)
600	2849.7	(b)

Applying linear interpolation for specific enthalpy,

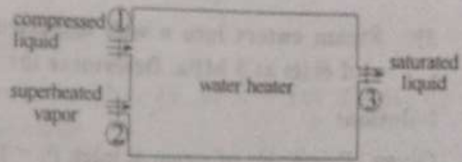
$$h_2 - (h_g)_a = \frac{(h_g)_b - (h_g)_a}{P_b - P_a}\ [P_2 - P_b)$$

$$\therefore h_2 = (h_g)_a + \frac{(h_g)_b - (h_g)_a}{P_b - P_a}\ (P_2 - P_a)$$

$$= 2860.1 + \frac{2849.7 - 2860.1}{600 - 400}\ (500\text{-}400) = 2854.9 \text{ kJ/kg}$$

For other properties at outlet 3, referring to the Table A2.1, h_f (500kPa) = 640.38 kJ/kg

Applying mass conservation and energy conservation for the device,



$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ - (i)

$\dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$ - (ii)

Substituting Equation (i) intoEquation (ii), we get

$(\dot{m}_1 + \dot{m}_2)\ h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$

or, $\dot{m}_1\ (h_3 - h_1) = \dot{m}_2\ (h_2 - h_3)$

$$\therefore \frac{\dot{m}_1}{\dot{m}_2} = \frac{h_2 - h_3}{h_3 - h_1} = \frac{2854.9 - 640.38}{640.38 - 209.33} = 5.1375$$

38. Warm atmospheric air containing water vapor enters the dehumidifier with an enthalpy of 90 kJ/kg at a rate of 210 kg/h. Heat is removed from the air as it passes over a bank of tubes through which cold water flows. Atmospheric moisture condenses on the tube drains from the dehumidifier with an enthalpy of 34 kJ/kg at a rate of 4 kg/h. Air leaving has an enthalpy of 23.8 kJ/kg. Velocities through the dehumidifier are quite low. Determine the rate of heat removal from the air stream through the dehumidifier.

Solution:

Given, Properties at inlet 1: h_1 = 90 kJ/kg, $\dot{m}_1$ = 210 kg/h

Properties at outlet 2: h_2 = 34 kJ/kg, $\dot{m}_2$ = 4 kg/h

Properties at outlet 3: h_3 = 23.8 kJ/kg,

Applying mass conservation equation for the device,

$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$

$\therefore \dot{m}_3 = \dot{m}_1 - \dot{m}_2$

= 210 − 4 = 206 kg/h

Applying steady state energy equation,

$\dot{Q}_{CV} = \dot{m}_{out}h_{out} - \dot{m}_{in}h_{in}$

$= (\dot{m}_2 h_2 + \dot{m}_3 h_3) - \dot{m}_1 h_1 = (4 \times 34 + 206 \times 23.8) - 210 \times 90$

$= -13861.2 \text{ kJ/h} = \frac{-13861.2}{3600} = -3.85 \text{ kW}$

39. Steam enters into a well insulated throttling valve at 10 MPa, 600° and exits at 5 MPa. Determine the final temperature of the steam.

Solution:

Given, Properties of steam at inlet: $P_1 = 10$ MPa = 10000 kPa, $T_1 = 600^0$C

Properties of steam at exit: $P_2 = 5$ MPa = 5000 kPa

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (10000 kPa) = 311.03° C. Here, $T > T_{sat}$, hence it is a superheated vapor. Now, referring to the Table A2.4,

$h_1 = 3624.7$ kJ/kg

Now, applying energy equation for throttling value,

$h_1 = h_2 = 3624.7$ kJ/kg

Then, referring to the Table A2.1, h_g (5000 kPa) = 2793.7 kJ/kg.

Here, $h > h_g$, hence it is a superheated vapor. Now referring to the Table A2... specific enthalpy of the steam which includes the specific enthalpy 3624.7 kJ/kg and corresponding temperature are listed as:

T, °C	h_g, kJ/kg	
550	3550.23	(a)
600	3666.2	(b)

Applying linear interpolation for temperature,

$$T_2 - T_b = \frac{T_a - T_b}{(h_g)_a - (h_g)_b}[h_2 - (h_g)_b]$$

$$\therefore T_2 = T_b + \frac{T_a - T_b}{(h_g)_a - (h_g)_b}[h_2 - (h_g)_b]$$

$$= 550 + \frac{600 - 550}{3666.2 - 3550.2}(3624.7 - 3550.2) = 582.112^0\text{C}$$

4.2 IOE Solutions

1. Air flows at the rate of 1.5 kg/s through a turbine, entering at 500 kPa, 150° C and with a velocity of 120 m/s and leaving at 100 kPa, 25° C and with a velocity of 60 m/s. Power produced by the turbine is 180 MW. Determine:

(a) Heat loss from the turbine and

(b) Diameters of inlet and exhaust pipe. [Take R = 287 J/kgK, C_p = 1005 J/kgK] ***(IOE 2070 Bhadra)***

Solution:

Given, Mass flow rate of air ($\dot{m}$) = 1.5 kg/s

Properties of air at inlet: $P_1 = 500$ kPa, $T_1 = 150^0$C, 150 + 273 = 423k, $\overline{V_1}$ = 120 m/s

Properties of air at exit: $P_2 = 100$ kPa, $T_2 = 25^0$C = 25 + 273 = 298 K, $\overline{V_2}$ = 60 m/s

Power produced by the turbine ($\dot{W}_{CV}$) = 180 MW = 180 × 10^3 kW

Now, applying steady state energy equation turbine,

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}[(h_2 - h_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2)]$$

For an ideal gas using $h_2 - h_1 = c_p(T_2 - T_1)$.

$$\dot{Q}_{CV} = \dot{W}_{CV} + \dot{m}[c_p(T_2 - T_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2)]$$

$$= 180 \times 10^3 + 1.5[1.005(298 - 423) + \frac{1}{2000}(60^2 - 120^2)]$$

$= 179.8$ MW

Specific volumes of air at inlet and outlet are given by

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 423}{500 \times 10^3} = 0.242802 \text{ m}^3/\text{kg}$$

$$v_2 = \frac{RT_2}{P_2} = \frac{287 \times 298}{100 \times 10^3} = 0.85526 \text{ m}^3/\text{kg}$$

Inlet area and exit area are given by

$$A_1 = \frac{\dot{m} v_1}{\overline{V_1}} = \frac{1.5 \times 0.242802}{120} = 0.00303503 \text{ m}^2$$

$$A_2 = \frac{\dot{m} v_2}{\overline{V_2}} = \frac{1.5 \times 0.85526}{60} = 0.0213815 \text{ m}^2$$

Then, inlet and exit diameters are given by

$$D_1 = 2\sqrt{\frac{A_1}{\pi}} = 2\sqrt{\frac{0.00303503}{\pi}} = 0.0622 \text{ m}$$

$$D_2 = 2\sqrt{\frac{A_2}{\pi}} = 2\sqrt{\frac{0.0213815}{\pi}} = 0.16499 \text{ m}$$

Note: Given data for this question, are not practicable since $\dot{Q}_{CV}$ is positive

2. A gas undergoes a thermodynamic cycle consisting of three processes:

Process 1-2: constant pressure, P = 1.4 bars, V_1 = 0.028 m³, W_{12} = 10.5 kJ

Process 2-3: compression with PV = constanU$_3$ = U$_2$,

Process 3-1: constant volume, $U_1 - U_3$ = -26.4 kJ

There are no significant changes in kinetic and potential energy.

(a) Sketch the cycle on a P-V diagram.

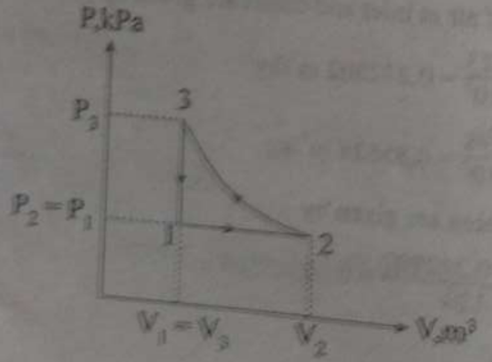
(b) Calculate the net work for the cycle, in kJ

(c) Calculate the heat transfer of process 1-2 in kJ

(d) Is this a power cycle of a refrigerator cycle? *(IOE 2069 Poush)*

Solution:

a) During constant pressure process 1 - 2, work transfer is positive hence it is a heating process. Therefore, its volume and temperature increases. During compression (PV = constant) process 2 - 3, temperature of the gas remains constant, volume decrease and pressure increases. During constant volume process 3 -1, there is decrease in internal in internal energy hence it is a cooling process.



b) Work transfer during process 1-2 is given as

$$W_{12} = P_1 (V_2 - V_1)$$

$$\therefore \; V_2 = \frac{W_{12}}{P_1} + V_1 = \frac{10.5 \times 10^3}{1.4 \times 10^5} + 0.028 = 0.1030 \text{ m}^3$$

Similarly, work transfer during process 2 - 3 is given as

$$W_{23} = P_2 V_2 \ln\left(\frac{V_3}{V_2}\right) = 100 \times 0.1030 \times \ln\left(\frac{0.028}{0.1030}\right) = -13.416 \text{ kJ}$$

Then, net work for the cycle is given by

$$W_{net} = W_{12} + W_{23} + W_{31} = 10.5 - 13.416 + 0 = -2.916 \text{kJ}$$

c) Heat transfer during process 1 - 2 is given as

$$Q_{12} = (\Delta U)_{12} + W_{12} = (U_2 - U_1) + W_{12} = (U_3 - U_1) + W_{12} = 26.4 + 10.5$$

$$= 36.9 \text{ kJ}$$

d) Since, net work is negative, given cycle is a refrigeration cycle.

3. Steam enters a turbine operating at a steady state with a mass flow rate of 46000 kg/h. The turbine develops a power output of 1000 kW. At the inlet, the pressure is 6000 kPa, the temperature is 4000° C, and the velocity is 10 m/s. At the exit, the pressure is 100 kPa, the quality is 0.9, and the velocity is 50 m/s. Calculate the rate of heat transfer between the turbine and surroundings in kW? *(IOE 2069 Ashad)*

Solution:

Given, Mass flow rate of steam ($\dot{m}$) = 4600 kg/h = $\frac{4600}{3600}$ = 1.278 kg/s

Properties of steam at inlet: P_1 = 6000 kPa, T_1 = 400°C, $\overline{V_1}$ = 10m/s

Properties of steam at exit: P_2 = 100 kPa, x_2 = 0.9, $\overline{V_2}$ = 20 m/s

Power output of the turbine ($\dot{W}_{CV}$) = 1000 kW

For the other properties of steam at the inlet of turbine, referring to the Table A2.1, T_{sat} (6000 kPa) = 275.62° C. Here, T>T_{sat}, hence it is a superheated steam. Now, referring to the Table A2.4, h_1 = 3177.0 kJ/kg

For other properties of steam at exit of the turbine, referring to the Table A2.1, h_f (100 kPa) = 417.51 kJ/kg, h_{fg} (100 kPa) = 2257.6 kJ/kg.

Therefore, specific enthalpy of steam at exit is given by

$$h_2 = h_f + x_2 h_{fg} = 417.51 + 0.9 \times 2257.6 = 2449.35 \text{ kJ/kg}$$

Now, applying steady state energy equation turbine,

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\,[(h_2 - h_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2)]$$

$$\therefore\ \dot{Q}_{CV} = \dot{W}_{CV} + \dot{m}\,[(h_2 - h_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2)]$$

$$= 1000 + 1.278\,[(2449.35 - 3177.0) + \frac{1}{2000}(50^2 - 10^2)]$$

$$= 71.597 \text{ kW}$$

Note: Given data for this question, are not practicable since $\dot{Q}_{CV}$ is positive

4. A piston cylinder device shown in figure below contains 2 kg of water initially at saturated liquid state 0f 1 MPa. There is heat transfer to the system until it hits the stops at which time its volume is 0.3 m³. There is further heat transfer to the device until water is completely vaporized. Sketch the process on P-v, T-v diagrams and determine total work and heat transfer. *(IOE 2068 Chaitra)*Solution:

Given, Mass of H_2O (m) = 2 kg

Initial state: P_1 = 1 MPa = 1000 kPa, saturated liquid

State 2: $V_2 = 0.3\ m^3$

State 3: Saturated vapor

Referring to the Table A2.1, v_f (1000 kPa) = 0.001127 m³/kg,

v_g (1000 kPa) = 0.1944 m³/kg, u_f (1000 kPa) = 761.75 kJ/kg.

Since state 1 is saturated liquid state, hence we can define state 1 as,

u_1 = 761.75 kJ/kg, v_1 = 0.001127 m³/kg

Specific volume at state 2 is given by

$$v_2 = \frac{V_2}{m} = \frac{0.3}{2} = 0.15\ m^3/kg$$

The system is heated until the piston hits the stops and the process occurs at constant pressure of 1000 kPa (Process 1 - 2).

Hence we can define state 2 as,

State 2: $v_2 = 0.15\ m^3$, P_2 1000 kPa

Here, $v_f < v_2 < v_g$, hence it is a two phase mixture.

But the system is heated until water is completely vaporised and the process occurs at constant volume (Process 2 - 3). Hence, we can define state 3 as,

State 3: v_3 = 0.15 m³/kg, saturated vapor

Referring to the Table A2.1, specific volume of saturated vapor which includes the specific volume m³/kg and corresponding specific internal energy are listed as:

v_g, m³/kg	u_g, kJ/kg	
0.1512	2590.5	(a)
0.1408	2592.3	(b)

Applying linear interpolation for specific internal energy,

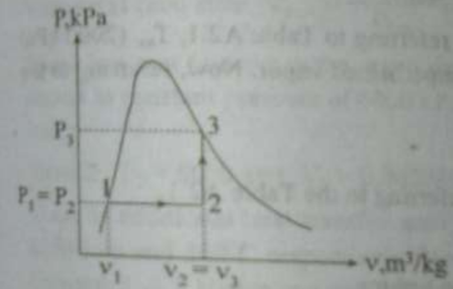
$$u_3 - (u_g)_a = \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

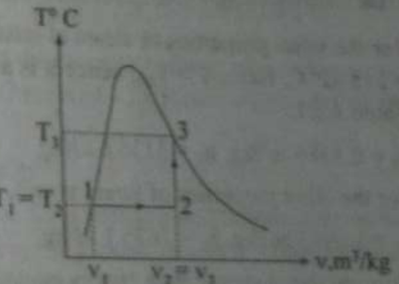
$$\therefore\ u_3 = (u_g)_a + \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_3 - (v_g)_a]$$

$$= 2590.5 + \frac{2592.3 - 2590.5}{0.1408 - 0.1512}(0.15 - 0.1512) = 2590.7077 \text{ kJ/kg}$$

Change in total internal energy is given by

$$\Delta U = m(u_3 - u_1) = 2\,(2590.5 - 761.75) = 3657.5 \text{ kJ}$$





∴ Total work transfer in the process is given by

$$W = W_{12} + W_{23} = P_1(V_2 - V_1) + 0 = mP_1(v_2 - v_1)$$

$$= 2 \times 1000 \times (0.15 - 0.001127) = 297.746 \text{ kJ}$$

And, Total heat transfer during the process is given by

$$Q = \Delta U + W = 3657.5 + 297.746 = 3955.246 \text{ kJ}$$

5. The mass rate of flow into a turbine is 1.5 kg/s and the heat transfer from the turbine is 8.5 kW. The following data are known for the steam entering and leaving the turbine.

	Inlet Conditions	Exit Conditions
Pressure	2.0 MPa	0.1 MPa
Temperature	350^0 C	
Quality		100%
Velocity	50 m/s	100m/s
Elevation above reference point	6 m	3 m

Determine the power output of the turbine and exit area of outlet pipe. ***(IOE 2068 Shrawan)***

Solution:

Given, Mass flow rate of steam ($\dot{m}$) = 1.5 kg/s

Properties of steam at inlet: P_1 = 2 MPa = 2000 kPa, $T_1 = 350°$ C, $\overline{V_1}$ = 50 m/s,

z_1 = 6 m

Properties of steam at exit: P_2 = 0.1 MPa = 100 kPa, saturated vapor, $\overline{V_2}$ = 100 m/s, z_2 = 3 m

Heat transfer from the turbine ($\dot{Q}_{CV}$) = -8.5 kW

For the other properties of steam at inlet, referring to Table A2.1, T_{sat} (2000 kPa) = 212.42° C. Here, T > T_{sat}, hence it is a superheated vapor. Now, referring to the Table A2.4,

$v_1 = 0.1386$ m³/kg, h_1 = 3136.6 kJ/kg

For the other properties of steam at exit, referring to the Table A2.1,

$v_2 = 1.6943$ m³/kg, h_2 = 2675.1 kJ/kg

Now, applying steady state energy equation turbine,

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2) + g(z_2 - z_1)\right]$$

$$\therefore \dot{W}_{CV} = \dot{Q}_{CV} + \dot{m}\left[(h_1 - h_2) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2) + g(z_1 - z_2)\right]$$

$$= -8.5 + 1.5\left[3136.6 - 2675.1) + \frac{1}{2000}(50^2 - 100^2) + 9.81(6\text{-}3)\right]$$

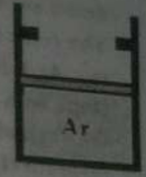
= 722.27 kW

Inlet area and exit area is given by

$$A_1 = \frac{\dot{m}\, v_1}{\overline{V_1}} = \frac{1.5 \times 0.1386}{50} = 0.21645 \text{ m}^2$$

$$A_2 = \frac{\dot{m}\, v_2}{\overline{V_2}} = \frac{1.5 \times 1.6943}{100} = 0.0254145 \text{ m}^2$$

6. **Argon (100 g) is in the piston-cylinder device shown in the figure below. The initial pressure is 6.0 MPa and temperature is 200^0 C. There is a heat transfer to the argon causing the piston to rise until it hits the stop. There is an additional heat transfer until the final pressure is 8.0 MPa and temperature is 800^0 C.** ***(IOE 2068 Bhadra)***



Solution:

Given, Mass of Argon (m) = 100 g = 0.1 kg

Initial state: P_1 = 6.0 MPa = 6000 kPa, T_1 = 200°C = 200 + 273 = 473 K

Final state: P_{final} = 8.0 MPa, = 8000 kPa, T_{final} = 800°C = 800 + 273 = 1073 K

Volume at initial state is given by

$$V_1 = \frac{mRT_1}{P_1} = \frac{0.1 \times 208 \times 473}{6000 \times 10^3} = 0.00164 \text{ m}^3$$

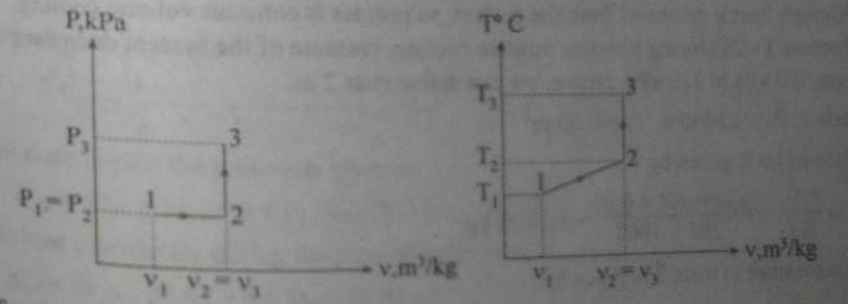
Volume at final state, $V_{final} = \frac{mRT_{final}}{P_{final}} = \frac{0.1 \times 208 \times 1073}{8000 \times 10^3} = 0.0027898 \text{ m}^3$

There is a heat transfer to the argon until the piston hits the stops and the process occurs at constant pressure of 6000 kPa (Process 1 - 2). Hence we can define state 2 as

State 2 : P_2 = 6000 kPa, V_2 = 0.0027898 m³

There is additional heat transfer until the final pressure and temperature become 8000 kPa and 800°C respectively. Hence, the process occurs at constant volume (Process 2 -3). Hence, we can define state 3 as

State 3: P_3 = 8000 kPa, T_3 = 1073 k, $V_3 = V_2$ =0.0027898 m³

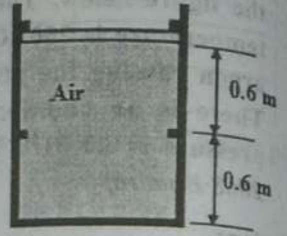


Total work done in the process is given by

$W = W_{12} + W_{23} = P_1 (V_2 - V_1) + 0 = 6000 \times (0.0027898 - 0.00164)$

$= 6.8988\ kJ = 6898.8\ J$

7. Air is contained in a vertical cylinder fitted with a frictionless piston and a set of stops as shown in figure below. The cross sectional area of the piston is 0.05 m^2. At initial condition, piston is in upper stops with pressure and temperature inside the cylinder as 0.3 MPa and 731^0 C respectively. Air is cooled as a result of heat transfer to the surroundings. The piston starts to move down at pressure 0.21 MPa. The cooling process continues until the temperature reaches 70^0 C.



(a) Draw P-V diagram for the process.

(b) Find the temperature of the air inside the cylinder when the piston reaches the lower stops.

(c) Calculate the heat transfer during the process. [For air R= 287 J/kgK, c_p= 1004 J/kgK, c_v= 717 J/kgK] ***(IOE 2068 Baishak)***

Solution:

Given, Cross sectional area of piston $(A_p) = 0.05\ m^2$

Initial state: $P_1 = 0.3\ MPa = 300\ kPa$, $T_1 = 731°C = 731 + 273 = 1004\ K$

Final state: $T_{final} = 70°C = 70 + 273 = 343\ K$

Volume at state 1 is given as

$V_1 = A_p \times x = 0.05 \times (0.6 + 0.6) = 0.06\ m^3$

Initial pressure of the system is 300 kPa but the piston starts to move down at pressure 210 kPa. Hence, during initial stage of cooling piston remains stationary although heat is removed from the system, so process is constant volume cooling. (Process 1 - 2) During constant volume cooling, pressure of the system decreases from 300 kPa to 210 kPa. Hence, we can define state 2 as,

State 2: $P_2 = 210\ kPa$, $V_2 = 0.06\ m^3$

Mass of air is given by

$$m = \frac{P_1V_1}{RT_1} = \frac{300 \times 10^3 \times 0.06}{287 \times 1004} = 0.0625\ kg$$

Temperature at state 2 is given by

$$T_2 = \frac{P_2V_2}{mR} = \frac{210 \times 10^3 \times 0.06}{0.0625 \times 287} = 702.44\ K = 429.44°C$$

But the required final temperature is 70° C, hence it should be further cooled to decrease temperature from 429.44° C to 70° C and the process occurs at constant pressure of 210 kPa (Process 2 - 3).Hence we can define state 3 as,

State 3: $P_3 = 210\ kPa$, $T_3 = 70°C$

Volume at state 3 is given

$$V_3 = \frac{mRT_3}{P_3} = \frac{0.0525 \times 287 \times 343}{210 \times 10^3} = 0.0292\ m^3$$

When the piston reaches the lower stops, volume is given by

$V = A_p \times 0.6 = 0.05 \times 0.6 = 0.03\ m^3$

Since, the volume at state 3 cannot become less than 0.03 m^3 Thus, $V_3 = 0.03\ m^3$

And, temperature of the air inside the cylinder when the piston reaches the lower stops is given by

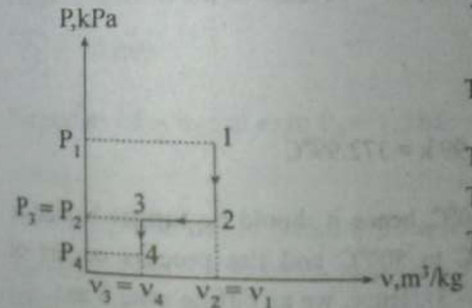
$$T_3 = \frac{P_3V_3}{mR} = \frac{210 \times 10^3 \times 0.03}{0.0625 \times 287} = 351.22\ k = 78.22°C$$

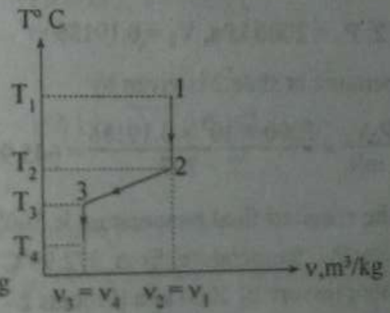
But the required final temperature is 70^0 C hence it is further cooled from temperature 78.22^0 C to 70^0 C and the process occurs at constant volume (Process 3-4). Hence we can define state 4 as,

State 4: $T_4 = 70^0 C$, $V_4 = 0.03$

Total change in internal energy during the process is given by

$\Delta U = mc_V (T_4 - T_1) = 0.0625 \times 717 \times (343 - 1004) = -29.62\ kJ$



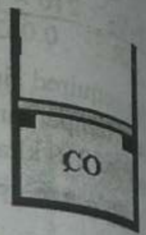


Work done during the process is given as

$W = W_{12} + W_{23} + W_{34} = 0 + P_2 (V_3 - V_2) + 0 = 210\ (0.03 - 0.06) = -6.3\ kJ$

Total heat transfer for during the process is given as

$Q = \Delta U + W = -29.62 + (-6.3) = -35.92\ kJ$



8. **Carbon monoxide (2 kg), contained in the piston-cylinder device as shown in the Figure P.10 is initially at a pressure of 1.0 MPa and a temperature of 50°C. Energy is added until the final temperature is 500°C and the pressure is 2.0 MPa. A pressure of 2.0 MPa is required to lift the frictionless piston from the stops. Show the process on P-V and T-V diagrams and determine the total work transfer and total heat transfer. [Take R= 297 J/kgK, c_v= 742 J/kgK]** ***(IOE 2068 Ashad)***

Solution:

Given, Mass of CO (m) = 2 kg

Initial state: P_1 = 1 MPa = 1000 kPa, T_1 = 50°C = 50 + 273 = 323 K

Final state: P_{final} = 2 MPa = 2000 kPa, T_{final} = 500°C = 500 + 273 = 773 K

Pressure required to lift the piston (P_{lift}) = 2 MPa = 2000 kPa

Initial volume of CO is given by

$$V_1 = \frac{mRT_1}{P_1} = \frac{2 \times 297 \times 323}{1000 \times 10^3} = 0.19186\ m^3$$

Initial pressure of the system is 1000 kPa and pressure required to lift the piston is 2000 kPa. Hence, during initial stage of heating piston remain stationary, although heat is supplied to the system, so process is constant volume heating (Process 1 -2). During constant volume heating, pressure of the system increase from 1000 kPa to 2000 kPa. Hence, we can define state 2 as

State 2: P_2 = 2000 kPa, V_2 = 0.19186 m³

Temperature at state 2 is given by

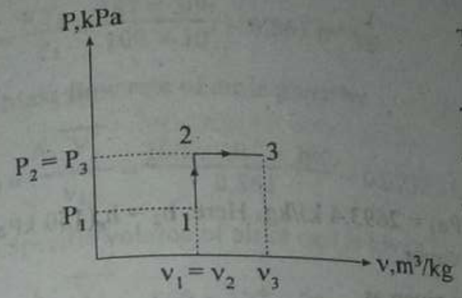
$$T_2 = \frac{P_2V_2}{mR} = \frac{2000 \times 10^3 \times 0.19186}{2 \times 297} = 645.99\ k = 372.99°C$$

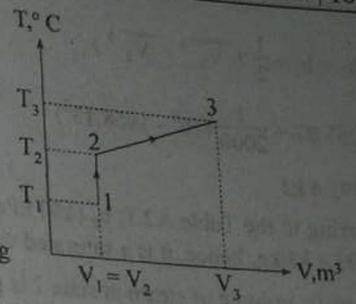
But, the required final temperature is 500°C, hence it should be further heated to increase the temperature from 372.99°C to 500°C and the process occurs at constant pressure of 2000 kPa (Process 2 - 3).Hence, we an define state 3 as,

State 3: P_3 = 2000 kPa, T_3 = 500°C

Volume at state 3 is given by

$$V_3 = \frac{mRT_3}{P_3} = \frac{2 \times 297 \times 773}{2000 \times 10^3} = 0.229581\ m^3$$





Total work transfer for the process is given by

$$W = W_{12} + W_{23} = 0 + P_2 (V_3 - V_2) = 200 \times (0.229581 - 0.19186) = 75.44\ kJ$$

Change in total internal energy is given as

$$\Delta U = m_{cv} (T_3 - T_1) = 2 \times 743 \times (773 - 323) = 668.7\ kJ$$

∴ Total heat transfer for the process is given by

$$Q = \Delta U + W = 668.7 + 75.44 = 744.14\ kJ$$

9. **Steam enters a nozzle operating at steady state with P_1 = 10 bar, T_1 = 400° C and velocity of 10 m/s. the steam flows through the horizontal adiabatic nozzle. At the exit, P_2 = 1.5 bar and the velocity of 1068. 13 m/s. The mass flow rate is 2 kg/s. Determine the exit area of the nozzle in m².**

Solution:

Given, Mass flow rate of steam ($\dot{m}$) = 2 kg/s

Properties of steam at inlet: P_1 = 10 bar = 1000 kPa, T_1 = 400°C

$\overline{V_1}$ = 10 m/s

Properties of steam at exit: P_2 = 1.5 bar = 150 kPa, $\overline{V_2}$ = 1068.13 m/s

$z_1 = z_2$

For other properties of steam at inlet, referring to the Table A2.1,

T_{sat} (1000 kPa) = 179.92°C. Here, T > T_{sat}, hence it is a superheated vapor. Now, referring to the Table A2.4,

h_1 = 3263.8 kJ/kg

Now applying energy equation for a horizontal adiabatic nozzle,

$$h_1 + \frac{1}{2}\overline{V_1}^2 = h_2 + \frac{1}{2}\overline{V_2}^2$$

$\therefore\ h_2 = h_1 + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2)$

$= 3263.87 + \frac{1}{2000}(10^2 - 1068.13^2)$

$= 2693.4$ kJ

Referring to the Table A2.1, h_g (150 kPa) = 2693.4 kJ/kg. Here, $h_2 = h_g$(150 k
= 2693.4 kJ/kg, hence, it is a saturated vapor.

$\therefore$ Specific volume of steam at state 2 is given as

$v_2 = v_g\ (150\ \text{kPa}) = 1.1593\ \text{m}^3/\text{kg}$

Now, exit area of the nozzle is given by

$$A_2 = \frac{\dot{m}\, v_2}{\overline{V_2}} = \frac{2 \times 1.1593}{1068.13} = 0.002171\ \text{m}^2$$

10. An adiabatic diffuser has air entering at 100 kPa, 300 K, with a veloc of 200 m/s. The inlet cross sectional area of the diffuser is 100 mm². A the exit, velocity is 20 m/s. Determine the exit temperature and pressur of the air. [Take C_p= 1005 J/kgK, R= 287 J/kgK] ***(IOE 2067 Mangsir)***

Solution:

Given, Properties of air at inlet: $P_1 = 100$ kPa, $T_1 = 300$ K, $\overline{V_1} = 200$ m/s

Properties of air at outlet: $\overline{V_2} = 20$ m/s

Inlet area $(A_1) = 100\ \text{mm}^2 = 100 \times 10^{-6}\ \text{m}^2$

Exit area $(A_2) = 860\ \text{mm}^2 = 860 \times 10^{-6}\ \text{m}^2$

Applying energy equation for an adiabatic diffuser,

$$h_2 - h_1 = \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2)$$

For an ideal gas using $h_2 - h_1 = c_p (T_2 - T_1)$

$$c_p (T_2 - T_1) = \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2)$$

$$\therefore\ T_2 = \frac{1}{2}\frac{(\overline{V_1}^2 - \overline{V_2}^2)}{c_p} + T_1$$

$$= \frac{1}{2}\frac{(200^2 - 20^2)}{1005} + 300 = 319.7\ \text{K}$$

Specific volume fo air at inlet is given by

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 300}{100 \times 10^3} = 0.861\ \text{m}^3/\text{kg}$$

$\therefore$ Mass flow rate of air is given by

$$\dot{m} = \frac{A_1\,\overline{V_1}}{v_1} = \frac{100 \times 10^{-6} \times 200}{0.861} = 0.02323\ \text{kg}$$

$\therefore$ Specific volume of air at exit is given by

$$v_2 = \frac{A_2\,\overline{V_2}}{\dot{m}} = \frac{860 \times 10^{-6} \times 20}{0.02323} = 0.74042\ \text{m}^3/\text{kg}$$

Hence, pressure of air at exit is given by

$$P_2 = \frac{RT_2}{v_2} = \frac{287 \times 319.7}{0.74042} = 123.92\ \text{kPa}$$

11. Consider the piston/ cylinder arrangement as shown below. When the piston rests on the lower stops, the enclosed volume is 400 L. When the piston reaches the upper stops, the volume is 600 L. The cylinder initially contains water at 100 kPa, 20% quality. It is heated until the water eventually exists as saturated vapor. It takes a pressure of 300 kPa to lift the piston. Sketch P-v and T-v diagrams and determine the work transfer and heat transfer for the overall process. ***(IOE 2069 Bhadra)***

Solution:

Given, Initial state: $P_1 = 100$ kPa, $x_1 = 20\% = 0.2$

$V_1 = 400\text{L} = 400 \times 40^{-3}\ \text{m}^3 = 0.4\ \text{m}^3$

Final state: saturated vapor

Pressure required to lift the piston $(P_{lift}) = 300$ kPa

Referring to the Table A2.1, v_f (100 kPa) = 0.001043 m³/kg,

v_{fg} (100 kPa) = 1.6933 m³/kg, u_f (100 kPa) = 417.41 kJ/kg

u_{fg} (100 kPa) = 2088.3 kJ/kg, T_{sat} (100 kPa) = 99.632°C

Specific volume and specific internal energy at state 1 are given as

$v_1 = v_f + x_1 v_{fg} = 0.001043 + 0.2 \times 1.6933 = 0.3397\ \text{m}^3/\text{kg}$

$u_1 = u_f + x_1 u_{fg} = 417.41 + 0.2 \times 2088.3 = 835.07\ \text{kJ/kg}$

Mass of H_2O is given by

$m = \frac{V_1}{v_1} = \frac{0.4}{0.3397} = 1.1775$ kg

Initial pressure of the system is 100 kPa and pressure required to lift the piston 300 kPa. Hence, during initial stage of heating piston remains stationary although heat is supplied to the system, so process is constant volume heating (Process 1 - 2). During constant volume heating, pressure of the system increases from 100 kPa to 300 kPa. Hence, we can define state 2 as

state 2: $P_2 = 300$ kPa, $v_2 = 0.3397$ m^3/kg

Referring to the Table A2.1, v_f (300 kPa) = 0.001073 m^3/kg,

v_g (300 kPa) = 0.6059 m^3/kg. Here $v_f < v < v_g$, hence it is a two phase mixture.

The maximum volume of cylinder when piston touches the upper stops V_{max} = 600 L = 600 × 10^{-3} m^3 = 0.6 m^3

Specific volume at state 3 is given as

$v_3 = \frac{V_3}{m} = \frac{0.6}{1.1775} = 0.5096$ m^3/kg

When the piston reaches the upper stops, its specific volume becomes 0.5096 m^3/kg. Hence it should be further heated to increases the specific volume from 0.3397 m^3/kg to 0.5096 m^3/kg and the process occurs at constant pressure of 300 kPa (Process 2 - 3). Hence we can define state 3 as,

State 3: $P_3 = 300$ kPa, $v_3 = 0.5096$ m^3/kg

Here, $v_f < v < v_g$, hence it is a two phase mixture.

But the final state is saturated vapor hence it should be further heated until it contains only saturated vapor and the process occurs at constant volume of 0.6 m^3 (Process 3 -4). Hence, we can define state 4 as,

State 4: $V_4 = 0.6$ m^3, $v_4 = 0.5096$ m^3/kg, saturated vapor.

Referring to Table A2.1, the specific volume of saturated vapor which includes the specific volume 0.5096 m^3/kg and corresponding specific internal energy is listed as:

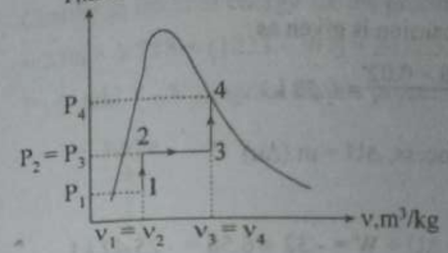
u_g, kJ/kg	v_g m^3/kg	
2548.9	0.5243	(a)
2551.3	0.4914	(b)

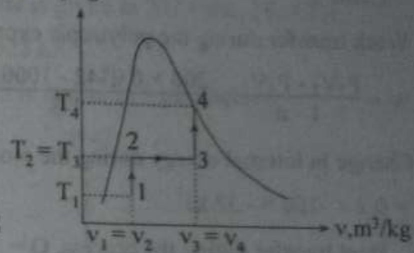
Applying linear interpolation for specific internal energy,

$$u_4 - (u_g)_a = \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_4 - (v_g)_a]$$

$$\therefore\ u_4 = (u_g)_a + \frac{(u_g)_b - (u_g)_a}{(v_g)_b - (v_g)_a}[v_4 - (v_g)_a]$$

$$= 2548.9 + \frac{2551.3 - 2548.9}{4.4914 - 0.5243}(0.5096 - 0.5243) = 2549.97 \text{ kJ/kg}$$





Total work transfer for the process is given by

$W = W_{12} + W_{23} + W_{34} = 0 + P_2(V_3 - V_2) + 0 = 300\ (0.6 - 0.4) = 60$ kJ

Change in total internal energy is given by

$\Delta U = m\ (u_4 - u_1) = 1.1775\ (2549.97 - 835.17) = 2019.295$ kJ

$\therefore$ Total heat transfer during the process is given by

$Q = \Delta U + W = 2019.295 + 60 = 2079.295$ kJ

4.3. Some Important Extra Questions

1. **A piston cylinder device contains 0.2 kg of a gas initially at P_1 = 1000 kPa and V_1 = 0.02 m^3. It undergoes polytropic expansion to a final pressure of 200 kPa during which the relation between pressure and volume is PV^3 = constant. If the specific internal energy of the gas decreases by 160 kJ/kg during the process, determine the heat transfer for the process.**

Solution:

Given, Mass of the gas (m) = 0.2 kg

Initial state: P_1 = 1000 kPa, V_1 = 0.02m^3

Final pressure: P_2 = 200 kPa

Process relation: PV^3 = constant

Change in specific internal energy (Δu) = -160 kJ/kg

$\therefore$ Volume of a gas at final state, $V_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{3}} V_1$

$= \left(\frac{1000}{200}\right)^{\frac{1}{3}} \times 0.02 = 0.0342\ m^3$

Work transfer during the polytropic expansion is given as

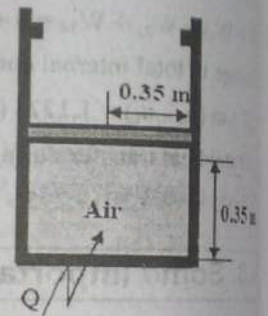
$$W = \frac{P_2V_2 - P_1V_1}{1 - n} = \frac{200 \times 0.0342 - 1000 \times 0.02}{1-3} = 6.58\ kJ$$

Change in internal energy during the process, $\Delta U = m\ (\Delta u)$

$= 0.2 \times -160 = -32\ kJ$

$\therefore$ Heat transfer during the process, $Q = \Delta U + W = -32 + 6.58 = -25.42\ kJ$

2. A piston cylinder device shown in figure below contains 3.06 kg of air initially at a temperature of 34° C. Heat is supplied to the system until it reaches to a final temperature of 950° C and a final pressure of 5 MPa. Sketch the process on P-V and T-V diagrams and determine the total work transfer and total heat transfer. [Take R = 287 J/kgK and c_v = 718 J/kgK]



Solution:

Given, Mass of air (m) = 3.06 kg

Initial state: $T_1 = 34°C = 34 + 273 = 307\ K$,

$V_1 = \pi\ (0.35)^2 \times 0.35 = 0.1347\ m^3$

Final state: $P_{final} = 5\ MPa = 5000\ kPa$, $T_{final} = 950 + 273 = 1223\ K$

Pressure of the at the initial state, $P_1 = \frac{mRT_1}{V_1} = \frac{3.06 \times 287 \times 307}{0.1347} = 2\ MPa$

Volume of air at the final state, $V_{final} = \frac{mR\ T_{final}}{P_{final}}$

$= \frac{3.06 \times 287 \times 1223}{5 \times 10^6} = 0.21481\ m^3$

Initial pressure of the system is 2 MPa and when heat is supplied to the system piston moves upward at constant pressure till it touches the stops (Process 1-2). The final required pressure is 5 MPa it should be further heated to increase the pressure from 2 MPa to 5 MPa and the process occurs at constant volume (Process 2-3). Hence, we can define state 2 and state 3 as

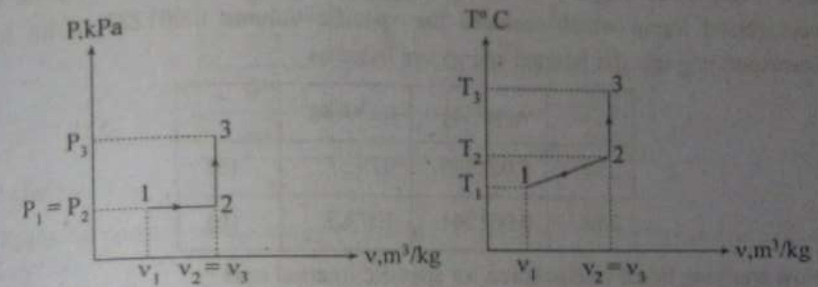
State 2: $P_2 = 2\ MPa = 2000\ kPa$, $V_2 = 0.21481\ m^3$

State 3: $P_3 = 5\ MPa = 5000\ kPa$, $V_3 = 0.21481\ m^3$, $T_3 = 950°C$

Change in internal energy for the process is given as $\Delta U = mc_v\ (T_3 - T_1)$

$= 3.06 \times 0.718 \times (1223 - 307) = 2012.525\ kJ$

P - V and T - V diagram for the process is shown in figure below.



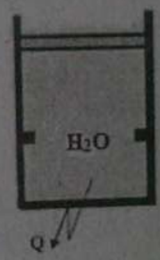
Total work transfer for the process is given as

$W = W_{12} + W_{23} = P_1\ (V_2 - V_1) = 2000\ (0.21481 - 0.13475) = 160.12\ kJ$

Total heat transfer for the process is given as

$Q = \Delta U + W = 160.12 + 2012.525 = 2172.645\ kJ$

3. Water (2 kg) is contained in a piston cylinder device shown in figure below. The mass of the piston is such that the H_2O exists at a pressure of 10 MPa and a temperature of 8000°C. There is a heat transfer from the device until the piston just rests on stops at which time the volume inside the cylinder is 2.45 × 10^{-3} m^3. Sketch the process on P-v and T-v diagrams and determine the total work transfer and total heat transfer.



Solution:

Given, Mass of H_2O (m) = 2 kg

Initial state: $P_1 = 10\ MPa$, $T_1 = 800°C$

Final state: $V_2 = 2.45 \times 10^{-3}\ m^3$

Referring to Table A2.1, T_{sat} (10000 kPa) = 311.03°C. Here $T > T_{sat}$, hence it is a superheated vapor. Then, referring to Table A2.4, $v_1 = 0.04863 m^3/kg$, $u_1 = 3627.2$ kJ/kg

When heat is transferred from the system piston slowly drops downward as cooling process occurs at constant pressure until it touches the stops. Hence, we can define state 2 as

State 2: $P_2 = 10$ MPa, and $v_2 = \frac{V_2}{m} = \frac{2.45 \times 10^{-3}}{2}$

$= 0.001225\ m^3/kg$

Referring to Table A2.1, v_f (10000 kPa) = 0.001452 m^3/kg. Here, $v < v_f$, hence it is a compressed liquid. Then referring to Table A2.3, specific volume of compressed liquid which includes the specific volume 0.001223 m^3/kg and corresponding specific internal energy are listed as

T,°C	v_f, m^3/kg	u_f, kJ/kg	
230	0.001199	975.55	(a)
250	0.001241	1073.3	(b)

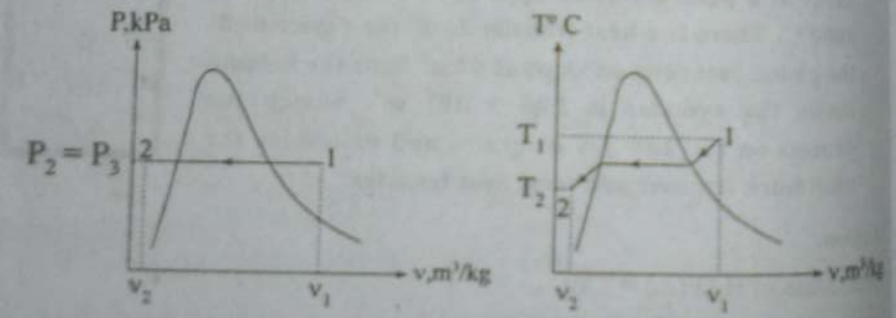
Now applying linear interpolation for specific internal energy,

$$u_2 - (u_f)_a = \frac{(u_f)_b - (u_f)_a}{(v_f)_b - (v_f)_a}\,[v_2 - (v_f)_a]$$

$$\therefore\ u_2 = (u_f)_a + \frac{(u_f)_b - (u_f)_a}{(v_f)_b - (v_f)_a}\,[v_2 - (v_f)_a]$$

$$= 975.55 + \frac{1073.3 - 975.55}{0.001241 - 0.001199}(0.001225 - 0.001199) = 1036.1\ kJ/kg$$

P - v and T - v diagrams for the process is shown in figure below.



Total work transfer for the process is given as

$W = P_1 (v_2 - v_1) = mP_1 (v_2 - v_1)$

$= 2 \times 10000 \times (0.001225 - 0.04863) = -948.1$ kJ/kg

Change in internal energy for the process is given as

$\Delta U = m (u_2 - u_1) = 2 (1036.1 - 3627.2) = -5182.2$ kJ

Total heat transfer for the process is given as

$Q = \Delta U + W = -948.1 - 5182.2 = -6130.3$ kJ

4. **A system undergoes a cycle consisting of four processes. Complete the missing table entries. (500, - 100, 300, 100, 200)**

Process	ΔU, kJ	W, kJ	Q, kJ
1-2	-500		0
2-3	0	-100	
3-4			400
4-1		300	500

Solution:

Applying control mass energy equation ($Q = \Delta U + W$) for process 1-2, 2-3, and 4-1

$W_{12} = Q_{12} - \Delta U_{12} = 0 + 500 = 500$ kJ

$Q_{23} = \Delta U_{23} + W_{23} = 0 - 100 = -100$ kJ

$\Delta U_{41} = Q_{41} - W_{41} = 500 - 300 = 200$ kJ

Now for the complete cycle

$\Sigma Q = \Sigma W$

or, $Q_{12} + Q_{23} + Q_{34} + Q_{41} = W_{12} + W_{23} + W_{34} + W_{41}$

or, $0 - 100 + 400 + 500 = 500 - 100 + W_{34} + 300$

$\therefore\ W_{34} = 100$ kJ

Applying control mass energy equation, we get

$\Delta U_{34} = Q_{34} - W_{34} = 400 - 100 = 300$ kJ

Alternatively,

Now for a complete cycle,

$(\Delta U)_{cycle} = 0$

or, $\Delta U_{12} + \Delta U_{23} + \Delta U_{34} + \Delta U_{41} = 0$

or, $-500 + 0 + (\Delta U)_{34} + 200 = 0$

$\therefore\ \Delta U_{34} = 300$ kJ

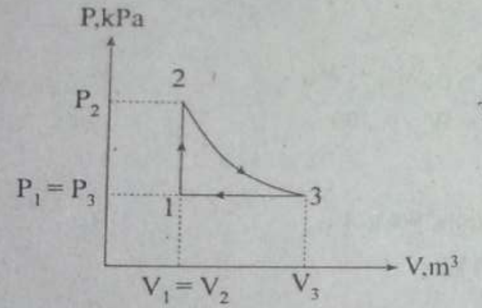
5. **A gas undergoes a thermodynamics cycle consistsing of the following three process.**

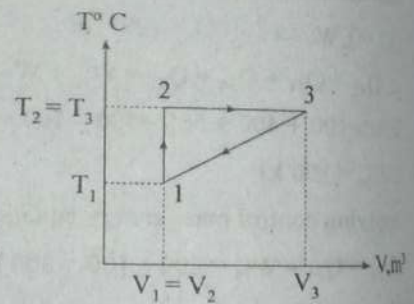
Process 1-2:	**Constant volume $V_1 = 0.08\ m^3$, $P_1 = 100$ kPa, $U_2 - U_1 = 140$ kJ**
Process 2-3:	**expansion with PV= constant, $U_3 = U_2$**
Process 3-1:	**constant pressure , $W_{31} = -56$ kJ**

(a) **Sketch the process on P-V and T-V diagram.**
(b) **Calculate net work for the cycle.**
(c) **Calculate net heat for the cycle.**
(d) **Calculate heat transfer for the process 2-3.**
(e) **Calculate the heat transfer for the process 3-1.**
(f) **Is this power cycle or a refrigerator cycle?**

Solution:

a) During constant volume process 1-2 , there is an increase in internal energy, hence it is a heating process during which pressure and temperature of the gas increases. During expansion (PV = constant) process 2 -3, temperature of the gas remains constant, volume increases and pressure decrease. During constant pressure process 3 -1, work transfer is negative, hence it is a cooling process. Therefore, its volume and temperature decrease P - V and T - V diagrams for the cycle are as shown below.





b) Work transfer during process 1- 3 is given as

$W_{31} = P_1 (V_1 - V_3)$

or, $-56 = 100 (0.08 - V_3)$

$\therefore\ V_3 = 0.64\ m^3$

Similarly, work transfer during process 2 -3 is given as

$$W_{23} = P_2V_2 \ln\left(\frac{V_3}{V_2}\right) = P_3V_3 \ln\left(\frac{V_3}{V_2}\right) = 100 \times 0.64 \times \ln\left(\frac{0.64}{0.08}\right) = 133.084\ kJ$$

Then, net work for cycle is given by

$\sum W = W_{12} + W_{23} + W_{31} = 0 + 133.084 - 56 = 77.084$ kJ

c) For a cycle, net work transfer is equal to net heat transfer, therefore

$\sum Q = \sum W = 77.084$ kJ

d) Heat transfer for process 2 - 3 is given as

$Q_{23} = \Delta U_{23} + W_{23} = 0 + 133.084 = 133.084$ kJ

e) Net heat transfer for the cycle is given as

$\sum Q = Q_{12} + Q_{23} + Q_{31}$

$Q_{12} = \Delta U_{12} + W_{12} = 140 + 0 = 140$ kJ

Then, heat transfer for the process 3 -1 is given as

$Q_{31} = \sum Q - (Q_{12} + Q_{23}) = 77.084 - (140 + 133.084) = -196$ kJ

Alternatively,

For complete cycle, ΔU cycle = 0

or, $\Delta U_{12} + \Delta U_{23} + \Delta U_{31} = 0$

or, $140 + 0 + \Delta U_{31} = 0$

$\therefore\ \Delta U_{31} = -140$ kJ

Then, heat transfer for the process 3-1 is given as

$Q_{31} = \Delta U_{31} + W_{31} = -140 - 56 = -196$ kJ

f) Since net work is positive, given cycle is a power cycle.

6. **Air flows at a rate of 1.2 kg/s through a compressor, entering at 100 kPa, 25° C, with a velocity of 60 m/s and leaving at 500 kPa, 150° C, with a velocity of 120 m/s. Heat lost by the compressor to the surrounding is estimated to be 20 kJ/kg. Calculate the power required to drive the compressor and diameters of inlet and exhaust pipes. [Take R = 287 J/kgK and c_p = 1005 J/kgK]** ***(IOE 2070 Chaitra, 2070 Ashad, 2069 Chaitra)***

Solution:

Given, Mass flow rate of air ($\dot{m}$) = 1.2 kg/s

Properties of air at inlet: $P_1 = 100$ kPa, $T_1 = 25 + 273 = 298$ K, $\overline{V_1} = 60$ m/s

Properties of air at outlet: $P_2 = 500$ kPa, $T_2 = 150 + 273 = 423$ K. $\overline{V_2} = 120$ m/s

Heat lost per unit mass of air ($\dot{q}_{cv}$) = - 20kJ/kg

$\therefore$ Heat transfer rate, $\dot{Q}_{CV} = \dot{m}\ \dot{q}_{CV} = 1.2 \times -20 = -24$ kW

Now, applying steady state energy equation

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\ [(h_2 - h_1) + \frac{1}{2}(\overline{V_2^2} - \overline{V_1^2}) + g(z_2 - z_1)]$$

For an ideal gas using $h_2 - h_1 = c_p(T_2 - T_1)$ and neglecting P.E., we get

$$\therefore \dot{W}_{CV} = \dot{Q}_{CV} - \dot{m}\ [c_p(T_2 - T_1) + \frac{1}{2}(\overline{V_2^2} - \overline{V_1^2})]$$

$$= -24 - 1.2\ [1005\ (423 - 298) + \frac{1}{2}(120^2 - 60^2) \times 10^3]$$

$$= -181.23 \text{ kW}$$

Specific volumes of air at inlet and outlet are given by

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 298}{100 \times 10^3} = 0.85526 \text{ m}^3/\text{kg}$$

$$v_2 = \frac{RT_2}{P_2} = \frac{287 \times 423}{500 \times 10^3} = 0.242802 \text{ m}^3/\text{kg}$$

Inlet area and exit area are given by

$$A_1 = \frac{\dot{m} v_1}{\overline{V_1}} = \frac{1.2 \times 0.85526}{60} = 0.0171052 \text{ m}^2$$

$$A_2 = \frac{\dot{m} v_2}{\overline{V_2}} = \frac{1.2 \times 0.242802}{120} = 0.00242802 \text{ m}^2$$

Then, inlet and exist diameters are given by

$$A_1 = \frac{\pi D_1^2}{4}$$

$$\therefore D_1 = \sqrt{\frac{4A_1}{\pi}} = \sqrt{\frac{4 \times 0.0171052}{\pi}} = 0.1476 \text{ m}$$

$$\text{And, } D_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4 \times 0.00242802}{\pi}} = 0.0556 \text{ m}$$

7. **A steam turbine develops 60 MW of power output. Mass flow rate of steam is found to be 80 kg/s. Properties of steam at inlet and exit of the turbine are as follows:**

Property	Inlet	Exit
Pressure	8 MPa	0.4 MPa
Temperature	50^0 C	
Quality		80 %
Velocity	50 m/s	150 m/s
Elevation above the reference level	10 m	5 m

(a) **Determine the rate at which heat is lost from the turbine surface.**

(b) **Determine the inlet and outlet areas.**

Solution:

Given, Mass flow rate of steam ($\dot{m}$) = 80 kg/s

Properties of steam at inlet: $P_1 = 8000$ kPa, $T_1 = 500°$C, $\overline{V_1} = 50$ m/s, $z_1 = 10$ m

Properties of steam at outlet: $P_2 = 400$ kPa, $x_2 = 0.8$, $\overline{V_2} = 150$ m/s, $z_2 = 5$ m

Power output of the turbine ($\dot{W}_{CV}$) = 60 MW

For other properties of steam at inlet, referring to Table A2.4,

$v_1 = 0.04174$ m^3/kg, $h_1 = 3398.52$ kJ/kg

For other properties of steam at outlet, referring to Table A2.1, v_f (400 kPa) = 0.001084 m^3/kg, v_{fg} (400 kPa) = 0.4614m^3/kg, h_f (400 kPa) = 604.91 kJ/kg, h_{fg} (400 kPa) = 2133.6 kJ/ kg

Therefore specific volume and specific enthalpy of steam at exit are given by

$$v_2 = v_f + x_2 v_{fg} = 0.001084 + 0.8 \times 0.4614 = 0.370204 \text{ m}^3/\text{kg}$$

$$h_2 = h_f + x_2 h_{fg} = 604.91 + 0.8 \times 2133.6 = 2311.79 \text{ m}^3/\text{kg}$$

a) Now, applying steady state energy equation as

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\ [(h_2 - h_1) + \frac{1}{2}(\overline{V_2^2} - \overline{V_1^2}) + g(z_2 - z_1)]$$

$$\therefore \dot{Q}_{CV} = \dot{W}_{CV} + \dot{m}\ [(h_2 - h_1) + \frac{1}{2}(\overline{V_2^2} - \overline{V_1^2}) + g(z_2 - z_1)]$$

$$= 60000 + 80\ [(2311.79 - 3398.52) + \frac{1}{2000}(150^2 - 50^2) + \frac{9.81 \times (5 - 10)}{1000}]$$

$$= -26.14 \text{ MW}$$

b) Inlet area and exit area are given by

$$A_1 = \frac{\dot{m}\, v_1}{\overline{V_2}} = \frac{80 \times 0.04174}{50} = 0.066784\ m^2$$

$$A_2 = \frac{\dot{m}\, v_2}{\overline{V_2}} = \frac{80 \times 0.370204}{150} = 0.197442\ m^2$$

8. Steam at 0.4 MPa and 200° C enters into an adiabatic nozzle with a velocity of 50 m/s and leaves the nozzle at 0.1 MPa and with a velocity of 75 m/s. Determine

(a) the exit temperature of the steam.

(b) the ratio of inlet diameter to the exit diameter.

Solution:

Given, Properties of steam at inlet: $P_1 = 400$ kPa, $T_1 = 200°C$, $\overline{V_1} = 50$ m/s

Properties of steam at outlet: $P_2 = 100$ kPa, $\overline{V_2} = 75$ m/s

For other properties of steam at inlet, referring to Table A2.1, T_{sat} (400 kPa) = 143.64°C. Here, $T_1 > T_{sat}$, hence it is a superheated vapor. Then referring to Table A2.4,

$v_1 = 0.5342\ m^3/kg$, $h_1 = 2860.1$ kJ/kg

Now, applying energy equation for an adiabatic nozzle

$$h_1 + \frac{1}{2}\overline{V_1^2} = h_2 + \frac{1}{2}\overline{V_2^2}$$

$$\therefore\ h_2 = h_1 + \frac{1}{2}(\overline{V_1^2} - V_2^2) = 2860.1 + \frac{1}{2}(50^2 - 75^2) \times 10^{-3} = 2858.54\ kJ/kg$$

Referring to Table A2.1, h_g (100 kPa) = 2675.1kJ/kg. Here, $h_2 > h_g$, it is a superheated steam. Then referring to Table A2.4, specific enthalpy of superheated vapor which includes the specific enthalpy of 2858.54 kJ/kg and corresponding specific volume and temperature are listed as

T,°C	v_g, m³/kg	h_g, kJ/kg	
150	1.9364	2776.1	(a)
200	2.1723	2874.8	(b)

Now applying linear interpolation for temperature and specific volume

$$T_2 - T_a = \frac{T_b - T_a}{(h_g)_b - (h_g)_a}[(h_2 - (h_g)_a]$$

$$\therefore\ T_2 = T_a + \frac{T_b - T_a}{(h_g)_b - (h_g)_a}[(h_2 - (h_g)_a]$$

$$= 150 + \frac{200 - 150}{2874.8 - 2776.1}(2858.54 - 2776.1) = 191.76°C$$

$$\text{And, } v_2 = (v_g)_a + \frac{(v_g)_b - (v_g)_a}{h_b - h_a}(h_2 - h_a)$$

$$= 1.9364 + \frac{2.1723 - 1.9364}{2874.8 - 2776.1} \times (2858.54 - 2776.1) = 2.1334\ m^3/kg$$

Applying conservation of mass equation,

$$\dot{m} = \frac{A_1 \overline{V_1}}{v_1} = \frac{A_2 \overline{V_2}}{v_2}$$

Then, ratio of inlet and exit area is given by

$$\frac{A_1}{A_2} = \frac{\overline{V_2}\, v_1}{\overline{V_1}\, v_2} = \frac{75}{50} \times \frac{0.5342}{2.1334} = 0.3756$$

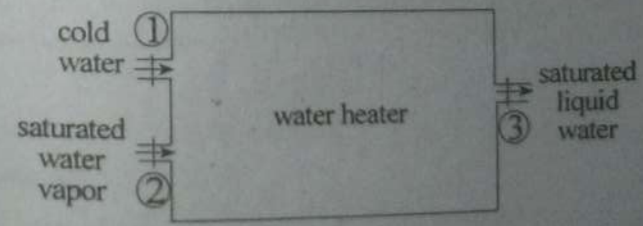
Therefore, ratio of inlet and exit diameter is given by

$$\frac{D_1}{D_2} = \sqrt{\frac{A_1}{A_2}} = \sqrt{0.3756} = 0.6129$$

9. In a water heat operating under steady state condition, water at 50° C flowing with a mass flow rate of 5 kg/s is mixed with the saturated vapor at 120° C. The mixture the heater as a saturated liquid water at 100° C. Determine the rate at which saturated water vapor must be supplied to the heater.

Solution:

Schematic diagram for the heating arrangement is shown in figure below.



Properties of water at inlets and outlet are given as

Properties at inlet 1: $\dot{m}_1 = 5$ kg/s, $h_1 = h_f\,(50°C) = 209.33$ kJ/kg

Properties at inlet 2: $h_2 = h_g\,(120°C) = 2706.2$ kJ/kg

Properties at outlet 3: $h_3 = h_f\,(100°C) = 419.06$ kJ/kg

Applying mass conservation and energy conservation for the device

$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ - (i)

$\dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$ - (ii)

Substituting Equation (i) into Equation (ii), we get

$(\dot{m}_1 + \dot{m}_2)\, h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2$

or, $\dot{m}_1 (h_3 - h_1) = \dot{m}_2 (h_2 - h_3)$

$$\therefore\ \dot{m}_2 = \dot{m}_1\left(\frac{h_3 - h_1}{h_2 - h_3}\right) = 5\left(\frac{419.06 - 209.33}{2706.2 - 419.06}\right) = 0.4585 \text{ kg/s}$$

Chapter 5 Second Law of Thermodynamics

5.1 Numerical Problems

1. **During an experiment a student claims that based on his measurements, a heat engine receives 300 kJ from a source at 500 K converts 160 kJ of it into work and rejects heat to the sink at 300 K. Are these data reasonable?**

Solution:

Given, Higher temperature (T_H) = 500 K

Lower temperature (T_L) = 300 K

Heat input (Q_H) = 300 kJ

Work output (W) = 160 kJ

Maximum possible efficiency of the engine operating between the given temperatures limits is given by

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{500} = 40\%$$

Efficiency of the engine according to the students claim is given as

$$\eta_{student} = \frac{W}{Q_H} = \frac{160}{300} = 53.33\%$$

Here, $\eta_{student} > \eta_{rev}$, hence these data are not reasonable.

2. **A heat engine receives 400 kJ from a source at a temperature of 1000 K. It rejects 150 kJ of heat to sink at a temperature of 300 K. The engine produces 250 kJ of work output. Is this cycle is a reversible, irreversible or impossible?**

Solution:

Given, Higher temperature (T_H) = 1000 K

Lower temperature (T_L) = 300 K

Heat rejected (Q_L) = 150 kJ

Work output (W) = 250 kJ

Efficiency of the engine is given by

Heat input is given by

$Q_H = Q_L + W = 150 + 250 = 400$ kJ

Maximum possible efficiency of the engine operating between the given temperature limits is given by

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1000} = 70\%$$

$$\eta_{HE} = \frac{W}{Q_H} = \frac{250}{400} = 62.5\%$$

Since, $\eta_{HE} < \eta_{rev}$, hence the cycle is irreversible.

3. **An inventor claims that a heat pump can maintain a building at 20° C. The heat loss from the room occurs at a rate of 1000 kJ/min and the heat pump requires 1 kW of power input. Evaluate his claim.**

Solution:

Given, Higher temperature (T_H) = 20°C = 20 + 273 = 293 K

Lower temperature (T_L) = 0°C = 0 + 273 = 273 K

Power input (W) = 1 kW

Rate of heat loss from the room ($\dot{Q}_H$) = 1000 kJ/min = $\frac{1000}{60}$ = 16.67 kW

Maximum possible COP of the heat pump operating between the given temperature limits is given by

$$(COP)_{rev,\,HP} = \frac{T_H}{T_H - T_L} = \frac{293}{293 - 273} = 14.65$$

COP of heat pump according to the inventors claim is given as $(COP)_{inventor} = \frac{Q_H}{W} = \frac{16.67}{1} = 16.67$

Here $(COP)_{inventor} > (COP)_{rev,\,HP}$, hence the given statement is not valid.

4. **During an experiment conducted in a room at 27° C, a student measures that a refrigerator consumes 2 kW of power and removes 36000 kJ of heat from the desired space at -23° C. The running time for the refrigerator during the experiment was 30 min. Are these data reasonable? Why?**

Solution:

Given, Higher temperature (T_H) = 27°C = 27 + 273 = 300 K

Lower temperature (T_L) = - 23°C = - 23 + 273 = 250 K

Power input ($\dot{W}$) = 2 kW

Heat removed from the desired space (Q_L) = 36000 kJ

The running time for the refrigerator (t) = 30 min = 30 × 60 = 1800 s

Maximum possible COP of the refrigerator operating between the given temperature limits is given by

$$(COP)_{rev,\,R} = \frac{T_L}{T_H - T_L} = \frac{250}{273 - 250} = 10.87$$

Rate of heat removed from the desired space is given as

$$\dot{Q}_L = \frac{Q_L}{t} = \frac{36000}{1800} = 20 \text{ kW}$$

COP of the refrigerator according to the student data is given as

$$(COP)_{student} = \frac{\dot{Q}_L}{\dot{W}} = \frac{20}{2} = 10$$

Here, $(COP)_{student} > (COP)_{rev,\,R}$, hence these data are not reasonable

5. **A power cycle operating between two reservoirs receives Q_H from a high temperature source at T_H=1000 K and rejects energy Q_L to a low temperature sink at T_L=300 K. For each of the following cases, determine whether the cycle operates reversibly, irreversibly or is impossible.**

   (a) Q_H=800kJ, W=600 kJ

   (b) Q_H=800kJ, Q_L=240 kJ

   (c) W=960 kJ, Q_L=640 kJ

   (d) η=50%

Solution:

Given, Higher temperature (T_H) = 1000 K

Lower temperature (T_L) = 300 K

Maximum possible efficiency of the heat engine operating between the given temperature limits is given by

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1000} = 70\%$$

a) Q_H = 800 kJ, W = 600 kJ

Efficiency of the heat engine is given as

$$\eta = \frac{W}{Q_H} = \frac{600}{800} = 75\%$$

Here, $\eta > \eta_{rev}$, hence it is impossible

b) $Q_H = 800$ kJ $Q_L = 240$ kJ

Work output is given as

$W = Q_H - Q_L = 800 - 240 = 560$ kJ

Therefore, efficiency of the heat engine is given by

$$\eta = \frac{W}{Q_H} = \frac{560}{800} = 70\ \%$$

Here, $\eta < \eta_{rev}$, hence it operates irreversibly.

c) $W = 960$ kJ, $Q_L = 640$ kJ

Heat added in a cycle is given as

$Q_H = W + Q_L = 960 + 640 = 1600$ kJ

Therefore, efficiency of the cycle is given by

$$\eta = \frac{W}{Q_H} = \frac{960}{1000} = 60\%$$

Here, $\eta < \eta_{rev}$, hence it operates irreversibly

d) $\eta = 50\%$

Here, $\eta < \eta_{rev}$, hence it operates irreversibly

6. A heat pump cycle operating between two reservoirs takes energy fro[m] the source at $T_L = 270$ K and supplies Q_H to a room at $T_L = 300$ K. I[n] each of the following cases, determine whether the cycle operat[es] reversibly, irreversibly or impossible.

(a) $Q_H = 1000$kJ, $W = 200$ kJ
(b) $Q_H = 2000$kJ, $Q_L = 1800$ kJ
(c) $W = 200$ kJ, $Q_L = 2000$ kJ
(d) COP=8.6

Solution:

Given, Higher temperature $(T_H) = 300$ K

Lower temperature $(T_L) = 270$ K

Maximum possible COP of the heat pump operating between the giv[en] temperature limits is given by

$$(COP)_{rev,\ HP} = \frac{T_H}{T_H - T_L} = \frac{300}{300 - 270} = 10$$

a) $Q_H = 1000$ kJ, $W = 200$ kJ

COP of the cycle is given as

$$(COP)_{HP} = \frac{Q_H}{W} = \frac{1000}{200} = 5$$

Here, $(COP)_{HP} < (COP)_{rev,\ HP}$, hence it operates irreversibly.

b) $Q_H = 2000$ kJ, $Q_L = 1800$ kJ

COP of the cycle is given by

$$(COP)_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{2000}{2000 - 1800} = 10$$

Here, $(COP)_{HP} = (COP)_{rev,\ HP}$, hence it operates reversibly.

c) $W = 200$ kJ, $Q_L = 2000$ kJ

Heat input is given as

$Q_H = W + Q_L = 200 + 2000 = 2200$

Therefore, COP of the cycle is given by

$$(COP)_{HP} = \frac{Q_H}{W} = \frac{2200}{200} = 11$$

Here, $(COP)_{HP} > COP_{rev}$ HP, hence it is impossible

d) COP = 8.6

Here $(COP)_{HP} < (COP)_{rev,\ HP}$, hence it operates irreversibly.

7. Find the power output and heat rejection rate for a heat engine operating on a Carnot cycle which receives heat at a rate of 6 kW at 327° C and rejects heat to 27° C.

Solution:

Given, Higher temperature $(T_H) = 327°C = 327 + 273 = 600$ K

Lower temperature $(T_L) = 27°C = 27 + 273 = 300$ K

Rate of heat input $(\dot{Q}_H) = 6$ kW

Efficiency of the Carnot cycle is given by

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{600} = 0.5 = 50\%$$

Also, efficiency of the cycle is given by

$$\eta = \frac{\dot{W}}{\dot{Q}_H}$$

Therefore, power output of Carnot cycle is given as

$W = \eta Q_H = 0.5 \times 6 = 3$ kW

Rate of heat rejection is given as

$Q_L = \dot{Q}_H - \dot{W} = 6 - 3 = 3$ kW

8. **An ideal heat engine has the same efficiency for a source and sink at 800 K and 400 K respectively for source and sink at T_H and 800 K. Determine T_H, in K.**

Solution:

Given,

Case I: Higher temperature (T_H) = 800 K

Lower temperature (T_L) = 400 K

Efficiency of an ideal heat engine is given by

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{800} = 0.5 = 50\%$$

case II: Lower temperature (T_L) = 800 K

Efficiency (η) = 0.5

Efficiency of an ideal heat engine is given by

$$\eta = 1 - \frac{T_L}{T_H}$$

$$\text{or, } 0.5 = 1 - \frac{800}{T_H}$$

$$\therefore T_H = \frac{800}{0.5} = 1600 \text{ K}$$

9. **A heat engine operates between a high temperature source T_H and a low temperature sink at 300 K. The engine develops 60 kWof power and rejects heat to the sink at the rate of 72 MJ/h. Determine the minimum theoretical value for T_H and T_L.**

Solution:

Given, Lower temperature (T_L) = 900 K

Power output ($\dot{W}$) = 60 kW

Heat rejected to the sink ($\dot{Q}_L$) = 72 MJ/h = $\frac{72 \times 10^3}{60 \times 60}$ kW = 20 kW

Efficiency of a heat engine is given by

$$\eta = \frac{\dot{W}}{\dot{Q}_H} = \frac{\dot{W}}{\dot{W} + \dot{Q}_L} = \frac{60}{60 + 20} = 0.75 - 75\%$$

Also, efficiency of the cycle is given by

$$\eta = 1 - \frac{T_L}{T_H}$$

$$\text{or, } 0.75 = 1 - \frac{300}{T_H}$$

$$\therefore T_H = \frac{300}{0.25} = 1200 \text{ K}$$

10. **A heat engine takes heat at a rate of 1200 kW from a high temperature source at 600^0 C rejects heat to the ambient at 25^0 C. Power output from the engine is 700 kW. Determine the engine efficiency and the energy rejected to the ambient. Compare both of these for a Carnot engine operating between the same temperature limits.**

Solution:

Given, Higher temperature (T_H) = 600°C = 600 + 273 = 873 K

Lower temperature (T_L) = 25°C = 25 + 273 = 298 K

Power output ($\dot{W}$) = 700 kW

Rate of heat input ($\dot{Q}_H$) = 1200 kW

Efficiency of the heat engine is given by

$$\eta_{HE} = \frac{\dot{W}}{\dot{Q}_H} = \frac{700}{1200} = 58.33\%$$

Heat rejected to the ambient is given as

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 1200 - 700 = 500 \text{ kW}$$

Now, efficiency of the Carnot engine operating between the given temperature limits is given by

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{298}{873} = 65.86\%$$

Work output from a Carnot engine is given by

$$\dot{W} = \eta_{rev} \times \dot{Q}_H = 0.6586 \times 1200 = 790.32 \text{ kW}$$

Therefore, heat rejected to the ambient for a Carnot engine is given by

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 1200 - 790.32 = 409.68 \text{ kW}$$

11. **An engine burns 0.5 kg of a fuel at 1800 K and rejects energy at an average temperature of 600 K. If the calorific value of the fuel is 42000 kJ/kg, determine the amount of work output that the engine can provide.**

Solution:

Given, Mass of fuel (m_f) = 0.5 kg

Higher temperature (T_H) = 1800 K

Lower temperature (T_L) = 600 K

Calorific value of the fuel (CV) = 42000 kJ/kg

Maximum possible efficiency of the engine operating between the temperature limits is given by

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{600}{1800} = 0.6667 = 66.67\%$$

Heat supplied to the engine is given as

$$\dot{Q}_H = m_f \times CV = 0.5 \times 42000 = 21000 \text{ kJ}$$

Therefore work output from the engine is given by

$$\dot{W} = \eta_{rev} \times \dot{Q}_H = 0.6667 \times 21000 = 14007 \text{ kJ}$$

12. A car engine having a thermal efficiency of 40 % produces 40 kW of power output. Determine the fuel consumption rate in kg/h, if the calorific value of the fuel is 42000 kJ/kg.

Solution:

Given, Efficiency of an engine (η) = 40% = 0.4

Power output of the engine ($\dot{W}$) = 40 kW

Calorific value of the fuel (CV) = 42000 kJ/kg

Rate at which heat is supplied to the engine is given by

$$\dot{Q}_H = \frac{\dot{W}}{\eta} = \frac{40}{0.4} = 100 \text{ kW}$$

Therefore, fuel consumption rate is given as

$$\dot{m}_f = \frac{\dot{Q}_H}{CV} = \frac{100}{42000} = \frac{1}{420} \text{ kg/s} = \frac{1}{420} \times 3600 = 8.57 \text{ kg/h}$$

13. A car engine consumes fuel at a rate of 30 L/h and delivers 80 kW of power output. If the calorific value of the fuel is 42000 kJ/kg and density of 0.8 g/cm³, determine the efficiency of the engine.

Solution:

Given, Power output ($\dot{W}$) = 80 kW

Calorific value of the fuel (CV) = 42000 kJ/kg

Fuel consumption rate ($\dot{V}_f$) = 30 L/h = $\frac{30 \times 10^{-3}}{3600}$ m^3/s

Density of fuel (ρ_f) = 0.8 g/cm³ = $\frac{0.8 \times 10^{-3}}{10^{-6}} \times \frac{30 \times 10^{-3}}{3600} = 0.00667$ kg/s

Rate at which heat is supplied to the engine is given by

$$\dot{Q}_H = m_f \times CV = 0.00667 \times 42000 = 280.14 \text{ kW}$$

Therefore, efficiency of the engine is given as

$$\eta = \frac{\dot{W}}{\dot{Q}_H} = \frac{80}{280.14} = 28.56\%$$

14. A heat engine has a solar collector receiving 0.25 kW/m², and provide a high temperature source at 400 K. The heat engine rejects heat to the ambient at 30° C. If the required power output is 2 kW, what is the minimum size of the solar collector?

Solution:

Given, Higher temperature (T_H) = 400 K

Lower temperature (T_L) = 30°C = 30 + 273 = 303 K

Power output ($\dot{W}$) = 2 kW

Rate at which heat received by the engines solar collector per unit area ($\dot{q}_H$) = 0.25 kW/m²

Efficiency of the engine is given as

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{303}{400} = 0.2425$$

Then, rate at which heat is supplied to the engine is given as

$$\dot{Q}_H = \frac{\dot{W}}{\eta} = \frac{2}{0.2425} = 8.25 \text{ kW}$$

Therefore, the size of the collector is given by

$$A = \frac{\dot{Q}_H}{\dot{q}_H} = \frac{8.25}{0.25} = 32.98 \text{ m}^2$$

15. An ideal engine can develop 27 kW power output while rejecting 15 kJ of heat per cycle. The engine operates between T_H = 1200 K and T_L = 300 K. Determine the minimum theoretical number of cycles per minute. An idealengine has an efficiency of 25 %. If the sink

temperature is reduced by 100^0 C, its efficiency gets doubled, determ[ine] its source and sink temperatures.

Solution:

Given, Power output ($\dot{W}$) = 27 kW

Heat rejected by the engine (Q_2/ cycle) = 15 kJ per cycle

Higher temperature (T_H) = 1200 K

Lower temperature (T_L) = 300 K

Efficiency of the ideal engine is given as

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1200} = 0.75$$

Then, rate of heat added is given as

$$\dot{Q}_H = \frac{\dot{W}}{\eta_{rev}} = \frac{27}{0.75} = 36 \text{ kW}$$

Rate of heat rejected is given as

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = 36 - 27 = 9 \text{ kW} = 9 \text{ kJ/S} = 9 \times 60 \text{ kJ/min} = 540 \text{ kJ/min}$$

Then, minimum theoretical number of cycles per minute

$$= \frac{\dot{Q}_L}{Q_L/\text{cycle}} = \frac{540}{15} = 36$$

16. **An ideal engine has an efficiency of 25%. If the sink temperature is reduced by100^0 C, its efficiency gets doubled ,determine its source and sink temperatures.**

Solution:

Given, Efficiency of an ideal engine (η_{rev}) = 25% = 0.25

Let the source temperature and the sink temperature of the engine be T_H and T_L respectively. Then its efficiency is given by

$$\eta_1 = 1 - \frac{T_L}{T_H}$$

$$\text{or,}0.25 = 1 - \frac{T_L}{T_L}$$

$$\therefore T_L = 0.75\, T_H \text{ .... (i)}$$

When the sink temperature is reduced by 100°C (= 100 K), its efficiency gets doubled i.e.,

$$\eta_2 = 1 - \frac{T_L - 100}{T_H}$$

$$\text{or,}0.5 = 1 - \frac{T_L - 100}{T_H}$$

$$\text{or, } \frac{T_L - 100}{T_H} = 0.5 \text{ ......(ii)}$$

Substituting equation (i) into equation (ii) we get,

$$\frac{0.75\, T_H - 100}{T_H} = 0.5$$

$$\text{or,}0.75\, T_H - 100 = 0.5\, T_H$$

$$\text{or,}0.25\, T_H = 100$$

$$\therefore T_H = 400 \text{ K}$$

Substituting T_H into equation (i) we get,

$$T_L = 0.75 \times 400 = 300 \text{ K}$$

17. **The difference between source and sink temperatures of an ideal heat engine is 450^0 C. If the work output of the engine is 1.5 the heat rejected, determine its thermal efficiency, source temperature and sink temperature.**

Solution:

Given, let the source temperature and the sink temperature of engine is T_H and T_L respectively.

When the difference between source and sink temperatures of an ideal engine is 450°C (= 450 K) then its efficiency is given by,

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{T_L}{T_L + 450}$$

Again, when the work output of the engine is 1.5 times the heat rejected then, heat input is given as

$$Q_H = W + Q_L = 1.5\, Q_L + Q_L = 2.5\, Q_L$$

Therefore, efficiency of the engine is given by

$$\eta = \frac{W}{Q_H} = \frac{1.5\, Q_L}{2.5\, Q_L} = 0.6 = 60\%$$

Substituting η into equation (i), we get

$$0.6 = 1 - \frac{T_L}{T_L + 450}$$

or, $\frac{T_L}{T_L + 450} = 0.4$

or, $T_L = 0.4\, T_L + 180$

or, $0.6\, T_L = 180$

$\therefore T_L = 300$ K

Then, $T_H = T_L + 450 = 300 + 450 = 750$ K

18. A heat pump requires a power input of 2.5 kW and maintains the temperature of a room at 25° C which loses heat at a rate of 20 kW to the colder ambient. What is the coefficient of performance of the heat pump?

Solution:

Given, Power input ($\dot{W}$) = 2.5 kW

Higher temperature (T_H) = 25°C = 25 + 273 = 298 K

Rate of heat loss by the heat pump to the colder ambient ($\dot{Q}_H$) = 20 kW

COP of the heat pump is then given by

$$COP = \frac{\dot{Q}_H}{\dot{W}} = \frac{20}{2.5} = 8$$

19. A building is maintained at 23° C by a heat pump when the surrounding temperature drops to -7° C. What is the minimum power required to drive the heat pump?

Solution:

Given, Higher temperature (T_H) = 23°C = 23 + 273 = 296 K

Lower temperature (T_L) = - 7°C = - 7 + 273 = 266 K

Rate at which heat is lost from the building ($\dot{Q}_H$) = 30 kW

COP of the heat pump is then given by

$$COP = \frac{T_H}{T_H - T_L} = \frac{296}{296 - 266} = 9.867$$

Then, power required to drive the heat pump is given as

$$\dot{W} = \frac{\dot{Q}_H}{COP} = \frac{30}{9.867} = 3.041 \text{ kW}$$

20. A heat pump having a COP of 5 maintains a building at a temperature of 24° C by supplying heat at a rate of 72000 kJ/h when the

surroundings is at 0° C. The heat pump runs 12 hour in a day and the electricity costs Rs 10/kWh.

(a) Determine the actual and minimum theoretical cost per day.

(b) Compare the actual operating cost with the cost of direct electric resistance heating.

Solution:

Given, COP of heat pump (COP) = 5

Higher temperature (T_H) = 24°C = 24 + 273 = 297 K

Lower temperature (T_L) = 0°C = 0 + 273 = 273 K

Rate at which heat is supplied by heat pump ($\dot{Q}_H$) = 72000 kJ/hr

$$= \frac{72000}{3600} = 20 \text{ kJ/s} = 20 \text{ kW}$$

Power required to drive the heat pump is given

$$\dot{W} = \frac{\dot{Q}_H}{COP} = \frac{20}{5} = 4 \text{ kW}$$

Maximum COP of the heat pump operating between the temperature limits is given by

$$(COP)_{rev,\,HP} = \frac{T_H}{T_H - T_L} = \frac{297}{297 - 273} = \frac{297}{24} = 12.375$$

Therefore, theoretical power required to drive the heat pump is given as

$$\dot{W}_{th} = \frac{\dot{Q}_H}{(COP)_{rev\,HP}} = \frac{20}{12.375} = 1.6162 \text{ kW}$$

a) The actual cost per day is given by

$C_{actual} = \dot{W} \times 12 \times 10 = 4 \times 12 \times 10 =$ Rs. 480

The minimum theoretical cost per day is given as

$C_{th} = \dot{W}_{th} \times 12 \times 10 = 1.6162 \times 12\ 10 =$ Rs. 193.944

b) The cost of direct electric resistance heating is given as

$C_{direct} = \dot{Q}_H \times 12 \times 10 = 20 \times 12 \times 10 =$ Rs. 2400

21. A building is maintained at a temperature at 25° C by a heat pump having a coefficient of performance of 2.5. It loses heat at a rate of 1 kW per degree temperature difference between the inside and the outside. It

the outside temperature is -10° C, determine the power required to drive the heat pump.

Solution:

Given, Higher temperature (T_H) = 25°C = 25 + 273 = 298 K

COP of heat pump (COP) = 2.5

Lower temperature (T_L) = -10°C = - 10 + 273 = 263 K

Heating rate ($\dot{Q}_H$) = 1 × (T_H - T_L) = 298 - 263 = 35 kW

COP of the heat pump is given by

$$(COP)_{HP} = \frac{\dot{Q}_H}{\dot{W}}$$

Then, the power required to drive the heat pump is given as

$$\dot{W} = \frac{\dot{Q}_H}{(COP)_{HP}} = \frac{35}{2.5} = 14 \text{ kW}$$

22. A heat pump having a coefficient of 50 % of the theoretical maximum maintains a house at a temperature of 20° C. The heat leakage from the house occurs at a rate of 0.8 kW per degree temperature difference. For a maximum power input of 1.5 kW, determine the minimum surroundings temperature for which the heat pump will be sufficient?

Solution:

Given, Higher temperature (T_H) = 20°C = 20 + 273 = 293 K

Heating rate ($\dot{Q}_H$) = 0.8 × (T_H - T_L) kW

Power input ($\dot{W}$) = 1.5 kW

Theoretical maximum COP of the heat pump operating between the given temperature limits is given by

$$(COP)_{rev,HP} = \frac{T_L}{T_H - T_L}$$

COP of the heat pump is given by $(COP)_{HP} = \frac{\dot{Q}_H}{\dot{W}}$

According to the question, COP of the heat pump is 50% of the theoretical maximum COP of heat pump.

$(COP)_{HP}$ = 50% of $(COP)_{rev,\ HP}$

or, $\frac{\dot{Q}_H}{\dot{W}} = 0.5 \times \left(\frac{T_H}{T_H - T_L}\right)$

or, $\frac{0.8\,(T_H - T_L)}{1.5} = 0.5 \times \left(\frac{T_H}{T_H - T_L}\right)$

or, $\frac{0.8\,(293 - T_L)}{1.5} = 0.5 \times \left(\frac{293}{293 - T_L}\right)$

or, $293 - T_L = 0.9375 \left(\frac{293}{293 - T_L}\right)$

$\therefore T_L = 276.4263 \text{ K} = 3.4263°C$

23. A heat pump maintains a room at a temperature of 20° C when the surrounding is at 5° C. The rate of heat loss from the room is estimated to be 0.6 kW degree temperature difference between inside and outside. If the electricity costs Rs 10/kWh, determine the minimum theoretical cost per day.

Solution:

Given, Higher temperature (T_H) = 20°C = 20 + 273 = 293 K

Lower temperature (T_L) = 5°C = 5 + 273 = 278 K

Heating rate ($\dot{Q}_H$) = 0.6 × (T_H - T_L) = 0.6 (293 - 278) = 9 kW

Theoretical maximum COP of heat pump operating between the temperature limits is given by

$$(COP)_{rev,\ HP} = \frac{T_H}{T_H - T_L} = \frac{293}{293 - 278} = 19.533$$

Again, COP of heat pump is given by

$$(COP)_{HP} = \frac{\dot{Q}_H}{\dot{W}}$$

Therefore, theoretical power required to drive the pump is given as

$$\dot{W} = \frac{\dot{Q}_H}{(COP)_{HP}} = \frac{9}{19.533} = 0.461 \text{ kW}$$

Assuming heat pump runs 12 hr/day, minimum theoretical cost per day at a rate of Rs 10/kWh = $\dot{W} \times 12 \times 10$ = Rs 55.32

24. An air conditioning unit rejects 5 kW to the ambient surroundings and requires a power input of 1.2 kW. Determine the rate of cooling and the coefficient of performance.

Solution:

Given, Power input ($\dot{W}$) = 1.2 kW

Heat rejected rate ($\dot{Q}_H$) = 5kW

Rate of cooling is given by

$\dot{Q}_L = \dot{Q}_H - \dot{W} = 5 - 1.2 = 3.8$ kW

COP of the air containing unit is given by

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}} = \frac{3.8}{1.2} = 3.1667$$

25. A refrigerator operates in a room a 22^0 C t. Heat must be taken from the desired space at a rate of 2.5 kW to maintain its temperature at -20^0 C. What is the minimum power required to drive the refrigerator?

Solution:

Given, Higher Temperature (T_H) = 22°C = 22 + 273 = 295 K.

Lower temperature (T_L) = - 20°C = -20 + 273 = 253 K

Rate of heat taken out from the desired space ($\dot{Q}_L$) = 2.5 kW

COP of the refrigerator is given by

$$(COP)_R = \frac{T_L}{T_H - T_L} = \frac{253}{295 - 253} = 6.024$$

Also COP of the refrigerator is given as

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}}$$

Therefore, minimum power required to drive the refrigerator is given by

$$\dot{W} = \frac{\dot{Q}_L}{(COP)_R} = \frac{2.5}{6.024} = 0.415 \text{ kW}$$

26. A refrigerator takes heat from a desired space maintained at -5^0 C at a rate of 100 kJ/min and rejects heat to the surroundings at 20^0 C. If the coefficient of performance of the refrigerator is 50 % of that of a reversible refrigerator cycle operating between the same temperature limits. Determine the power required to drive the cycle.

Solution:

Given, Lower temperature (T_L) - 5°C = - 5 + 273 = 268 K

Rate of heat taken out from the desired space ($\dot{Q}_L$) = 100 kJ/min = $\frac{100}{60}$ kJ/s = 1.667 kW

Higher temperature (T_H) = 20°C = 20 + 273 = 293 K

COP of a reversible refrigerator cycle operating between the given temperature limits is given by

$$(COP)_{rev.R} = \frac{T_L}{T_H - T_L} = \frac{268}{293 - 268} = 10.72$$

According to the question, COP of the refrigerator is 50% of that of a reversible refrigerator cycle operating between the same temperature limits.

i.e. $(COP)_R$ = 50% of $(COP)_{rev.R} = 0.5 \times 10.72 = 5.36$

Then, COP of the refrigeration is given by

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}}$$

Therefore, the power required to drive the cycle is given as

$$\dot{W} = \frac{\dot{Q}_L}{(COP)_R} = \frac{1.667}{5.36} = 0.3109 \text{ kW}$$

27. An air conditioning unit having a COP of 4 maintains a hall at 20^0 C on a day when the outside temperature is 35^0 C. The thermal load consists of heat energy entering through the walls at a rate of 600 kJ/min and from the occupants, computers and lighting at a rate of 120 kJ/min. Determine the power required to drive the unit and compare it with the minimum theoretical power required.

Solution:

Given, COP of an air conditioning unit $(COP)_R = 4$

Higher temperature (T_H) = 35°C = 35 + 273 = 308 K

Lower temperature (T_L) = 20°C = 20 + 273 = 293 K

Rate at which heat is taken out from the desired space ($\dot{Q}_L$)

$= (600 + 120)$ kJ/min $= \frac{720}{60}$ kJ/s = 12kW

Maximum COP of an air conditioning unit (Working as refrigerator) operating between the temperature limits is given by

$$(COP)_{rev,R} = \frac{T_L}{T_H - T_L} = \frac{293}{308 - 293} = 19.53$$

COP of an air condition unit is given by $(COP)_R = \frac{\dot{Q}_L}{\dot{W}}$

Therefore, the power required to drive the unit is given as

$$\dot{W} = \frac{\dot{Q}_L}{(COP)_R} = \frac{12}{4} = 3 \text{ kW}$$

Again, the minimum theoretical power required is given by

$$\dot{W}_{min} = \frac{\dot{Q}_L}{(COP)_{rev,R}} = \frac{12}{19.53} = 0.6144 \text{ kW}$$

28. A refrigerator having a COP of 4 maintains the freezer compartment at -3° C by removing heat at a rate of 10800 kJ/h and rejects heat to the surroundings at 27° C.

(a) Determine the power input to the refrigerator and compare it with minimum theoretical power input.

(b) If the electricity costs Rs 10/kWh, determine the actual and minimum theoretical cost per day for effective operation of 12 h/day.

Solution:

Given, COP of a refrigerator $(COP)_R = 4$

Lower temperature $(T_L) = -3°C = -3 + 273 = 270$ K

Higher temperature $(T_H) = 27°C = 27 + 273 = 300$ K

Rate of which heat is removed from the freezer compartment

$$(\dot{Q}_L) = 10800 \text{ kJ/h} = \frac{10800}{3600} \text{ kJ/s} = 3 \text{ kW}$$

Maximum COP of a refrigerator operating between the temperature limits is given by

$$(COP)_{rev,R} = \frac{T_L}{T_H - T_L} = \frac{270}{300 - 270} = 9$$

Again, COP of a refrigeration is given by

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}}$$

a) Therefore, the power input to the refrigerator is given by

$$\dot{W} = \frac{\dot{W}}{(COP)_{rev R}} = \frac{3}{9} = 0.333 \text{ kW}$$

b) Cost = Rs. 10/ kWh

Time (T) = 12 h/day

Therefore, the total actual cost per day for effective operation of 12h/day is given by

$$C_{actual} = \text{cost} \times T \times \dot{W} = 10 \times 12 \times 0.75 = \text{Rs. } 90$$

And the total minimum theoretical cost per day is given by

$$C_{min} = \dot{W}_{min} \times \text{cost} \times T$$

$$= 0.333 \times 10 \times 12 = 39.6 \text{ kW}$$

29. An air conditioning unit having COP 50 % of the theoretical maximum maintains a house at a temperature of 20° C by cooling it against the surrounding temperature. The house gains energy at a rate of 0.8 kW per degree temperature difference. For a maximum work input of 1.8 kW, determine the maximum surrounding temperature for which it provides sufficient cooling.

Solution:

Given, Lower temperature $(T_L) = 20°C = 20 + 273 = 293$ K

Rate at which heat is removed from a house $(\dot{Q}_L) = 0.8 (T_H - T_L)$ kW

Power input $(\dot{W}) = 1.8$ kW

Theoretical maximum COP of an air conditioning unit operating between the temperature limits is given by

$$(COP)_{rev,R} = \frac{T_L}{T_H - T_L}$$

Again, COP of an air conditioning unit is given by

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}} = \frac{0.8 (T_H - T_L)}{1.8}$$

According to the question, COP of an air conditioning unit is 50% of the theoretical maximum COP of an air conditioning unit i.e.,

$$(COP)_R = 50\% \text{ of } (COP)_{rev,R}$$

$$\text{or, } \frac{0.8 (T_H - T_L)}{1.8} = 0.5 \times \left(\frac{T_H}{T_H - T_L}\right)$$

$$\text{or, } \frac{0.8 (T_H - 293)}{1.8} = 0.5 \left(\frac{293}{T_H - 293}\right)$$

$$\text{or, } (T_H - 293)^2 = \frac{0.5 \times 1.8 \times 293}{0.8} = 329.625$$

Solving, we get

$T_H = 293 + 18.156 = 311.156\ K = 38.156°C$

30. A heat pump heats a house in the winter and then reverses to cool it in the summer. The room temperature should be 22° C in the winter and 26° C in the summer. Heat transfer through the walls and ceilings is estimated to be 3000 kJ/h per degree temperature difference between the inside and outside.

(a) Determine the power required to run it in the winter when the outside temperature decrease to 0° C.

(b) If the unit is run by the same power as calculated in (a) throughout the year, determine the maximum outside summer temperature for which the house can be maintained at 26° C.

Solution:

Given,

a) In winter:

Given, Higher temperature $(T_H) = 22°C = 22 + 273 = 295\ K$

Lower temperature $(T_L) = 0°C = 0 + 273 = 273\ K$

Rate at which heat is supplied to the house $(\dot{Q}_H) = 3000 \times (T_H - T_L)$ kJ/h =

$\frac{3000}{3600}(295 - 273)\ kW = 18.33\ kW$

COP of the heat pump is given by

$$(COP)_{HP} = \frac{T_H}{T_H - T_L} = \frac{295}{295 - 273} = 13.41$$

Again, COP of the heat pump is given as

$$(COP)_{HP} = \frac{\dot{Q}_H}{\dot{W}}$$

Therefore, the power required to run a heat pump in the winter is given as

$$\dot{W} = \frac{\dot{Q}_H}{(COP)_{HP}} = \frac{18.33}{13.41} = 1.367\ kW$$

b) In summer:

Lower temperature $(T_L) = 26°C = 26 + 273 = 299\ K$

Power input $(\dot{W}) = 1.367\ kW$

Rate at which heat is removed from the house $(\dot{Q}_L) = 18.33\ kW$

COP of the refrigerator is given by

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}} = \frac{18.33}{1.367} = 13.41$$

Again, COP of the refrigerator is given as

$$(COP)_R = \frac{T_L}{T_H - T_L}$$

$$or,\ 13.47 = \frac{299}{T_H - 299}$$

$or, T_H - 299 = 22.197$

$\therefore T_H = 321.197\ K = 48.197\ °C$

31. An air conditioning unit with a power input of 1.5 kW has a COP of 3while working as a cooling unit in summer and 4 while working as heating unit in winter. It maintains a hall at 22° C year round, which exchanges heat at a rate of 0.8 kW per degree temperature difference with the surroundings. Determine the maximum and minimum outside temperature for which the unit is sufficient.

Solution:

Given, Power input $(\dot{W}) = 1.5\ kW$

When an air conditioning unit is working as a cooling unit (Refrigerator) in summer:

COP of an air conditioning unit $(COP)_R = 3$

Lower temperature $(T_L) = 22°C = 22 + 273 = 295\ K$

Rate at which heat is removed from a hall $(\dot{Q}_L) = 0.8\ (T_H - T_L)\ kW$

COP of an air conditioning unit is given by

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}}$$

$$or,\ 3 = \frac{0.8\ (T_H - T_L)}{1.5} = \frac{0.8\ (T_H - 295)}{1.5}$$

$or, T_H - 295 = 5.625$

$\therefore T_H = 300.625\ K = 27.625°C$

When an air conditioning unit is working as heating unit (heat pump) in winter:

COP of an air conditioning unit, $(COP)_{HP} = 4$

Higher temperature $(T_H) = 22°C = 22 + 273 = 295\ K$

Rate at which heat is supplied in a hall ($\dot{Q}_H$) = 0.8 (T_H - T_L) kW

Then, COP of an air conditioning unit is given as

$$(COP)_{HP} = \frac{\dot{Q}_H}{\dot{W}}$$

$$\text{or, } 4 = \frac{0.8\,(T_H - T_L)}{1.5} = \frac{0.8\,(295 - T_L)}{1.5}$$

or, $295 - T_L = 7.5$

$\therefore\ T_L = 287.5\text{ K} = 14.5°C$

32. A rigid vessel consists of 0.4 kg of hydrogen initially at 200 kPa and 27° C. Heat is transferred to the system from a reservoir at 600 K until its temperature reaches 450 K. Determine the heat transfer , the change in entropy of hydrogen and the amount of entropy produced.

Solution:

Given, Mass of hydrogen (m) = 0.4 kg

Initial state: P_1 = 200 kPa, T_1 = 27°C = 27 + 273 = 300 K

Temperature of the reservoir (T_i) = 600 K

R = 4.124 kJ/kgK, c_p = 14.307 kJ/kgK, c_v = 10.183 kJ/kgK

Final state: T_2 = 450 K, $V_2 = V_1$

Work transfer during the process is given as $W = W_{12} = 0$

Change in total internal energy is given as $\Delta U = mc_v\,(T_2 - T_1) = 0.4 \times 10.183\,(450 - 300) = 610.98$ kJ

Therefore, the heat transfer during the process is given as

$Q = \Delta U + W = 610.98 + 0 = 610.98$ kJ

Then, the change in entropy of hydrogen is given by

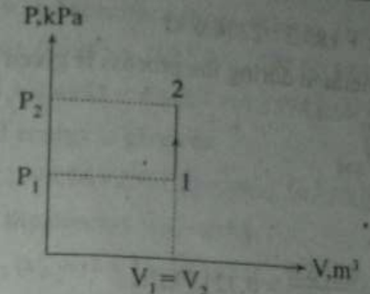
$$S_2 - S_1 = mc_v \ln\left(\frac{T_2}{T_1}\right) + mR\ln\left(\frac{V_2}{V_1}\right)$$

$$= 0.4 \times 10.183 \times \ln\left(\frac{450}{300}\right) + 0 = 1.6515\text{ kJ/K}$$

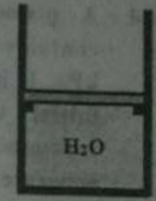
The amount of entropy produced is given by

$$S_{gen} = (dS)_{CM} - \Sigma\left(\frac{Q_i}{T_i}\right)_{CM}$$

$$= (S_2 - S_1) - \frac{Q}{T_i} = 1.6515 - \frac{610.98}{600} = 0.6332\text{ kJ/K}$$



33. A piston cylinder device as shown in figure below contains 2 kg of water at 2 MPa and 300° C. Heat is added from the source at 800° C to the water until its temperature reaches 800° C. Determine the total entropy generated during the process.



Solution:

Given, Mass of H_2O (m) = 2 kg

Initial state: P_1 = 2 MPa, T_1 = 300°C

Final state: $T_{final} = T_2$ = 800°C

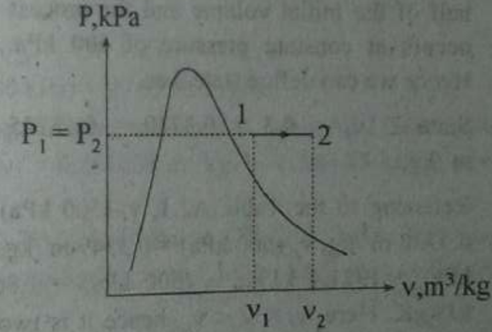
Temperature of the source (T_i) = 800°C = 800 + 273 = 1073 K

Referring to the Table A2.1, T_{sat} (2000 kPa) = 212.42°C . Here, T > T_{sat} (2000 kPa), hence it is a superheated steam. Now, referring to the Table A2.4, s_1 = 6.7651 kJ/kgK, v_1 = 0.1254 m³/kg, u_1 = 2771.8 kJ/kg

Heat is added until the temperature reaches to 800°C and the process occurs at constant pressure of 2000 kPa. Hence, we can define state 2 as,

State 2: P_2 = 2000 kPa, T_2 = 800°C.

Here, T > T_{sat}, hence it is a superheated vapor. Now, referring to the Table A2.4, s_2 = 8.1771 kJ/kgK, v_2 = 0.2467 kg/m³, u_2 = 3657.5 kJ/kg



Change in total internal energy is given as

$\Delta U = m\,(u_2 - u_1) = 2 \times (3657.5 - 2771.8) = 1771.4$ kJ

Total work transfer for the process is given by

$W = P_2\,(V_2 - V_1) = mP_2\,(v_2 - v_1) = 2 \times 2000\,(0.2467 - 0.1254) = 185.2$ kJ

Heat transfer during the process is given by

$Q_i = \Delta U + W = 1771.4 + 185.2 = 2256.6$ kJ

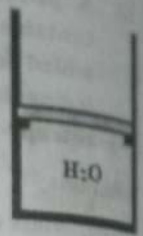
Then, total entropy generated during the process is given by

$$S_{gen} = (dS)_{CM} - \Sigma\left(\frac{Q_i}{T_i}\right)_{CM}$$

$$= m\,(s_2 - s_1) - \left(\frac{Q_i}{T_i}\right)$$

$$= 2\,(8.1771 - 6.7651) - \frac{2256.6}{1073} = 0.7211 \text{ kJ/K}$$

34. A piston cylinder device shown in figure below contains 1 kg of water at saturated vapor state 500 kPa. It is cooled so that volume reduces to half of the initial volume because of heat transfer to the surrounding at 20° C. Determine the total entropy generated during the process.



Solution:

Given, Mass of H_2O (m) = 1 kg

Initial state: P_1 = 500 kPa, Saturated vapor

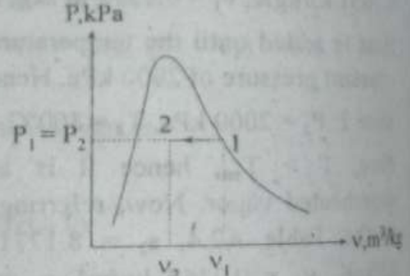
Final state: $V_2 = 0.5\ V_1$

Temperature of the surrounding (T_{sur}) = 20°C = 20 + 273 = 293 K

Referring to the Table A2.1, $v_1 = 0.3749$ m³/kg, $u_1 = 2561.2$ kJ/kg

$s_1 = 6.8214$ kJ/kgK

It is cooled until the volume reduces to half of the initial volume and the process occurs at constant pressure of 500 kPa. Hence we can define state 2 as,



State 2: $v_2 = 0.5 \times 0.3749 = 0.18745$ m³/kg,

Referring to the Table A2.1, v_f (500 kPa) = 0.001093 m³/kg, v_{fg} (500 kPa) = 0.3738 m³/kg, v_g (500 kPa) = 0.3749 m³/kg, u_f (500 kPa) = 639.84 kJ/kg, u_{fg} (500 kPa) = 1921.4 kJ/kg, s_f (500 kPa) = 1.8610 kJ/kgK, s_{fg} (500 kPa) = 4.9604 kJ/kgK. Here, $v_f < v < v_g$, hence it is two phase mixture. Quality at state 2 is given as

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.18745 - 0.001093}{0.3738} = 0.4985$$

Specific internal energy and specific entropy at state 2 are given as

$u_2 = u_f + x_2\,u_{fg} = 639.84 + 0.4985 \times 1921.4 = 1597.66$ kJ/kg

$s_2 = s_f + x_2\,s_{fg} = 1.8610 + 0.4985 \times 4.9604 = 4.3338$ kJ/kgk

Change in total internal energy is given as

$\Delta U = m\,(u_2 - u_1) = 1\times(1597.66 - 2561.2) = -963.54$ kJ/kg

Total work transfer for the process is given by

$W = P_2\,(V_2 - V_1) = mP_2\,(v_2 - v_1) = 1 \times (0.18745 - 0.3749) = -93.275$ kJ/kg

Heat transfer during the process is given by

$Q_i = \Delta U + W = -963.54 - 93.275 = -1057.625$kJ

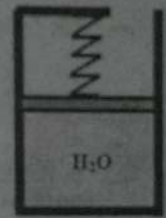
Then, total entropy generated during the process is given by

$$S_{gen} = (dS)_{CM} - \Sigma\left(\frac{Q_i}{T_i}\right)_{CM}$$

$$= m\,(s_2 - s_1) - \frac{Q_i}{T_i} = 1 \times (4.3338 - 6.8214) - \frac{(-1057.265)}{293}$$

$= 1.1208$ kJ/K

35. A piston cylinder device loaded with linear spring as shown in figure below contains 0.5 kg of water at 100 kPa and 25° C. Heat is transferred from source at 750° C until water reaches to a final state at 1000 kPa and 600° C. Determine the total entropy generated during the process.



Solution:

Given, Mass of H_2O (m) = 0.5 kg

Initial state: P_1 = 100 kPa, T_1 = 25°C

Final state: T_{final} = 600°C, P_{final} = 1000 kPa

Temperature of the source (T_i) = 750°C = 750 + 273 = 1023 K

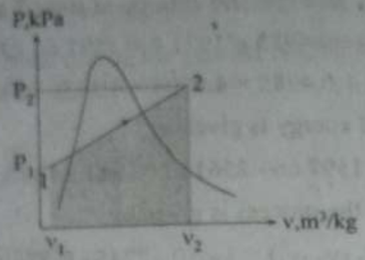
Referring to the Table A2.1, T_{sat} (100 kPa) = 99.632°C. Here, $T < T_{sat}$, hence it is a compressed liquid. Now, referring to Table A2.2 (since 100 kPa is not available in TableA2.3), $v_1 = 0.001003$ m³/kg, $u_1 = 104.75$ kJ/kg, $s_1 = 0.3670$ kJ/kg

Again, referring to the Table A2.1, T_{sat} (1000 kPa) = 179.92°C. Here, $T > T_{sat}$, hence it is a superheated vapor. Now, referring to the Table A2.4, we can define state 2 as,

State 2: $v_2 = 0.4011$ m³/kg, $u_2 = 3297.0$ kJ/kg, $s_2 = 8.0292$ kJ/kgK

Change in total internal energy is given by

$\Delta U = m\,(u_2 - u_1) = 0.5\,(3297.0 + 104.75) = 1700.875$ kJ



Total work transfer for the process is given by

$$W = W_{12} = \frac{1}{2}(P_1 + P_2)(V_2 - V_1) = \frac{1}{2} m (P_1 + P_2)(v_2 - v_1)$$

$$= \frac{1}{2} \times 0.5 \times (1000 + 100)(0.4011 - 0.001003) = 110.027 \text{ kJ}$$

Then, total heat transfer for the process is given by

$$Q = \Delta U + W = 1700.875 + 110.027 = 1810.902 \text{ kJ}$$

Therefore the total entropy generated during the process is given by

$$S_{gen} = (S_2 - S_1) - \Sigma\left(\frac{Q_i}{T_i}\right)_{CM} = m(s_2 - s_1) - \frac{Q}{T_i}$$

$$= 0.5(8.0292 - 0.3670) - \frac{1810.982}{1023} = 2.0609 \text{ kJ/K}$$

36. A piston cylinder device shown in figure below contains 1.5 kg of water initially at 100 kPa with 10 % of quality. The mass of the piston is such that a pressure of 500 kPa is required to lift the piston. Heat is added to the system from a source at 500° C until its temperature reaches 400° C. Determine the total entropy generation during the process.

Solution:

Given, Mass of H_2O (m) = 1.5 kg

Initial state: $P_1 = 100$ kPa, $x_1 = 0.1$

Final state: $T_{final} = 400°C$

Temperature of the source (T_i) = 500°C = 500 + 273 = 773 K

Pressure required to lift the piston (P_{lift}) = 500 kPa

Referring to the Table A2.1, v_f (100 kPa) = 0.001043 m³/kg, v_{fg} (100 kPa) = 1.6933 m³/kg, u_f (100 kPa) = 417.41 kJ/kg, u_{fg} (100 kPa) = 2088.3 kJ/kg, s_f (100 kPa) = 1.3027 kJ/kgK, s_{fg} (100 kPa) = 6.0562 kJ/kgK, T_{sat} (100 kPa) = 99.63°C

Therefore, specific volume, specific internal energy and specific entropy are given by

$$v_1 = v_f + x_1 v_{fg} = 0.001043 + 0.1 \times 1.6933 = 0.170373 \text{ m}^3/\text{kg}$$

$$u_1 = u_f + x_1 u_{fg} = 417.41 + 0.1 \times 2088.3 = 626.24 \text{ kJ/kg}$$

$$s_1 = s_f + x_1 s_{fg} = 1.3027 + 0.1 \times 6.0562 = 1.90832 \text{ kJ/kgK}$$

Initial pressure of the system is 100 kPa and pressure required to lift the piston is 500 kPa. Hence, during initial stage of heating piston remains stationary although heat is supplied to the system, so process is constant volume heating (Process 1-2). During constant volume heating, pressure of the system increases from 100 kPa to 500 kPa. Hence, we can define state 2 as

State 2: $P_2 = 500$ kPa, $v_2 = 0.170373$ m³/kg

Referring to the Table A2.1, v_f (500 kPa) = 0.001093 m³/kg

v_g (500 kPa) = 0.3749 m³/kg. Here $v_f < v < v_g$, hence it is two phase mixture.

∴ Temperature at state 2, $T_2 = T_{sat}$ (500 kPa) = 151.87°C.

But the final required temperature is 400°C, hence it should be further heated to increase the temperature from 151.87 °C to 400°C and the process occurs at constant pressure of 500 kPa (Process 2-3). Hence, we can define state 3 as,

State 3: $P_3 = 500$ kPa, $T_3 = 400°C$

Referring to the Table A 2.1, T_{sat} (500 kPa) = 151.87°C. Here, $T > T_{sat}$, hence it is a superheated vapor. Now, referring to the Table A2.4, pressure of a superheated vapor which includes pressure 500 kPa and the corresponding specific volume, specific internal energy and specific entropy are listed as:

P, kPa	v_g, m³/kg	u_g, m³/kg	s_g, m³/kgK	
400	0.7726	2964.3	7.8982	(a)
600	0.5137	2961.9	7.7076	(b)

Applying linear interpolation for specific volume, specific internal energy, and specific entropy

$$v_3 - (v_g)_a = \frac{(v_g)_b - (v_g)_a}{P_b - P_a}(P_3 - P_a)$$

$$\therefore v_3 = (v_g)_a + \frac{(v_g)_b - (v_g)_a}{P_b - P_a}(P_3 - P_a)$$

$$= 0.7726 + \frac{0.5137 - 0.7726}{600 - 400}(500 - 400) = 0.64315 \text{ m}^3/\text{kg}$$

$u_3 = (u_g)_a + \frac{(u_g)_b - (u_g)_a}{P_b - P_a}(P_3 - P_a)$

$= 2964.3 + \frac{2961.9 - 2964.3}{600 - 400}(500 - 400) = 2963.1$ kJ/kg

And, $s_3 = (s_g)_a + \frac{(s_g)_b - (s_g)_a}{P_b - P_a}(P_3 - P_a)$

$= 7.8982 + \frac{7.7076 - 7.8982}{600 - 400}(500 - 400) = 7.8029$ kJ/kgK

Change in total internal energy is given by

$\Delta U = m(u_3 - u_1) = 1.5\,(2963.1 - 626.24) = 3505.29$ kJ

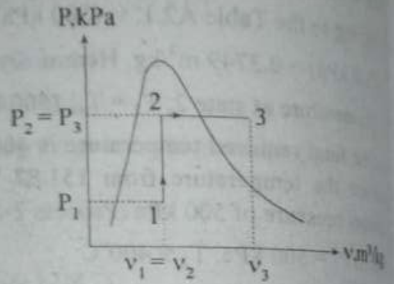
Work transfer during the process is given by

$W = W_{12} + W_{23} = 0 + P_2(V_3 - V_2)$

$= mP_2(v_3 - v_2)$

$= 1.5 \times 500 \times (0.64315 - 0.170373)$

$= 354.583$ kJ



Then, total work transfer is given by

$Q = \Delta U + W = 3505.29 + 354.583$

$= 3859.873$ kJ

Therefore, total entropy generated during the process is given by

$S_{gen} = (\Delta S)_{CM} - \Sigma\left(\frac{Q_i}{T_i}\right)_{CM}$

$= m(s_3 - s_2) - \frac{Q}{T_i} = 1.5 \times (7.8029 - 1.90832) - \frac{3859.873}{773} = 3.8485$ kJ/K

37. Water is contained in a piston cylinder device with two set of stops as shown in figure below is initially at 1 MPa and 400° C. The limiting volume are $V_{min} = 1\ m^3$ and $V_{max} = 2\ m^3$. Thw weight of the piston is such that a pressure of 400 kPa is required to support the piston. The system is cooled to 100° C by allowing system to reject heat to the surrounding at 25° C. Sketch the process on P-v and T-v diagrams and determine the total entropy generated during the process.

Solution:

Given, Minimum volume (V_{min}) = 1 m³

Maximum volume (V_{max}) = 2 m³

Initial state: P_1 = 1 MPa = 1000 kPa, T_1 = 400°C

Final state: T_{final} = 100°C

Temperature of the surrounding (T_i) = 25°C = 25 + 273 = 293 K

Pressure required to support the piston ($P_{support}$) = 400 kPa

Referring to the Table A2.4, T_{sat} (1000 kPa) = 179.92°C. Here $T > T_{sat}$, hence it is a superheated vapor. Then, referring to the table A2.4, $v_1 = 0.3066$ m³/kg, u_1 = 2957.2 kJ/kg, s_1 = 7.4648 kJ/kgK

Mass of H_2O is given as

$m = \frac{V_i}{v_1} = \frac{2}{0.3066} = 6.5232$ kg

Minimum specific volume of H_2O is given by

$v_{min} = \frac{V_{min}}{m} = \frac{2}{6.5232} = 0.1533$ m³/kg

Initial pressure of the system is 1000 kPa but the pressure required to support the piston is 400 kPa. Hence, during initial state of cooling piston remains stationary although heat is removed from the system, so the process is constant volume cooling (Process 1-2). During constant volume cooling, pressure of the system decreases from 1000 kPa to 4000 kPa. Hence we can define state 2 as,

State 2: P_2 = 400 kPa, v_2 = 0.3066 m³/kg

Referring to the Table A2.1, v_f (400 kPa) = 0.001084 m³/kg, v_g (400 kPa) = 0.4625 m³/kg. Here $v_f < v < v_g$, hence it is a two phase mixture.

∴ Temperature at state 2, $T_2 = T_{sat}$ (400 kPa) = 143.64 °C

But the final required temperature is 100°C, hence it should be further cooled to decrease the temperature and the process occurs at constant pressure of 400 kPa (Process 2-3). Hence we can define state 3 as,

State 3: P_3 = 400 kPa, v_3 = 0.1533 m³/kg

Here, $v_1 < v_3 < v_g$, hence it is a two phase mixture.

∴ Temperature at state 3, $T_3 = T_{sat}$ (400 kPa) = 143.64 °C

It is further cooled to decrease temperature from 143.6°C to 100°C and the process occurs at constant volume (Process 3-4).Hence, we can define state 4 as,

State 4: v_4 = 0.1533 m³/kg, T_4 = 100°C

Referring to the Table A,2.2, v_f (100°C) = 0.001043 m²/kg, v_g (100°C) = 1.6943 m³/kg, v_{fg} (100°C) = 1.6933 m³/kg, u_f (100°C) = 417.41 kJ/kg, u_{fg} (100°C) = 2088.3 kJ/kg, s_f (100°C) = 1.3027 kJ/kgK

s_{fg} (100°C) = 6.0562 kJ/kgK. Here, $v_f < v_4 < v_g$, hence, it is a two phase mixture

Quality at state 4 is given as

$$x_4 = \frac{v_4 - v_f}{v_{fg}} = \frac{0.1533 - 0.001043}{1.6933} = 0.0899$$

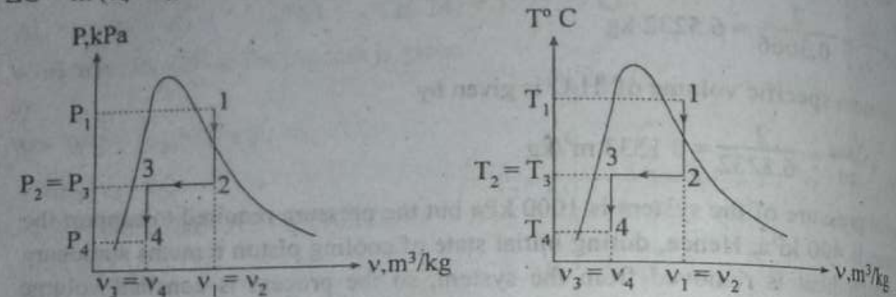
Then, specific internal energy and specific entropy are given as

$u_4 = u_f + x_4 u_{fg} = 417.41 + 0.0899 \times 2088.3$ kJ/kg 605.148 kJ/kg

$s_4 = s_f + x_4 s_{fg} = 1.3027 + 0.0899 \times 6.0562 = 1.8472$ kJ/kgK

Change in total internal energy is given by

$\Delta U = m (u_4 - u_1) = 6.5232 (605.148 - 2957.2) = -15342.91$ kJ



Work transfer during the process is given by

$W = W_{12} + W_{23} + W_{34} = 0 + P_2 (V_3 - V_2) + 0 = 400 \times (1 - 2) = - 400$ kJ

Therefore, total heat transfer is given by

$Q = \Delta U + W = - 15342.91 - 400 = - 15742.92$ kJ

Then, total entropy generated during the process is given by

$$S_{gen} = (\Delta S)_{CM} - \Sigma \left(\frac{Q_i}{T_i}\right)_{CM}$$

$$= m (s_4 - s_1) - \frac{Q}{T_i} = 6.5232 (1.8472 - 7.4648) - \frac{(-15742.92)}{298} = 16.1838 \text{ kJ/K}$$

38. 2 kg water at 100° C is mixed with 4 kg of water at 20° C in an isolated system. Calculate the net change in entropy due to the mixing process. [Take specific heat of water c=4.18 kJ/K]

Solution:

Given, Mass of water 1 (m_1) = 2 kg

Initial temperature of water 1 (T_1) = 100°C = 100 + 273 = 373 K

Mass of water 2 (m_2) = 4 kg

Initial temperature of water 2 (T_2) = 20°C = 20 + 273 = 293 K

Let, T_3 be the equilibrium temperature, then heat lost by water 1 is absorbed by the water 2, i.e.

$m_1 c (T_1 - T_3) = m_2 c (T_3 - T_2)$

or, $2 \times (100 - T_3) = 4 \times (T_3 - 20)$

or, $100 - T_3 = 2T_3 - 40$

or, $3T_3 = 140$

$\therefore T_3 = 46.667°C = 46.667 + 273 = 319.667$ K

Then, change in entropy of the water 1 is given by

$$(\Delta S)_1 = m_1 c \ln\left(\frac{T_3}{T_1}\right) = 2 \times 4.18 \times \ln\left(\frac{319.667}{373}\right) = - 1.2899 \text{ kJ/K}$$

Also, change in entropy of the water 2 is given by

$$(\Delta S)_2 = m_2 c \ln\left(\frac{T_3}{T_2}\right) = 4 \times 4.18 \times \ln\left(\frac{319.667}{293}\right) = 1.4564 \text{ kJ/kgK}$$

Therefore, the net change in entropy is given by

$(\Delta S)_{net} = (\Delta S)_1 + (\Delta S)_2 = - 1.2899 + 1.4564 = 0.1665$ kJ/K

39. Block A (m_A = 0.5 kg, c_A=1kJ/kgK) and block B (m_B = 1 kg, c_B = 0.5kJ/kgK) which are initially at 100° C and 500° C respectively are brought in contact inside an isolated system. Determine the change in entropy when they reach to a final state of thermal equilibrium.

Solution:

Given, Mass of Block A (m_A) = 0.5 kg

Specific heat capacity of Block A (c_A) = 1 kJ/kgK

Mass of Block B (m_B) = 1 kg

Specific heat capacity of Block B (c_B) = 0.5 kJ/kgK

Initial temperature of Block A (T_{A1}) = 100°C = 100 + 273 = 373 K

Initial temperature of Block B (T_{B1}) = 500°C = 500 + 273 = 773 K

Let, T_2 be the final equilibrium temperature then, the heat lost by Block B is absorbed by the Block A, i.e.

$m_B \times c_B (T_{B1} - T_2) = m_A \times c_A (T_2 - T_{A1})$

or, $1 \times 0.5 \times (773 - T_2) = 0.5 \times 1 \times (T_2 - 373)$

or, $773 - T_2 = T_2 - 373$

or, $2T_2 = 1146$

$\therefore T_2 = 573$ K

Then, change in entropy of Block A is given by

$$(\Delta S)_A = m_A c_A \ln\left(\frac{T_2}{T_{A1}}\right) = 0.5 \times 1 \times \ln\left(\frac{573}{373}\right) = 0.2147 \text{ kJ/K}$$

Change in entropy of Block B is given by

$$(\Delta S)_B = m_B c_B \ln\left(\frac{T_2}{T_{B1}}\right) = 1 \times 0.5 \times \ln\left(\frac{573}{773}\right) = -0.1497 \text{ kJ/K}$$

Therefore, the net change in entropy is given by

$$(\Delta S)_{net} = (\Delta S)_A + (\Delta S)_B = 0.2147 - 0.1497 = 0.065 \text{ kJ/K}$$

40. A lump of steel (c_s = 0.5 kJ/kgK) of mass 10 kg at 727° C is dropped in 100 kg of oil (c_o = 3.5 kJ/kgK) at 27° C Determine the net change in entropy.

1 kg of air enclosed in an isolated box with volume V_1, pressure P_1 and temperature T_1 is allowed to expand freely until its volume increases to $V_2 = 2V_1$. Determine the change in entropy. [Take R=287 J/kgK]

Solution:

Given, Mass of steel (m_s) = 10 kg

Specific heat capacity of steel (c_s) = 0.5 kJ/kgK

Initial temperature of steel (T_{1s}) = 727°C = 727 + 273 = 1000 K

Mass of oil (m_o) = 100 kg

Specific heat capacity of oil (c_o) = 3.5 kJ/kgK

Initial temperature of oil (T_{1o}) = 27°C = 27 + 273 = 300 K

Let, T_2 be the final equilibrium temperature then, the heat lost by steel is absorbed by oil i.e.,

$$m_s c_s (T_{1s} - T_2) = m_o c_o (T_2 - T_{1o})$$

or, $10 \times 0.5 \times (1000 - T_2) = 100 \times 3.5 \times (T_2 - 300)$

or, $1000 - T_2 = 70 (T_2 - 300)$

or, $1000 - T_2 = 70 T_2 - 21000$

or, $71 T_2 = 22000$

$\therefore T_2 = 309.589$ K

Then, change in entropy of steel is given by

$$(\Delta S)_s = m_s c_s \ln\left(\frac{T_2}{T_{1s}}\right) = 10 \times 0.5 \times \ln\left(\frac{309.859}{1000}\right) = -5.8582 \text{ kJ/K}$$

Change in entropy of oil is given by

$$(\Delta S)_o = m_o c_o \ln\left(\frac{T_2}{T_{1o}}\right) = 100 \times 3.5 \times \ln\left(\frac{309.859}{300}\right) = 11.3172 \text{ kJ/K}$$

Therefore, the net change in entropy is given as

$$(\Delta S)_{net} = (\Delta S)_s + (\Delta S)_o = -5.8582 + 11.3172 = 5.459 \text{ kJ/K}$$

41. 1 kg of air enclosed in an isolated box with volume V_1, pressure P_1 and temperature T_1 is allowed to expand freely until its volume inceeases to $V_2 = 2V_1$. Determine the change in entropy. [Take R = 287 JlkgK]

Solution:

Given, Mass of air (m) = 1 kg

Final volume (V_2) = 2 V_1

Since the temperature of system is constant,

$$P_1 V_1 = P_2 V_2$$

$$\therefore P_2 = \frac{P_1 V_1}{V_2} = P_1 \times \frac{V_1}{2V_1} = \frac{P_1}{2}$$

Then, change in entropy is given by

$$\Delta S = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right)$$

$$= 0 - 1 \times 287 \times \ln\left(\frac{P_1}{2 \times P_1}\right) = -287 \times \ln\left(\frac{1}{2}\right) = 198.93 \text{ kJ/k}$$

42. A rigid cylinder contains nitrogen initially at 100 kPa, 300 K and 0.005 m³. It is heated reversibly until its temperature reaches 400 K. Determine the entropy change of nitrogen during the process. [Take R = 297 J/kgK]

Solution:

Given, Initial state: P_1 = 100 kPa, T_1 = 300 k, V_1 = 0.005 m³

Final state: T_2 = 400k, $V_2 = V_1$ = 0.005 m³

For a constant volume (reversible isochoric) heating process,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\therefore P_2 = P_1 \times \frac{T_2}{T_1} = 100 \times \frac{400}{300} = 133.33 \text{ kPa}$$

Mass of N_2 is given by

$$m = \frac{P_1 V_1}{RT_1} = \frac{100 \times 10^3 \times 0.005}{287 \times 300} = 0.0058 \text{ kg}$$

Then, change in entropy is given by $\Delta S = mc_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right)$

$$= 0.0058 \times 718 \times \ln\left(\frac{400}{300}\right) + 0 = 1.198 \text{ kJ/K}$$

43. 5 kg of air initially at 150 kPa and 27° C is heated reversibly at constant pressure to 227° C. Determine the entropy change of the nitrogen during the process. [Take c_p=1005 J/kgK]

Solution:

Given, Mass of air (m) = 5 kg

Initial state: P_1 = 150 kPa, T_1 = 27°C = 27 + 273 = 300 K

Final state: T_2 = 227°C = 227 + 273 = 500 K

Process: constant pressure

Then, change in entropy is given by

$$\Delta S = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right) = 5 \times 1005 \times \ln\left(\frac{500}{300}\right) - 0 = 2.5669 \text{ kJ/K}$$

44. 1kg of air initially at 400 kPa and 500 K expand polytropically until its pressure reduces to 100 kPa. Determine the entropy change of air during the process. [Take R=297 J/kgK, c_p=1005 J/kgK]

Solution:

Given, Mass of air (m) = 1 kg

Initial state: P_1 = 400 kPa, T_1 = 500 K

Final state: P_2 = 100 kPa

R = 287 J/kgK, c_p = 1005 J/kgK, c_v = 718 J/kgK

$$\gamma = \frac{c_p}{c_v} = \frac{1005}{718} = 1.4$$

For polytropic process, PV^n = constant

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 500 \times \left(\frac{100}{400}\right)^{\frac{1.4-1}{1.4}} = 336.475 \text{ K}$$

Then, change in entropy is given by

$$\Delta S = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right)$$

$$= 1 \times 1005 \times \ln\left(\frac{336.475}{500}\right) - 1 \times 287 \times \ln\left(\frac{100}{400}\right) = -0.1982 \text{ J/K}$$

45. 0.5 m^3 of air at 600 kPa and 500 K expands reversibly to 100 kPa. Determine the change in entropy when it under goes the following process:

(a) PV= constant

(b) PV^3=constant

(c) Adiabatic process [Take R=297 J/kgK, c_p=1005 J/kgK]

Solution:

Given, Initial state: V_1 = 0.5 m^3, P_1 = 600 kPa, T_1 = 500 K

Final state: P_2 = 100 kPa

Then, mass of air is given by

$$m = \frac{P_1V_1}{RT_1} = \frac{600 \times 10^3 \times 0.5}{287 \times 500} = 2.091 \text{ kg}$$

a) For PV = constant,

$$V_2 = V_1\left(\frac{P_1}{P_2}\right) = 0.5\left(\frac{600}{100}\right) = 3 \text{ m}^3$$

Therefore, change in entropy is given by

$$\Delta S = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right) = 0 - 2.091 \times 287 \times \ln\left(\frac{100}{600}\right) = 1.075 \text{ kJ/K}$$

b) For PV^3 = constant

$$T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{3-1}{3}} \times T_1 = \left(\frac{100}{600}\right)^{\frac{2}{3}} \times 500 = 151.427 \text{ K}$$

Then, change in entropy is given by

$$\Delta S = mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right)$$

$$= 2.091 \times 1005 \times \ln\left(\frac{151.427}{500}\right) - 2.091 \times 297 \times \ln\left(\frac{100}{600}\right) = -1.4352 \text{ kJ/K}$$

c) For reversible adiabatic process, Q = 0

$\therefore \Delta S = 0$

46. Air at a pressure of 100 kPa and 27° C is compressed by an air compressor to a pressure of 1500 kPa. Determine the work required per kg of air for the compressor assuming process to be reversible and adiabatic.

Solution:

Given, Initial state: P_1 = 100 kPa, T_1 = 27°C = 27 + 273 = 300 K

Final state: P_2 = 1500 kPa

Process: reversible and adiabatic

Temperature at state 2 is given by

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 300\left(\frac{1500}{100}\right)^{\frac{1.4-1}{1.4}} = 650.35 \text{ K}$$

Applying energy equation for an adiabatic compressor,

$$\dot{W}_{CV} = \dot{m}\,(h_1 - h_2)$$

For an ideal gas using $h_1 - h_2 = c_p (T_1 - T_2)$

$$w_{CV} = \frac{\dot{W}_{CV}}{\dot{m}} = c_p (T_1 - T_2) = 1.005 \times (300 - 650.35) = -352.102 \text{ kJ/kg}$$

Therefore work required per kg of air for the compressor, $w_{CV} = -352.102$ kJ/kg

47. Steam at 2.5 MPa and 500° C and with a velocity of 100 m/s enters into an well insulated turbine and exits at 200 kPa and with a velocity of 150 m/s. The work developed per kg of steam is claimed to be

(a) 650 kJ/kg

(b) 680 kJ/kg

Solution:

Given, Properties of steam at inlet: $P_1 = 2.5$ MPa = 2500 kPa, $T_1 = 500°C$, $\overline{V_1} =$ 100 m/s

Properties of steam at exit: $P_2 = 200$ kPa , $\overline{V_2} = 150$ m/s

Process: Isentropic

For other properties of steam at inlet, referring to the table A2.1,

T_{sat} (2500 kPa) = 223.99°C. Here, $T > T_{sat}$, hence it is a superheated vapor.

$h_1 = 3462.2$ kJ/kg, $s_1 = 7.3235$ kJ/kgK

Since entropy remains constant during isentropic process, entropy at the turbine exit is $s_2 = 7.3235$ kJ/kg

Referring to the Table A 2.1, s_f (200 kPa) = 1.5304 kJ/kgK, s_{fg} (200 kPa) = 5.5968 kJ/kgK, s_g (200 kPa) = 7.1272 kJ/kgK. Here, $s_2 > s_g$, hence it is a superheated vapor. Now, referring to the Table A2.4, specific entropy of superheated vapor which includes specific entropy 7.3235 kJ/kgK and corresponding specific enthalpy are listed as:

h_g, kJ/kg	s_g, kJ/kgK	
2726.6	7.2793	(a)
2870.0	7.5059	(b)

Applying linear interpolation for specific enthalpy,

$$h_2 - (h_g)_a = \frac{(h_g)_b - (h_g)_a}{(s_g)_b - (s_g)_a}[s_2 - (s_g)_a]$$

$$\therefore h_2 = (h_g)_a + \frac{(h_g)_b - (h_g)_a}{(s_g)_b - (s_g)_a}[s_2 - (s_g)_a]$$

$$= 2768.6 + \frac{2870.0 - 2768.6}{7.5059 - 7.2793}(7.3235 - 7.2793) = 2788.3788 \text{ kJ/kg}$$

Now applying energy equation for an adiabatic turbine,

$$\dot{W}_{CV} = \dot{m}\left[(h_1 - h_2) + \frac{1}{2}\left(\overline{V_1}^2 - \overline{V_2}^2\right) + g(z_1 - z_2)\right]$$

Neglecting P.E., we get

$$w_{CV} = \frac{\dot{W}_{CV}}{\dot{m}} = (h_1 - h_2) + \frac{1}{2}\left(\overline{V_1}^2 - \overline{V_2}^2\right) + 0$$

$$= (3462.2 - 2788.3788) + \frac{1}{2000}(100^2 - 150^2)$$

$$\therefore (w_{CV})_{max} = 667.57 \text{ kJ/kg}$$

a) Given, $w_{CV} = 650$ kJ/kg

Since $(w_{CV})_{max} > w_{CV}$, hence the claim is valid.

b) Given, $w_{CV} = 680$ kJ/kg

Since, $(w_{CV})_{max} < w_{CV}$, hence the claim is invalid.

48. Steam at 5 MPa and 400° C enters into a turbine at a rate of 2 kg/s and exits at a pressure of 400 kPa. Assuming the process to be reversible and adiabatic, determine the power output.

Solution:

Given, Properties of steam at state 1: $P_1 = 5$ MPa = 5000 kPa, $T_1 = 400°C$

Mass flow rate of steam ($\dot{m}$) = 2 kg/s

Properties of steam at exit: $P_2 = 400$ kPa

Process: reversible and adiabatic (isentropic)

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (5000 kPa) = 263.98°C. Here, $T > T_{sat}$, hence it is a superheated vapor. Now referring to the Table A2.4,

$\therefore h_1 = 3195.5$ kJ/kg and $s_1 = 6.6456$ kJ/kgK

Since, entropy remains constant during isentropic process, entropy at the turbine exit is $s_2 = 6.6456$ kJ/kgK

Referring to the Table A2.1, s_f (400 kPa) = 1.7770 kJ/kgK

s_{fg} (400 kPa) = 5.1191 kJ/kgK, s_g (400 kPa) = 6.8961 kJ/kgK

h_f (400 kPa) = 604.91 kJ/kgk, h_{fg} (400 kPa) = 2133.6 kJ/kg

Here, $s_f < s_2 < s_g$, hence condition of steam at turbine exit is a two phase mixture. Therefore, quality of the two phase mixture is given by

$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{6.6456 - 1.7770}{5.1191} = 0.9511$$

Therefore, specific enthalpy of steam at the turbine exit is given by

$$h_2 = h_f + x_2 h_{fg} = 604.91 + 0.9511 \times 2133.6 = 2634.177 \text{ kJ/kg}$$

Now applying energy equation for adiabatic turbine,

$$\dot{W}_{CV} = \dot{m}\,[(h_1 - h_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + g(z_1 - z_2)]$$

$$= 2\,[(3195.5 - 2634.177) + 0 + 0] = 1122.65 \text{ kW}$$

49. Steam enters a turbine at 1.5 MPa and 300° C and with a velocity of 60 m/s, expands in a reversible adiabatic process and exits at 200 kPa with a velocity of 150 m/s. Determine the specific work output.

Solution:

Given, Properties of steam at inlet: P_1 = 1.5 MPa = 1500 kPa, T_1 = 300°C

$\overline{V_1}$ = 60 m/s

Properties of steam at outlet: P_2 = 200 kPa, $\overline{V_2}$ = 150 m/s

Process: reversible adiabatic (isentropic)

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (1500 kPa) = 198.33°C. Here, T > T_{sat}, hence it is a superheated vapor. Now, Referring to the Table A2.4,

$\therefore$ h_1 = 3036.9 kJ/kg and s_1 = 6.9168 kJ/kgK

Since entropy remains constant during isentropic process, entropy at the turbine exits is s_2 = 6.9168 kJ/kgK

Referring to the Table A2.1, s_f (200 kPa) = 1.5304 kJ/kgK, s_{fg} (200 kPa) = 5.5968 kJ/kgK, s_g (200 kPa) = 7.1272 kJ/kgK, h_f (200 kPa) = 504.80 kJ/kg, h_{fg} (200 kPa) = 2201.7 kJ/kg. Here, $s_f < s_2 < s_g$, hence the condition of steam at turbine exit is a two phase mixture. Therefore, quality of the mixture is given by

$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{6.9168 - 1.5304}{5.5968} = 0.96241$$

Therefore, the specific enthalpy of steam at the turbine exit is given by

$$h_2 = h_f + x_2 h_{fg} = 504.80 + 0.96241 \times 2201.7 = 2623.738 \text{ kJ/kg}$$

$$\dot{W}_{CV} = \dot{m}\,[(h_1 - h_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + g(z_1 - z_2)]$$

$$\therefore \frac{\dot{W}_{CV}}{\dot{m}} = w_{CV} = (h_1 - h_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + 0$$

$$= (3036.9 - 2623.738) + \frac{1}{2000}(60^2 - 150^2) = 403.712 \text{ kJ/kg}$$

50. Steam enters into a turbine at 2 MPa and 300° C and exits at 20 kPa. If the power output of the turbine is 1 Mw, determine the mass flow rate of steam. Assume reversible adiabatic process.

Solution:

Given, Properties of steam at inlet: P_1 = 2 MPa = 2000 kPa, T_1 = 300°C

Properties of steam at exit: P_2 = 20 kPa

Power output ($\dot{W}_{CV}$) = 1 MW = 1000 kW

Process: reversible adiabatic (isentropic)

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (2000 kPa) = 212.42°C. Here T > T_{sat} (2000 kPa), hence the condition of steam at turbine inlet is a superheated vapor. Now, referring to the Table A2.4,

h_1 = 3022.7 kJ/kg, s_1 = 6.7651 kJ/kgK

Since entropy remains constant during isentropic process entropy at the turbine exit is s_2 = 6.7651 kJ/kgK

Referring to the Table A2.1, s_f (20 kPa) = 0.8321 kJ/kgK, s_{fg} (20 kPa) = 7.0747 kJ/kgK, s_g (20 kPa) = 7.9068 kJ/kg, h_f (20 kPa) = 251.46 kJ/kg, h_{fg} (20 kPa) = 2357.4 kJ/kg. Here, $s_f < s_2 < s_g$, hence the condition of steam at turbine exit is a two phase mixture. Therefore, the quality of the steam at turbine exit is given by

$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{6.7651 - 0.8321}{7.0747} = 0.8386$$

Therefore, specific enthalpy of steam at turbine exit is given by

$$h_2 = h_f + x_2 h_{fg} = 254.46 + 0.8386 \times 2357.4 = 2228.376 \text{ kJ/kg}$$

Now applying the energy equation for an adiabatic turbine,

$$\dot{W}_{CV} = \dot{m}\,[(h_1 - h_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + g(Z_1 - Z_2)]$$

$$\therefore \dot{m} = \frac{\dot{W}_{CV}}{(h_1 - h_2) + 0 + 0} = \frac{1000}{(3022.7 - 2228.376)} = 1.259 \text{ kg/s}$$

51. Steam enters a nozzle at 1.5 MPa and 300° C and with a velocity of 50 m/s, undergoes a reversible adiabatic process and exits at 200 kPa. Determine the exit velocity.

Solution:

Given, Properties of steam at inlet: $P_1 = 1.5$ MPa = 1500 kPa, $T_1 = 300°C$, $\overline{V_1}$ = 50 m/s

Properties of steam at exit: $P_2 = 200$ kPa

Process: reversible adiabatic

For other properties of steam at inlet referring to the Table A2.1, T_{sat} (1500 kPa) = 198.33°C. Here, $T > T_{sat}$, hence the condition of steam at nozzle inlet is superheated vapor. Now, referring to the Table A2.4,

$h_1 = 3036.9$ kJ/kg, $s_1 = 6.9168$ kJ/kgK

Since entropy remains constant during isentropic process, entropy at the nozzle exit is $s_2 = 6.9168$ kJ/kgK

Referring to the Table A2.1, s_f (200 kPa) = 1.5304 kJ/kgK, s_{fg} (200 kPa) = 5.5968 kJ/kgK, s_g (200 kPa) = 7.1272 kJ/kgK, h_f (200 kPa) = 504.80 kJ/kg, h_{fg} (200 kPa) = 2201.7 kJ/kg. Here, $s_f < s_2 < s_g$, hence the condition of steam at nozzle exit is two phase mixture. Therefore, quality of the steam at turbine exit is given by

$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{6.9168 - 1.5304}{5.5968} = 0.96241$$

Therefore, specific enthalpy of the steam at nozzle exit is given by

$$h_2 = h_f + x_2 h_{fg} = 504.8 + 0.96241 \times 2201.7 = 2623.7381 \text{ kJ/kg}$$

Now applying energy equation for an adiabatic nozzle,

$$h_1 + \frac{1}{2}\overline{V_1}^2 = h_2 + \frac{1}{2}\overline{V_2}^2$$

$$\therefore \overline{V_2} = \sqrt{2(h_1 - h_2) + \overline{V_1}^2} = \sqrt{2000\,(2036.9 - 2623.7381) + 50^2}$$

= 910.398 m/s

52. A compressor receives air at 100 kPa and 27° C and requires a power input of 60 kW. If the mass flow rate of the air is 0.1 kg/s, determine the maximum exit pressure of the compressor.

Solution:

Given, Properties of air at inlet: $P_1 = 100$ kPa, $T_1 = 27°C = 27 + 273 = 300$ K

Power input ($\dot{W}_{cv}$) = - 60 kW

Mass flow rate of air ($\dot{m}$) = 0.1 kg/s

Now applying energy equation for an adiabatic compressor,

$$\dot{W}_{CV} = \dot{m}\left[(h_1 - h_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + g(z_1 - z_2)\right]$$

For an ideal gas using $h_1 - h_2 = c_p (T_1 - T_2)$ and neglecting kinetic energy and potential energy,

$$\dot{W}_{CV} = \dot{m}\, c_p (T_1 - T_2) + 0 + 0$$

or, $-60 = 0.1 \times 1.005\,(300 - T_2)$

$\therefore T_2 = 897.015$ K

Now for an isentropic process, pressure of air at compressor exit is given by

$$P_2 = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} P_1 = \left(\frac{897.015}{300}\right)^{\frac{1.4}{1.4-1}} \times 100 = 4622.475 \text{ kPa}$$

53. Air at 100 kPa and 25° C enters into a diffuser at a velocity of 150 m/s and exits with a velocity of 40 m/s. Assuming the process to be reversible and adiabatic, determine the exit pressure and temperature of the air.

Solution:

Given, Properties of air at inlet: $P_1 = 100$ kPa, $T_1 = 25°C = 25 + 273 = 298$ K, $\overline{V_1} = 150$ m/s

Properties of air at state 2: $\overline{V_2} = 40$ m/s

Process: reversible and adiabatic (isentropic)

Applying energy equation for an adiabatic diffuser,

$$h_2 - h_1 = \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2)$$

For an ideal gas, using $h_2 - h_1 = c_p (T_2 - T_1)$,

$$c_p (T_2 - T_1) = \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2)$$

$$\therefore T_2 = \frac{1}{2}\frac{(\overline{V_1}^2 - \overline{V_2}^2)}{cp} + T_1 = \frac{(150^2 - 40^2)}{1005} + 298 = 308.398 \text{ K}$$

Now for an isentropic process, pressure of air at diffuser exit is given by

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 100 \times \left(\frac{308.398}{298}\right)^{\frac{1.4}{1.4-1}} = 112.75 \text{ kPa}$$

54. Air at 200 kPa and 1000 K with very low velocity enters into a nozzle and exits at a pressure of 100 kPa. Assuming the process to be isentropic, determine the exit velocity.Solution:

Given, Properties of air at inlet: $P_1 = 200$ kPa, $T_1 = 1000$ K, $\overline{V_1} = 0$ m/s

Properties air at exit: $P_2 = 100$ kPa

Process: isentropic (reversible and adiabatic)

For an isentropic process, temperature of air at nozzle exit is given by

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma}{\gamma-1}} = 1000\left(\frac{100}{200}\right)^{\frac{1.4-1}{1.4}} = 820.34 \text{ K}$$

Applying energy equation for an adiabatic nozzle,

$$h_2 - h_1 = \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2)$$

For an ideal gas, using $h_2 - h_1 = c_p(T_2 - T_1)$

$$c_p(T_2 - T_1) = \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2)$$

$$\therefore \overline{V_2} = \sqrt{\overline{V_1}^2 - 2\,cp\,(T_2 - T_1)} = \sqrt{0 - 2 \times 1005\,(820.34 - 1000)}$$

$= 600.9298$ m/s

55. Air enters into an insulated turbine at 500 kPa and 527° C and exit at 100 kPa and 267° C. Determine the work developed per kg of air and whether the process is internally reversible, irreversible or impossible.

Solution:

Given, Properties of air at inlet: $P_1 = 500$ kPa, $T_1 = 527°C = 527 + 273 = 800$ K

Properties of air at outlet: $P_2 = 100$ kPa, $T_2 = 267°C = 267 + 273 = 540$ K

Applying energy equation for an adiabatic turbine,

$$\dot{W}_{CV} = \dot{m}\,[(h_1 - h_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + g(z_2 - z_1)]$$

For an ideal gas using $h_1 - h_2 = c_p(T_1 - T_2)$ and neglecting k.E. and P.E.,

$$\therefore w_{CV} = \frac{\dot{W}_{CV}}{\dot{m}} = c_p(T_1 - T_2) + 0 + 0 = 1.005\,(800 - 540) = 261.3 \text{ kJ/kg}$$

56. Determine whether it is possible to compress air adiabatically from 100 kPa and 27° C to

(a) 400 kPa, 150° C and

(b) 400 kPa, 2000° C

Solution:

Given, Initial state: $P_1 = 100$ kPa, $T_1 = 27°C = 27 + 273 = 300$ K

a) Final state: $P_2 = 400$ kPa, $T_2 = 150°C = 150 + 273 = 423$ K

Process: adiabatic

Change in entropy per unit mass of air is given by

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

$$= 1.005 \times \ln\left(\frac{423}{300}\right) - 0.287 \times \ln\left(\frac{400}{100}\right) = -0.05256 \text{ kJ/kgK}$$

For adiabatic process, $Q = 0$

The entropy generation per unit mass of air is given by

$$s_{gen} = (s_2 - s_1) - \Sigma\left(\frac{q_i}{T_i}\right)_{CV} = (s_2 - s_1) - 0 = -0.05256 \text{ kJ/kgK}$$

Since, s_{gen} is negative hence the process is impossible.

b) Final state: $P_2 = 400$ kPa, $T_2 = 200°C = 200 + 273 = 473$ K

Change in entropy per unit mass of air is given by

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

$$= 1.005 \times \ln\left(\frac{473}{300}\right) - 0.287 \times \ln\left(\frac{400}{100}\right) = 0.059723 \text{ kJ/kgK}$$

Therefore, the entropy generation per unit mass of air is given by

$$s_{gen} = (s_2 - s_1) - \Sigma\left(\frac{q_i}{T_i}\right)_{CV}$$

$$= (s_2 - s_1) - 0 = 0.059723 - 0 = 0.059723 \text{ kJ/kgK}$$

Since, $S_{gen} > 0$, hence the process is possible and irreversible.

57. Air is compressed isothermally from 100 kPa and 27° C to 1000 kPa by supplying 175 kJ/kgK of work. Determine whether it is a reversible, irreversible or an impossible process.

Solution:

Given, Initial state: $P_1 = 100$ kPa, $T_1 = 27°C = 27 + 273 = 300$ K

Final state: $P_2 = 1000$ kPa

Work input per kg of air to the compressor (w_{CV}) = - 175 kJ/kg

Process: Isothermal ($T_1 = T_2$)

Applying steady state energy equation for a compressor,

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\,(h_2 - h_1)$$

Therefore, heat lost per unit mass of air to the surrounding is given as

$q_{CV} = w_{CV} + (h_1 - h_2)$

For an ideal gas using $h_2 - h_1 = c_p (T_2 - T_1)$,

$q_{CV} = w_{CV} + c_p (T_2 - T_1)$

$\therefore q_{CV} = w_{CV} + 0 = -175$ kJ/kg

Change in entropy per unit mass of air is given by

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right) = 0 - 0.287 \times \ln\left(\frac{1000}{100}\right) = -0.66084 \text{ kJ/kgK}$$

Then, the entropy generation per unit mass of air is given by

$$s_{gen} = (s_2 - s_1) - \Sigma\left(\frac{q_i}{T_i}\right)_{CV}$$

$$= (s_2 - s_1) - \frac{q_{CV}}{T_i} = -0.66084 - \left(\frac{-175}{300}\right) = -0.07751 \text{ kJ/kgK}$$

Since, $s_{gen} < 0$, hence the process is impossible.

58. Steam enters an adiabatic nozzle at 4 MPa, 400° C and with a velocity of 50 m/sand exits at 2 MPa and with a velocity of 300 m/s. If the nozzle has an inlet area of 8 cm², determine

(a) the exit temperature of steam from the nozzle, and

(b) the rate of entropy generation for the process

Solution:

Given, Properties of steam at inlet: $P_1 = 4$ MPa, $T_1 = 400°C$, $\overline{V_1} = 50$ m/s

Properties of steam at outlet: $P_2 = 2$ MPa $= 2000$ kPa, $\overline{V_2} = 300$ m/s

Inlet area $(A_1) = 8\text{ cm}^2 = 8 \times 10^{-4}\text{ m}^2$

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (4000 kPa) = 250.39°C. Here, $T > T_{sat}$, hence the condition of steam at nozzle inlet is superheated vapor. Now referring to the Table A2.4,

$h_1 = 3213.4$ kJ/kg, $s_1 = 6.7688$ kJ/kgK, $v_1 = 0.0734\text{ m}^3/\text{kg}$

Now applying energy equation for an adiabatic nozzle,

$$h_1 + \frac{1}{2}\overline{V_1}^2 = h_2 + \overline{V_2}^2$$

$$\therefore h_2 = h_1 + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) = 3213.4 + \frac{1}{2000}(50^2 - 300^2) = 3169.65 \text{ kJ/kg}$$

Referring to the Table A2.1, h_g (2000 kPa) = 2798.7 kJ/kg. Here, $h_2 > h_g$, hence, the condition of steam at nozzle exit is also superheated vapor. Now, referring to the Table A2.4, specific enthalpy of a superheated vapor which includes the specific enthalpy 3169.65 kJ/kg and corresponding temperature and specific entropy are listed as:

T°,C	h_g, kJ/kg	s_g, kJ/kgK	
350	3136.6	6.9556	(a)
400	3247.5	7.1269	(b)

Applying linear interpolation for temperature and specific entropy,

$$T_2 - T_a = \frac{T_b - T_a}{(h_g)_b - (h_g)_a}[h_2 - (h_g)_a]$$

$$\therefore T_2 = T_a + \frac{T_b - T_a}{(h_g)_b - (h_g)_a}[h_2 - (h_g)_a]$$

$$= 350 + \frac{400 - 350}{3247.5 - 3136.6}(3169.65 - 3136.6) = 364.9°C$$

Similarly, $s_2 = (s_g)_a + \frac{(s_g)_b - (s_g)_a}{(h_g)_b - (h_g)_a}[h_2 - (h_g)_a]$

$$= 6.9556 + \frac{7.1269 - 6.9556}{3247.5 - 3136.6}(3169.65 - 3136.6) = 7.00665 \text{ kJ/kgK}$$

For adiabatic turbine, $Q_{CV} = 0$

Mass flow rate of steam is given by

$$\dot{m} = \frac{A_1 \overline{V_1}}{v_1} = \frac{8 \times 10^{-4} \times 50}{0.0734} = 0.545 \text{ kg}$$

Therefore, rate of entropy generation for the process is given by

$$\dot{S}_{gen} = (\dot{S}_{out} - \dot{S}_{in}) - \Sigma\left(\frac{Q_i}{T_i}\right)$$

$$= \dot{m}(s_2 - s_1) - \frac{Q_{CV}}{T_i} = 0.545(7.00665 - 6.7688) - 0 = 0.1293 \text{ kW/K}$$

59. Air enters a compressor operating steadily at 100 kPa, 27° C and with a volumetric flow rate of 1.2 m³/min and exits at 400 kPa, 177° C. The power required to drive the compressor is 3.6 kW. Determine

(a) the heat transfer rate from the compressor surface and

(b) the rate of entropy generation if heat is transferred to the surrounding at 20° C.

Solution:

Given, Properties of air at inlet: $P_1 = 100$ kPa, $T_1 = 27°C = 27 + 273 = 300$ K

Volumetric flow rate ($\dot{V}$) = 1.2 m³/min = $\frac{1.2}{60}$ m³/s = 0.02 m³/s

Properties of air at exit: P_2 = 400 kPa, T_2 = 177°C = 177 + 273 = 450 K

Power required to drive the compressor ($\dot{W}_{CV}$) = - 3.6 kW

Temperature of the surrounding (T_{sur}) = 20°C = 20 + 273 = 293 K

Specific volume of air at compressor inlet is given as

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 300}{100 \times 10^3} = 0.861 \text{ m}^3/\text{kg}$$

Then, mass flow rate of air is given by

$$\dot{m} = \frac{V}{v_1} = \frac{0.02}{0.861} = 0.02323 \text{ kg}$$

Now applying energy equation for a compressor,

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\ [(h_2 - h_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2) + g(z_2 - z_1)]$$

For an ideal gas using $h_2 - h_1 = c_p (T_2 - T_1)$ and neglecting K.E. and P.E.,

$$\dot{Q}_{CV} = \dot{W}_{CV} + \dot{m}\ [c_p (T_2 - T_1) + 0 + 0]$$

$$= -3.6 + 0.02323\ [1.005\ (450 - 300)] = -0.0981 \text{ kW} = -98.1 \text{ W}$$

Then, change in entropy is given by

$$\Delta S = S_2 - S_1 = \dot{m}\, c_p \ln\left(\frac{T_2}{T_1}\right) - \dot{m}\, R \ln\left(\frac{P_2}{P_1}\right)$$

$$= 0.02323 \times 1.005 \times \ln\left(\frac{450}{300}\right) - 0.02323 \times 0.287 \times \ln\left(\frac{400}{100}\right)$$

$$= 0.0002236 \text{ kW/kgK} = 0.2236 \text{ W/K}$$

Therefore, rate of entropy generation is given by

$$\dot{S}_{gen} = (\dot{S}_{out} - \dot{S}_{in}) - \Sigma\left(\frac{\dot{Q}_i}{T_i}\right)$$

$$= (\dot{S}_2 - \dot{S}_1) - \frac{\dot{Q}_{CV}}{T_{Sur}} = 0.2236 - \left(\frac{-98.1}{293}\right) = 0.55841 \text{ W/K}$$

60. Air enters a nozzle operating steadily at 2 MPa, 327° C and with a velocity of 50 m/sand exits at 100 kPa, 27° C and with a velocity of 500 m/s. Determine

(a) the heat loss per kg of air from the nozzle surface and

(b) the rate of entropy generation per kg of air if heat is transferred to the surrounding at 20° C.

Solution:

Given, Properties of air at inlet: P_1 = 2MPa = 2000 kPa, T_1 = 327°C = 327 + 273 = 600 K, $\overline{V_1}$ = 50 m/s

Properties of air at exit: P_2 = 100 kPa, T_2 = 27°C = 27 + 273 = 300 K, $\overline{V_2}$ = 500 m/s

Temperature of the surrounding (T_{sur}) = 20°C = 20 + 273 = 293 K

Applying steady state energy equation for a nozzle,

$$\dot{Q}_{CV} = \dot{m}\ [(h_2 - h_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2) + 2(z_2 - z_1)]$$

For an ideal gas using $h_2 - h_1 = c_p (T_2 - T_1)$ and neglecting P.E.,

$$\frac{\dot{Q}_{CV}}{\dot{m}} = q_{CV} = c_p (T_2 - T_1) + \frac{1}{2}(\overline{V_2}^2 - \overline{V_1}^2) + 0$$

$$\therefore q_{cv} = 1.005\ (300 - 600) + \frac{1}{2000}(500^2 - 50^2) = -177.75 \text{ kJ/kg}$$

Then, change in entropy per kg of air is given by

$$\Delta s = s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$= 1.005 \times \ln\left(\frac{300}{600}\right) - 0.287 \times \ln\left(\frac{100}{2000}\right) = 0.1632 \text{ kJ/kgK}$$

Therefore, the entropy generation per kg of air is given by

$$s_{gen} = (s_{out} - s_{in}) - \Sigma\left(\frac{q_i}{T_i}\right)_{CV}$$

$$\therefore s_{gen} = (s_2 - s_1) - \frac{q_{CV}}{T_i} = 0.1632 - \frac{(-177.75)}{293} = 0.7699 \text{ kJ/kgK}$$

61. Steam enters into a well insulated throttling valve at 10 MPa and 600 °C and exits at 5 MPa. Determine the change in entropy per unit mass of the steam.

Solution:

Given, Properties of steam at inlet: P_1 = 10 MPa = 1000 kPa, T_1 = 600°C

Properties of steam at exit: P_2 = 5 MPa = 5000 kPa

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (1000 kPa) = 311.03°C. Here, T > T_{sat}, hence, the condition of steam at inlet of throttling valve is superheated vapor. Then, referring to Table A2.4,

$h_1 = 3624.7$ kJ/kg, $s_1 = 6.9022$ kJ/kgK

Since, enthalpy remains constant during throttling process specific enthalpy at the throttling valve exist is $h_2 = 3624.7$ kJ/kg

Referring to the Table A2.1, h_g (5000 kPa) = 2793.7 kJ/kg. Here $h_2 > h_g$, hence the condition of steam at throttling valve exit is superheated vapor. Now, referring to the Table A2.4, specific enthalpy of superheated vapor which includes specific enthalpy 3624.7 kJ/kg and corresponding specific entropy are listed as:

h_g, kJ/kg	s_g, kJ/kgK	
3550.2	7.1218	(a)
3666.2	7.2586	(b)

Then applying linear interpolation for specific entropy,

$$s_2 - (s_g)_a = \frac{(s_g)_b - (s_g)_a}{(h_g)_b - (h_g)_a}[h_2 - (h_g)_a]$$

$$\therefore s_2 = (s_g)_a + \frac{(s_g)_b - (s_g)_a}{(h_g)_b - (h_g)_a}[h_2 - (h_g)_a]$$

$$= 7.1218 + \frac{7.2586 - 7.1218}{3666.2 - 3550.2}[3624.7 - 3550.2] = 7.2097 \text{ kJ/kgK}$$

Therefore, change in entropy per unit mass is given by

$\Delta s = s_2 - s_1 = 7.2097 - 6.9022 = 0.3075$ kJ/kgK

62. Air at 1 MPa and 327 ^{0}C is throttled to the pressure of 100 kPa. Determine the change in entropy per unit mass of air.

Solution:

Given, Properties of air at inlet: $P_1 = 1$ MPa = 1000 kPa, $T_1 = 327°C = 327 + 273 = 600$ K

Properties of air at exit: $P_2 = 100$ kPa

Then applying energy equation for a throttling valve,

$h_2 - h_1 = 0$

For an ideal gas using $h_2 - h_1 = c_p (T_2 - T_1)$

$c_p (T_2 - T_1) = 0$

$\therefore T_2 = T_1 = 600$ K

Therefore, change in entropy per unit mass is given by

$$\Delta s = s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$= 0 - 0.287 \times \ln\left(\frac{100}{1000}\right) = 0.6608 \text{ kJ/kgK}$$

63. Steam enters a turbine at 2 MPa and 300^0 C and exits at 20 kPa. If the specific work output from the turbine is 650 kJ/kg of steam, determine the isentropic efficiency of the turbine.

Solution:

Given, Properties of steam at inlet: $P_1 = 2$ MPa = 2000 kPa, $T_1 = 300°C$

Properties of steam at exit: $P_2 = 20$ kPa

Specific work output from the turbine (w_{real}) = 650 kJ/kg

Process: Isentropic (reversible and adiabatic)

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (2000 kPa) = 212.42°C, Here, T > T_{Sat}, hence, the condition of steam at turbine inlet is superheated vapor. Now, referring to the Table A2.4,

$h_1 = 3022.7$ kJ/kg, $s_1 = 6.7651$ kJ/kgK

Since the entropy remains constant during an isentropic process, specific entropy of steam at turbine exit is $s_2 = 6.7651$ kJ/kgK

Referring to the Table A 2.1, s_g (20 kPa) = 7.9068 kJ/kgK, s_f (20 kPa) = 0.8321 kJ/kgK, s_{fg} (20 kPa) = 7.0747 kJ/kgK, h_f (20 kPa) = 251.46 kJ/kg, h_{fg} (20 kPa) = 2357.4 kJ/kg. Here, $s_f < s_2 < s_g$, hence the condition of steam at turbine exit is a two phase mixture. Then, quality of mixture is given by

$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{6.7651 - 0.8321}{7.0747} = 0.8386$$

Therefore, specific enthalpy of steam at the turbine exit is given by

$h_2 = h_f + x_2 h_{fg} = 251.46 + 0.8386 \times 2357.4 = 2228.376$ kJ/kg

Now applying energy equation for an adiabatic turbine,

$$\dot{W}_{CV} = \dot{m}\left[(h_1 - h_2) + \frac{1}{2}(\overline{V_1}^2 - \overline{V_2}^2) + g(z_1 - z_2)\right]$$

Neglecting K.E and P.E, we get

$$w_{CV} = \frac{\dot{W}_{CV}}{\dot{m}} = (h_1 - h_2) + 0 + 0 = 3022.7 - 2228.376$$

$\therefore w_{isen} = 794.324$ kJ/kg

Therefore, isentropic efficiency of the turbine is given by

$$\eta_{isen} = \frac{W_{real}}{W_{isen}} = \frac{650}{794.324} = 81.83\%$$

64. Steam enters an adiabatic turbine at 5 MPa, 500 °C and with a velocity of 50 m/s and exits at 50 kPa, 100 °C and with a velocity of 150 m/s. If the power output of the turbine is 50 MW, determine

(a) The mass flow rate of steam flowing through the turbine, and
(b) The isentropic efficiency of the turbine.

Solution:

Given, Properties of steam at inlet: $P_1 = 5$ MPa = 5000 kPa, $T_1 = 500$°C $\overline{V_1} = 50$ m/s

Properties of steam at exit: $P_2 = 50$ kPa, $T_2 = 100$° C, $\overline{V_2} = 150$ m/s

Power output of the turbine ($\dot{W}_{real}$) = 50 MW = 50000 kW

Process: Isentropic (reversible and adiabatic)

For other properties of steam at turbine inlet, referring to the Table A2.1, T_{sat} (5000 kPa) = 263.98°C. Here, $T > T_{sat}$, hence the condition of steam at turbine inlet is superheated vapor. Now, referring to the Table A2.4,

$h_1 = 3433.9$ KJ/kg, $s_1 = 6.9760$ KJ/KgK

For other properties of steam at turbine exit, referring to the Table A2.1, T_{sat} (50 kPa = 81.33°C. Here, $T > T_{sat}$, hence, the condition of steam at turbine exit is superheated vapor. Now, referring to the Table A2.4, $h_{2r} = 2682.1$ kJ/kg

Now applying energy equation for an adiabatic turbine,

$$\dot{W}_{real} = \dot{m}\ [(h_1 - h_{2r}) + \frac{1}{2}(\overline{V_1^2} - \overline{V_2^2}) + g(z_1 - z_2)]$$

Neglecting P.E., we get

$$\dot{m} = \frac{\dot{W}_{real}}{(h_1 - h_{2r}) + \frac{1}{2}(\overline{V_1^2} - \overline{V_2^2}) + 0}$$

$$= \frac{50000}{(3433.9 - 2682.1) + \frac{1}{2000}(50^2 - 150^2)} = 67.404 \text{ kg/s}$$

To calculate the isentropic power output of the turbine:

The entropy remains constant during isentropic process, hence specific entropy of steam at turbine exit is $s_2 = 6.9760$ kJ/kgK

Referring to the Table A2.1, s_f (50 kPa) = 1.0912 kJ/kgK, s_g (50 kPa) = 7.5928 kJ/kg K, s_{fg} (50 kPa) = 6.5016 kJ/kgK, h_f (50 kPa) = 340.54 kJ/kg, h_{fg} (50 kPa) = 2304.8 kJ/kg. Here, $s_f < s_2 < s_g$, hence the condition of steam at isentropic turbine exit is a two phase mixture. Then, the quality of two phase mixture at turbine exit is given by

$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{6.9760 - 1.0912}{6.5016} = 0.90513$$

Therefore, specific enthalpy of stem at turbine exit is given by

$$h_2 = h_f + x_2 h_{fg} = 340.54 + 0.90513 \times 2304.8 = 2426.684 \text{ kJ/kg}$$

Now applying energy equation for an isentropic turbine,

$$\dot{W}_{isen} = \dot{m}\ [(h_1 - h_2) + \frac{1}{2}(\overline{V_1^2} - \overline{V_2^2}) + g(z_1 - z_2)]$$

Neglecting P.E., we get

$$\dot{W}_{isen} = 67.404\ [(3433.9 - 2426.684) + \frac{1}{2000}(50^2 - 150^2) + 0]$$

$$= 67216.3474 \text{ kW}$$

Therefore, isentropic efficiency of the turbine is given by

$$\eta_{isen} = \frac{\dot{W}_{real}}{\dot{W}_{isen}} = \frac{50000}{67216.3474} = 0.74387 = 74.387\%$$

65. Air enters a gas turbine at 1 MPa and 1500 K and exits at 100 kPa. If its isentropic efficiency is 80 %, determine the turbine exit temperature.

Solution:

Given, Properties of are at inlet: $P_1 = 1$ Mpa = 1000kPa, $T_1 = 1500$ K

Properties of air at exit: $P_2 = 100$kPa

Isentropic efficiency (η_{isen}) = 80% = 0.8

Process: Isentropic (reversible and adiabatic)

Then, temperature of air at turbine exit is given by

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} = 1500 \times \left(\frac{100}{10000}\right)^{\frac{1.4 - 1}{1.4}} = 776.92 \text{ K}$$

Now applying energy equation for an adiabatic turbine,

$$\dot{W}_{isen} = \dot{m}\ [(h_1 - h_2) + \frac{1}{2}(\overline{V_1^2} - \overline{V_2^2}) + g(z_1 - z_2)]$$

For ideal gas using $h_1 - h_2 = c_p(T_1 - t_2)$ and neglecting K.E. and P.E., we get,

$$w_{isen} = \frac{\dot{W}_{isen}}{\dot{m}} = c_p (T_1 - T_2) + 0 + 0 = 1.005\,(1500 - 776.92) = 726.695 \text{ kJ/kg}$$

Therefore, isentropic efficiency of the turbine is given by

$$\eta_{isen} = \frac{w_{real}}{w_{isen}}$$

$$\therefore w_{real} = \eta_{isen} \times w_{isen} = 0.8 \times 726.695 = 581.356 \text{ kJ/kg}$$

Now applying energy equation for a turbine,

$$w_{real} = \dot{m}\,[(h_1 - h_2) + \frac{1}{2}(\overline{V_1^2} - \overline{V_2^2}) + g\,(z_1 - z_2)]$$

For an ideal gas using $h_1 - h_2 = c_p (T_1 - T_2)$ and neglecting

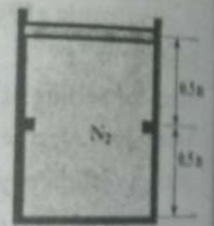
$\therefore$ K.E. and P.E., we get,

$$w_{isen} = \frac{\dot{W}_{isen}}{\dot{m}} = c_p (T_1' - T_2) + 0 + 0$$

$$\therefore T_2' = T_1 - \frac{w_{real}}{c_p} = 1500 - \frac{581.356}{1.005} = 921.54 \text{ K}$$

5.2 IOE Solutions

1. A piston cylinder device shown in figure below contains 1 kg of Nitrogen initially at a pressure of 250 kPa and a temperature of 500° C. Heat is lost from the system till its temperature reaches 40° C. Sketch the pressure on P-V and T-V diagrams and determine the energy generation. Assume that surrounding is at 20° C. [Take P= 297 J/kgK, C_v= 743 J/kgK]. (IOE 2070 Bhadra)



Solution:

Given, Mass of N_2 (m) = 1 kg

Initial state: $P_1 = 250$ kPa, $T_1 = 500°C = 500 + 273 = 773$ K

Final state: $T_{final} = 40°C = 40 + 273 = 313$ K

Temperature of the surrounding $(T_{sur}) = 20°C = 20 + 273 = 293$ K

Volume of N_2 at initial state is given by

$$V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 297 \times 773}{250 \times 10^3} = 0.918324 \text{ m}^3$$

If heat is lost by the system, piston drops downward and process (Process 1- 2) occurs at constant pressure of 250 kPa and volume decreases to half of the initial volume. Hence, the temperature of the system when the piston just hits the stop is calculated as

$$T_2 = \frac{V_2}{V_1} \times T_1 = \frac{1}{2} \times 773 = 386.5 \text{ K} = 113.5°C$$

But the required final temperature is 40°C, hence it is further cooled to decrease the temperature from 113.5°C to 40°C and the process occurs at constant volume (Process 2 - 3). Hence we can define state 3 as,

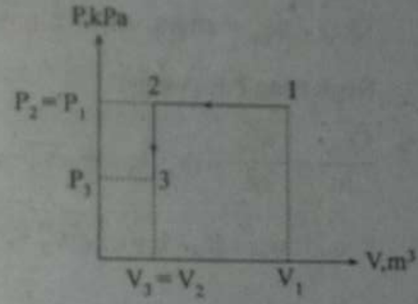
State 3: $T_3 = 313$ K, $V_3 = V_2 = \frac{V_1}{2} = \frac{0.918324}{2} = 0.459162 \text{ m}^3$

$\therefore$ Pressure of N_2 at final state, $P_3 = \frac{mRT_3}{V_3} = \frac{1 \times 297 \times 313}{0.439162} = 202.46$ kPa

Then, change in total internal energy is given by

$$\Delta U = m\,(u_3 - u_1) = mc_p\,(T_3 - T_1)$$

$$= 1 \times 743 \times (313 - 773) = -341.78 \text{ kJ}$$



Work transfer during the process is given by

$$W = W_{12} + W_{23} = P_1\,(V_2 - V_1) + 0$$

$$= 250\,(0.459162 - 0.918324)$$

$$= -114.791 \text{ kJ}$$

Total heat transfer during the process is given by

$$Q = \Delta U + w = -341.78 - 114.791 = -456.571 \text{ kJ}$$

Then change in entropy for the process is given by

$$\Delta S = mc_v \ln\left(\frac{T_3}{T_1}\right) + mR \ln\left(\frac{V_3}{V_1}\right)$$

$$= 1 \times 743 \times \ln\left(\frac{313}{773}\right) + 1 \times 297 \times \ln\left(\frac{0.459162}{0.918324}\right)$$

$$\therefore S_2 - S_1 = -0.52323 \text{ kJ/K}$$

Therefore, rate of entropy generation is given by

$$S_{gen} = (dS)_{CM} - \Sigma\left(\frac{Q_i}{T_i}\right)_{CM}$$

$$= (S_2 - S_1) - \frac{Q}{T_{sur}} = -0.52323 - \left(\frac{-456.571}{293}\right) = 1.035033 \text{ kJ/kg}$$

2. **The conditions of steam at entrance and exit of a turbine are: h_1 = 3456.5 kJ/kg, s_1 = 7.2338 kJ/kgK, and velocity of 150 m/s; h_2 = 27[illegible] kJ/kg, s_2 = 7.4665 kJ/kgK, velocity of 100 m/s respectively. The work output per kg of the steam flow is 600 kJ. Heat transfer between of 500 K. Determine the entropy generation per kg steam flow. (IOE 20[illegible] Bhadra)**

Solution:

Given, Properties of steam at inlet: h_1 = 3456.5 kJ/kg, s_1 = 7.2338 kJ/kgK

$\overline{V_1}$ = 150 m/s

Properties of steam at exit: h_2 = 2792.8 kJ/kg, s_2 = 7.4665 kJ/kgK, $\overline{V_2}$ = 100 m/s

Work output per kg of steam (w_{cv}) = 600 kJ

Temperature of the surrounding (T_{sur}) = 500 K

Applying energy equation for a turbine,

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}\left(\overline{V_2}^2 - \overline{V_1}^2\right) + g(z_2 - z_1)\right]$$

Neglecting P.E., we get

$$\frac{\dot{Q}_{CV}}{\dot{m}} + \frac{\dot{W}_{CV}}{\dot{m}} = (h_2 - h_1 + \frac{1}{2}\left(\overline{V_2}^2 - \overline{V_1}^2\right) + 0$$

$$\therefore q_{cv} = w_{cv} + [h_2 - h_1) + \frac{1}{2}\left(\overline{V_2}^2 - \overline{V_1}^2\right)]$$

$$= 600 + [(2792.8 - 3456.5) + \frac{1}{2000}(100^2 - 150^2)]$$

$$= -69.95 \text{ kJ}$$

Therefore, entropy generation per kg steam flow is given by

$$s_{gen} = (s_{out} - s_{in}) - \Sigma\left(\frac{q_i}{T_i}\right)_{CV}$$

$$= (s_2 - s_1) - \frac{q_{cv}}{T_{sur}} = (7.4665 - 7.2338) - \left(\frac{-69.95}{500}\right) = 0.3726 \text{ kJ/kgK}$$

3. **Steam enters an adiabatic turbine at 10 MPa and 550° C. Exit conditions are 0.06 MPa and a quality of 96%. Determine the isentropic efficiency and actual work output for a mass flow rate of 10 kg/s. *(IOE 2069 Poush)***

Solution:

Given, Properties of steam at inlet: P_1 = 10 MPa = 10000 kPa, T_1 = 550°C

Properties of steam at outlet: P_2 = 0.06 MPa = 60 kPa, x_{2r} = 96% = 0.96

Mass flow rate of steam ($\dot{m}$) = 10 kg/s

Process: isentropic (reversible and adiabatic)

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (10000 kPa) = 311.03°C. Here, T > T_{sat}, hence the conditions of steam at turbine inlet is superheated vapor. Now referring to the Table A2.4,

h_1 = 3500.9 kJ/kg, s_1 = 6.7561 kJ/kg

Since, entropy remains constant during isentropic process, specific entropy at turbine exit is s_2 = 6.7561 kJ/kg

Referring to the Table A.1, s_f (60 kPa) = 1.1454 kJ/kgK,

s_{fg} (60 kPa) = 6.3856 kJ/kgK, s_g (60 kPa) = 7.5310 kJ/kgK

h_f (60 kPa) = 359.90 kJ/kg, h_{fg} (60 kPa) = 2293.1 kJ/kg. Here, $s_f < s_2 < s_g$, hence the condition of steam at turbine exits is a two phase mixture. Then, quality of steam at turbine exit is given by

$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{6.7561 - 1.1454}{6.3856} = 0.8786$$

Then, specific enthalpy of steam at turbine exit is given by

$h_2 = h_f + x_2 h_{fg}$ = 259.90 + 0.8786 × 2293.1 = 2374.618 kJ/kg

Now applying energy equation for an isentropic turbine,

$$\dot{W}_{CV} = \dot{m}\left[(h_1 - h_2) + \frac{1}{2}\left(\overline{V_1}^2 - \overline{V_2}^2\right) + g(z_1 - z_2)\right]$$

Neglecting K.E. and P.E., we get

$\dot{W}_{CV}$ = 10 × (h_1 - h_2) + 0 + 0 = 10 (3500.9 - 2374.618)

$\therefore$ $\dot{W}_{isen}$ = 11262.82 kW

Again, specific enthalpy of steam at turbine exit is given by

$h_{2r} = h_f + x_{2r} h_{fg}$ = 359.90 + 0.96 × 2233.1 = 2561.276 kJ/kg

Now applying energy equation for an adiabatic turbine,

$$\dot{W}_{CV} = \dot{m}\left[(h_1 - h_{2r}) + \frac{1}{2}\left(\overline{V_1}^2 - \overline{V_2}^2\right) + g(z_1 - z_2)\right]$$

Neglecting K.E. and P.E., we get

$\dot{W}_{CV}$ m (h_1 - h_{2r}) + 0 + 0

$\dot{W}_{CV}$ = 10 (3500.9 - 2561.276) = 9396.24 kW

Therefore, isentropic efficiency of the turbine is given by

$$\eta_{iT} = \frac{\dot{W}_{actual}}{\dot{W}_{isen}} = \frac{9396.24}{11262.82} = 0.8343 = 83.43\%$$

Actual work output per kg of steam is given as

$$W_{actual} = \frac{\dot{W}_{actual}}{\dot{m}} = \frac{9396.24}{10} = 939.624 \text{ kJ}$$

4. **A heat engine working on Carnot cycle converts one-fifth of the heat input into work. When the temperature of the sink is reduced by 80°C, the efficiency gets doubled. Make calculations for the temperature of source and sink. *(IOE 2069 Ashad)***

Solution:

Let the source temperature and the sink temperature of the heat engine be T_H and T_L respectively. Also let heat input and work output of the heat engine be Q_H and W respectively.

$Q_H = 5W$

Then efficiency of the the heat engine is given by

$$\eta = \frac{W}{Q_H}$$

$$\text{or, } 1 - \frac{T_L}{T_H} = \frac{W}{5W} = \frac{1}{5} = 0.2$$

$$\text{or, } \frac{T_L}{T_H} = 0.8 \therefore T_L = 0.8\, T_H$$

When the sink temperature is reduced by 80°C (= 80 K), its efficiency get doubled i.e.,

$$\eta = 1 - \frac{T_L}{T_H}$$

$$\text{or, } 2 \times 0.2 = 1 - \frac{T_L - 80}{T_H}$$

$$\text{or, } \frac{T_L - 80}{T_H} = 0.6$$

$$\text{or, } 0.8\, T_H - 80 = 0.6\, T_H$$

$$\text{or, } 0.2\, T_H = 80$$

$$\therefore T_H = 400 \text{ K}$$

And, $T_L = 0.8 \times 400 = 320$ K

5. **Steam enters into a turbine at a rate of 2 kg/s with P_1 = 2 MPa, T_1 = 600°C and exits at P_2 = 9 kPa. Find:**

(a) **Power output if the turbine is isentropic,**
(b) **Power output if isentropic efficiency of the turbine is 80% and**
(c) **Outlet enthalpy of steam from the real turbine. *(IOE 2068 Chaitra)***

Solution:

Given, Mass flow rate of steam ($\dot{m}$) = 2 kg/s

Properties of steam at inlet: P_1 = 2MPa = 2000 kPa, T_1 = 600°CS

Properties of steam at outlet: P_2 = 9 kPa

For other properties of steam at inlet, referring to Table A2.1

T_{sat} (2000 kPa) = 212.42°C. Here, T > T_{sat}, hence, the condition of steam at turbine inlet is a superheated vapor. Now, referring to the Table A2.4,

h_1 = 3690.2 kJ/kg s_1 = 7.7024 kJ/kgK

Since entropy remains constant during isentropic process, entropy at the turbine exit is s_2 = 7.7024 kJ/kgK

Referring to Table A2.1, s_l (9 kPa) = 0.6223 kJ/kgK, s_{lg} (9kPa) = 7.5629 kJ/kgK, s_g (9kPa) = 8.1852 kJ/kgK, h_l (9kPa) = 183.27 kJ/kg, h_{lg} (9 kPa) = 2396.8 kJ/kg

Here, $s_l < s_2 < s_g$, hence the condition of steam at turbine exits is a two phase mixture. Therefore, quality of the two phase mixture is given by

$$x_2 = \frac{s_2 - s_l}{s_{lg}} = \frac{7.7024 - 0.6223}{7.5629} = 0.9362$$

Therefore, specific enthalpy of steam at the turbine exit is given by

$$h_2 = h_l + x_2 h_{lg} = 183.27 + 0.9362 \times 2396.8 = 2427.154 \text{ kJ/kg}$$

Now, applying energy equation for an isentropic turbine

$$\dot{W}_{isen} = \dot{m}(h_1 - h_2) = 2 \times (3690.2 - 2427.154)$$

$$= 2526.092 \text{ kW}$$

Isentropic efficiency of the turbine is given by

$$\eta_{iT} = \frac{\dot{W}_{real}}{\dot{W}_{isen}}$$

Therefore, power output from the real turbine is given by

$$\dot{W}_{real} = \eta_{iT} \times \dot{W}_{isen} = 0.8 \times 2526.092 = 2020.8736 \text{ kW}$$

Power output from the real turbine can also be given as

$$\dot{W}_{real} = \dot{m}(h_1 - h_{2r})$$

Therefore, outlet enthalpy of steam from the real turbine is given by

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$$h_{2r} = h_1 - \frac{\dot{W}_{real}}{\dot{m}} = 3690.5 - \frac{2020.8736}{2} = 2680.063 \text{ kJ/kg}$$

6. **Two kg of water at 90⁰ C is mixed with three kg of water at 10⁰ C in an isolated system. Calculate the change of entropy due to the mixing process. [C_p for water= 4.18 kJ/kgK] *(IOE 2068 Shrawan)***

Solution:

Given, Mass of water 1 (m_1) = 2 kg

Initial temperature of water 1 (T_1) = 90° C = 90 + 273 = 363 K

Mass of water 2 (m_2) = 3 kg

Initial temperature of water 2 (T_2) = 10° C = 10 + 273 = 283 K

Let T_3 be the equilibrium temperature then heat lost by water 1 is absorbed by the water 2, i.e.

$m_1c(T_1 - T_3) = m_2c(T_3 - T_2)$

or, $2 \times 4.18 \times (363 - T_3) = 3 \times 4.18 \times (T_3 - 283)$

or, $726 - 2T_3 = 3T_3 - 849$

or, $5T_3 = 1575$

$\therefore T_3 = 315 \text{ K} = 42°C$

Then, change in entropy of the water 1 is given by

$$(\Delta S)_1 = m_1c \ln\left(\frac{T_3}{T_1}\right) = 2 \times 4.18 \times \ln\left(\frac{315}{363}\right) = -1.1857 \text{ kJ/K}$$

Change in entropy of the water 2 is given by

$$(\Delta S)_2 = m_2c \ln\left(\frac{T_3}{T_2}\right) = 3 \times 4.18 \times \ln\left(\frac{315}{283}\right) = 1.34336 \text{ kJ/K}$$

Therefore, the net change in entropy is given by

$(\Delta S)\text{ net} = (\Delta S)_1 + (\Delta S)_2 = -1.1857 + 1.34336 = 0.15766$ kJ/kg

7. **Steam enters an adiabatic turbine at 10 MPa and 510⁰ C. Exit conditions are 0.06 MPa and quality of 96%. Determine the isentropic efficiency and actual work for a mass flow rate of 10 kg/s. *(IOE 2068 Baishak)***

Solution:

Given, Mass flow rate of steam ($\dot{m}$) = 10 kg/s

Properties of steam at inlet: P_1 = 10 MPa = 10000 kPa, T_1 = 510°C

Properties of steam at exit: P_2 = 0.06 MPa = 60 kPa, x_{2r} = 0.96

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} (10000 kPa) = 311.03°C. Here, $T > T_{sat}$, hence the condition of steam at turbine inlet is a superheated vapor. Now, referring to the Table A2.4, temperature of a superheated steam which includes temperature 510°C and corresponding specific enthalpy and specific entropy are listed as:

T,°C	h_g, kJ/kg	s_g, kJ/kgk	
500	3374.0	6.5971	(a)
550	3500.9	6.7561	(b)

Now, applying linear interpolation for specific enthalpy and specific entropy

$$h_1 - (h_g)_a = \frac{(h_g)_b - (h_g)_a}{T_b - T_a}(T_1 - T_a)$$

$$\therefore h_1 = (h_g)_a + \frac{(h_g)_b - (h_g)_a}{T_b - T_a}(T_1 - T_a)$$

$$= 3374.0 + \frac{3500.9 - 3374.0}{550 - 500}(510 - 500) = 3475.52 \text{ kJ/kg}$$

Similarly, $s_2 = (s_g)_a + \frac{(s_g)_b - (s_g)_a}{T_b - T_a}(T_1 - T_a)$

$$= 6.5971 + \frac{(6.7561 - 6.5971)}{550 - 500}(510 - 500) = 6.6289 \text{ kJ/kgK}$$

Since, entropy remains constant during isentropic process, entropy at turbine exits is s_2 = 6.6289 kJ/kgK

Referring to the Table A2.1, s_f (60 kPa) = 1.1454 kJ/kgK,

s_{fg} (60 kPa) = 6.3856 kJ/kgK, s_g (60 kPa) = 7.5310 kJ/kgK,

h_f (60 kPa) = 359.90 kJ/kg, h_{fg} (60 kPa) = 2293.1 kJ/kgK

Here, $s_f < s_2 < s_{fg}$, hence the condition of steam at turbine exits if a two phase mixture. Then quality of steam at isentropic turbine exits is given by

$$x_2 = \frac{s_1 - s_f}{s_{fg}} = \frac{6.6289 - 1.1454}{6.3856} = 0.8587$$

Then specific enthalpy of steam at isentropic turbine exits is given by $h_2 = h_f + x_2h_{fg}$ = 359.90 + 0.8587 × 2293.1 = 2328.945 kJ/kg

Now applying energy equation for an isentropic turbine,

$\dot{W}_{isen} = \dot{m}(h_2 - h_2)$ = 10 (3475.52 - 2328.945) = 11465.75 kW

Again, specific enthalpy of steam at real turbine is given by

$h_{2r} = h_f + x_{2r}h_{fg}$ = 359.9 + 0.96 × 2293.1 = 2561.276 kJ/kg

Then, power output from the real turbine si given as

$\dot{W}_{real} = \dot{m}\,(h_1 - h_{2a}) = 10 \times (3475.52 - 2561.276) = 9142.44$ kW

Therefore, isentropic efficiency of the turbine is given by

$$\eta_{isT} = \frac{\dot{W}_{real}}{\dot{W}_{isen}} = \frac{9142.44}{11465.75} = 0.7974 = 79.74\%$$

Actual work is given by

$$w_{real} = \frac{\dot{W}_{real}}{\dot{m}} = \frac{9142.44}{10} = 914.244 \text{ kJ}$$

8. **Steam enters the nozzle at 1 MPa, 300° C, with a velocity of 30 m/s. The pressure of the steam at the nozzle exit is 0.3 MPa. Determine the exit velocity of the steam from the nozzle, assuming a reversible and adiabatic steady flow process. *(IOE 2067 Ashad)***

Solution:

Given, Properties of steam at inlet: P_1 = 1 MPa = 1000 kPa, T_1 = 300°C, $\overline{V_1}$ = 30 m/s

Properties of steam at exit: P_2 = 0.3 MPa = 300 kPa

Process: reversible and adiabatic

For other properties of steam at inlet, referring to the Table A2.1, T_{sat} = 179.92°C. Here, $T > T_{sat}$, hence the condition of steam at nozzle exit is a superheated vapor. Now referring to the Table A2.4,

h_1 = 3050.6 kJ/kg, s_1 = 7.1219 kJ/kgK

Since the entropy remains constant during the isentropic process, entropy of steam at turbine exit is s_2 = 7.1219 kJ/kgK

Referring to the Table A2.1, s_g (300 kPa) = 6.9921 kJ/kgK.

Here, $s_2 > s_g$, hence the condition of steam at nozzle exit is a superheated vapor. Now, referring to the Table A2.4, specific entropy of a superheated vapor which includes specific entropy 7.1219 kJ/kgK and corresponding specific enthalpy are listed as:

s_g, kJ/kgK	h_g, kJ/kg	
7.0779	2760.9	(a)
7.3108	2865.1	(b)

Now, applying linear interpolation for specific enthalpy,

$$h_2 - (h_g)_a = \frac{(h_g)_b - (h_g)_a}{(s_g)_b - (s_g)_a}[s_2 - (s_g)_a]$$

$$h_2 = (h_g)_a + \frac{(h_g)_b - (h_g)_a}{(s_g)_b - (s_g)_a}[s_2 - (s_g)_a]$$

$$= 2760.9 + \frac{(2865.1 - 2760.9)}{(7.3108 - 7.0779)}(7.1219 - 7.0779) = 2780.5857 \text{ kJ/kg}$$

Now applying energy equation for adiabatic nozzle,

$$h_1 + \frac{1}{2}\overline{V_1}^2 = h_2 + \frac{1}{2}\overline{V_2}^2$$

$$\therefore \overline{V_2} = \sqrt{2\,(h_1 - h_2) + \overline{v_1}^2}$$

$$= \sqrt{2(3050.6 - 2780.5857) + 30^2}$$

$$= 37.948 \text{ m/s}$$

9. **A cold storage is to be maintained at -5° C while the surroundings are at 35° C. The heat leakage from the surroundings into the cold storage is estimated to be 50 kW. The actual COP of the refrigeration plant is half of an ideal plant working between the same temperatures. Find the power required to drive the plant. *(IOE 2067 Chaitra)***

Solution:

Given, Lower temperature (T_L) = -5°C = - 5 + 273 = 268 K

Higher temperature (T_H) = 35°C = 35 + 273 = 308 K

Rate at which heat is taken out from cold storage ($\dot{Q}_L$) = 50 kW

COP of the ideal refrigerant plant operating between the temperature limits is given by

$$(COP)_{rev,\,R} = \frac{T_L}{T_H - T_L} = \frac{268}{308 - 268} = 6.7$$

COP of the refrigerant plant is half of an ideal plant

$$\therefore (COP)_R = \frac{1}{2}\,(COP)_{rev,\,R} = \frac{1}{2} \times 6.7 = 3.35$$

COP of the refrigerant plant is given by

$$(COP)_R = \frac{\dot{Q}_L}{\dot{W}}$$

Therefore, power required to drive the plant is given as

$$\dot{W} = (COP)_R \times \dot{Q}_L = 3.37 \times 50 = 167.5 \text{ kW}$$

10. A Carnot engine operates between two reservoirs at temperature T_L and T_H. The work output of the engine is 0.6 times the heat rejected. The difference in temperature between the source and the sink is 200° C. Calculate the thermal efficiency, source temperature and the sink temperature. *(IOE 2067 Mangsir)*

Solution:

Given, $T_H - T_L = 200°C$ (200 K)

$W = 0.6 \times (\dot{Q}_L)$

Efficiency of a Carnot engine operating between two reservoirs at temperature T_H and T_L is given by

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = \frac{T_H - T_L}{T_H} = \frac{200}{T_H} \quad ...(i)$$

Also efficiency of Carnot engine is given as

$$\eta_{rev} = \frac{\dot{W}}{\dot{Q}_H} = \frac{0.6 \times \dot{Q}_L}{\dot{Q}_H} = 0.6\frac{T_L}{T_H} \quad ....(ii)$$

Substituting equation (ii) into equation (i) we get,

$$\frac{200}{T_H} = 0.6\frac{T_L}{T_H}$$

$$\therefore\ T_L = \frac{200}{0.6} = 333.33 \text{ K}$$

And $T_H = 333.33 + 200 = 533.33$ K.

$$\text{Efficiency } (\eta_{rev}) = 1 - \frac{T_L}{T_H} = 1 - \frac{333.33}{533.33} = 0.375 = 37.5\%$$

11. Steam at 700 kPa with a quality of 0.96, is throttled down to 350 kPa. Calculate the change of entropy per unit mass of steam. *(IOE 2067 Mangsir)*

Solution:

Given, Properties of steam at inlet: $P_1 = 700$ kPa, $x_1 = 0.96$

Properties of steam at exit : $P_2 = 350$ kPa

For other properties of steam at inlet, referring to the Table A2.1, h_f (700 kPa) = 697.35 kJ/kg, h_{fg} (700 kPa) = 2066.0 kJ/kg, s_f (700 kPa) = 1.9925 kJ/kgK, s_{fg} (700 kPa) = 4.7154 kJ/kgK

Specific enthalpy and specific entropy of mixture at inlet are given as

$$h_1 = h_f + x_1 h_{fg} = 697.35 + 0.96 \times 2066.0 = 2680.71 \text{ kJ/kg}$$

$$s_1 = s_f + x_1 s_{fg} = 1.9925 + 0.96 \times 4.7154 = 6.5193 \text{ kJ/kgK}$$

Since enthalpy remains constant during throttling process, enthalpy of steam at throttling valve exit is $h_2 = 2680.71$ kJ/kg

Referring to the Table A2.1, h_f (350 kPa) = 584.48 kJ/kg, h_{fg} (350 kPa) = 2147.9 kJ/kg, h_g (350kPa) = 2732.4 kJ/kg, s_f (350 kPa) = 1.7278 kJ/kgK, s_{fg} (350 kPa) = 5.2129 kJ/kgK

Here, $h_f < h_2 < h_g$, hence the condition of steam at valve exit is a two phase mixture. Then, quality of a two phase mixture at exit is given by

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{2680.71 - 584.48}{2147.9} = 0.9759$$

Therefore, specific entropy of two phase mixture at exit is given by $s_2 = s_f + x_2 s_{fg}$ $= 1.7278 + 0.9759 \times 5.2129 = 6.8151$ kJ/kgK

Then, change of entropy per unit mass is given as

$$\Delta s = s_2 - s_1 = 6.8151 - 6.5193 = 0.2958 \text{ kJ/kgK}$$

12. A control mass system consists of ice and water 12 kg of water, at 37° C is mixed with 8 kg of ice at –27° C. Assuming the process of mixing is adiabatic, find the change of entropy. Latent heat of ice= 336kJ/kg, C_p for water= 4.2 kJ/kgK. *(IOE 2070 Magh)*

Solution:

Given, mass of water $(m_w) = 12$ kg

Initial temperature of water $(T_{w1}) = 37° C = 37 + 273 = 310$ K

Mass of ice $(m_i) = 8$ kg

Initial temperature of ice $(T_{i1}) = -27° C = -27 + 273 = 246$ K

Heat required for melting all ice into water at 0° C is given as

$= m_i c_i (0 - T_{i1}) + m_i L = 8 \times 0.205 \times (0 + 27) + 8 \times 336 = 2732.28$ kJ

Heat available from water before freezing is given as

$= m_w c_w (T_{w1} - 0) = 12 \times 4.2 \times (37 - 0) = 1864.8$ kJ

Here, heat available from water is less than heat required to melt all the ice. Hence all ice does not melt. Only certain amount of ice will melt and final temperature of mixture will be 0° C.'

Let, m be the amount og ice that will melt then heat lost by water is absorbed by ice, i.e.,

$m_w c_w (T_{w1} - 0) = m_i c_i (0 - T_{i1}) + mL$

$8 \times 0.205 \times (0 + 27) = 8 \times 0.205 \times (0 + 27) + m \times 336$

$m = 5.4182$ kg

Change in entropy of water is given by

$$(\Delta S)_w = m_w c_w \ln\left(\frac{273}{T_{wi}}\right) = 4 \times 4.2 \times \ln\left(\frac{273}{310}\right) = -6.4059 \text{ kJ/K}$$

Then change in entropy of the ice is given by the summation of the change in entropy of ice when its temperature increase from T_{i1} to 273 K and change in entropy of ice during melting of 5.4182 kg of ice i.e.,

$$(\Delta S)_i = m_i c_i \ln\left(\frac{273}{T_{i1}}\right) + \frac{mL}{273}$$

$$= 8 \times 0.205 \times \ln\left(\frac{273}{240}\right) + \frac{5.4182 \times 336}{273} = 6.8393 \text{ kJ/K}$$

Net change in entropy due to mixing process is then given by

$$\Delta S = (\Delta S)_w + (\Delta S)_i = -6.4059 + 6.8393 = 0.43345 \text{ kJ/K}$$

5.3 Some Important Extra Questions

1. **An inventor makes the following claims. Determine whether the claims are valid or not and explain why or why not.**
   - (a) **A petrol engine operating between temperatures 2000° C and 500° C will produce 1.2 kW of power output consuming 0.15 kg/h of petrol having a calorific value of 42500 kJ/kg.**
   - (b) **A heat pump supplies heat to a room maintained at 22° C at a rate of 50000 kJ/h. The inventor claims a work input of 5000 kJ/h is sufficient when the surroundings is at -2° C.**
   - (c) **A refrigerator maintains -5° C in the refrigerator which is kept in a room where the temperature is 30° C and has a COP of 8.**

Solution:

a) Given, Higher Temperature (T_H) = 2000°C = 2000 + 273 = 2273 K

Lower temperature (T_1) = 500°C = 500 + 273 = 773 K

Power output ($\dot{W}$) = 1.2 kW

Fuel consumption rate ($\dot{m}_f$) = 0.15 kg/h

Calorific value of fuel (CV) = 42500 kJ/kg

Maximum possible efficiency of the engine operating between the given temperature limits is given by

$$\eta_{rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{773}{2273} = 65.992 \%$$

Rate at which heat is supplied to the engine is given as

$$\dot{Q}_H = \dot{m}_f \cdot CV = \frac{0.15}{3600} \times 42500 = 1.7708 \text{ kW}$$

Therefore, efficiency of the engine according to the inventor's claim's given as

$$\eta_{inventor} = \frac{\dot{W}}{\dot{Q}_H} = \frac{1.2}{1.7708} = 67.765 \%$$

Hence, $\eta_{inventor} > \eta_{rev}$, hence given statement is not valid.

b) Given, higher temperature (T_H) = 22°C = 22 + 273 = 295 K

Lower temperature (T_L) = - 2°C = - 2 + 273 = 271 K

Heating rate ($\dot{Q}_H$) = 50000 kJ/h

Power input ($\dot{W}$) = 8000 kJ/h

Maximum possible COP of the heat pump operating between the given temperature limits is given by

$$(COP)_{rev, HP} = \frac{T_H}{T_H - T_L} = \frac{295}{295 - 271} = 12.29$$

COP of the heat pump according to the inventor's claim is given as (COP)

$$\text{inventor} = \frac{Q_H}{W} = \frac{50000}{8000} = 6.25$$

Here, $(COP)_{inventor} < (COP)_{rev, HP}$, hence the given statement is valid and the heat pump is running under the irreversible conditions.

c) Given, Higher temperature (T_H) = 30°C = 30 + 273 = 303 K

Lower temperature (T_L) = - 5°C = - 5 + 273 = 268 K

COP according to the inventor's claim (COP) inventor = 8

Maximum possible COP is the refrigerator operating between the given temperature limits is given by

$$(COP)_{rev,R} = \frac{T_L}{T_H - T_L} = \frac{268}{303 - 268} = 7.657$$

Here, $(COP)_{inventor} > (COP)_{rev,R}$, hence the given statement is not valid.

2. **An ideal engine has a efficiency of 25 %. If the sink temperature is reduced by 100° C, its efficiency gets doubled, determine its source and sink temperatures.**

Solution:

Let the source temperature and the sink temperature of the engine be T_H and T_L respectively. Then its efficiency is given by

$$\eta_1 = 1 - \frac{T_L}{T_H}$$

or, $0.25 = 1 - \frac{T_L}{T_H}$

or, $0.75 = \frac{T_L}{T_H}$

$\therefore T_L = 0.75\ T_H$ ....(i)

When the source temperature is increased by 200°C (= 200K), its efficiency gets doubled. i.e.,

$$\eta_2 = 1 - \frac{T_L}{T_H + 200}$$

or, $0.5 = 1 - \frac{0.75 \times T_H}{T_H + 200}$ ....(ii)

Substituting equation (i) into equation (ii),

$$0.5 = 1 - \frac{T_L}{T_H + 200}$$

or, $0.5T_H + 100 = 0.75T_H$

$\therefore T_H = 400$ K

Substituting T_H into Equation (i), we get

$T_L = 0.75 \times 400 = 300$ K

3. **A heat pump has a coefficient of performance that is 80% of the theoretical maximum. It maintains a hall at 20°C, which leaks energy 1 kW per degree temperature difference to the ambient. For a maximum of 1.5Kw power input, determine the minimum outside temperature for which the heat pump is sufficientSolution:**

Given, Higher temperature (T_H) = 20°C = 20 + 273 = 293 K

Heating rate ($\dot{Q}_H$) = 1 × (T_H - T_L) = (T_H - T_L) kW

Power input ($\dot{W}$) = 1.5 kW

Actual COP of the heat pump is 80% of the theoretical maximum (reversible COP), i.e.,

$$(COP)_{actual,\ HP} = 0.8 \times (COP)_{rev,\ HP}$$

or, $\frac{\dot{Q}_H}{\dot{W}} = 0.8 \times \frac{T_H}{T_H - T_L}$

or, $\frac{T_H - T_L}{1.5} = 0.8 \times \frac{T_H}{T_H - T_L}$

or, $(T_H - T_L)^2 = 1.2\ T_H = 1.2 \times 293 = 351.6$

or, $T_H - T_L$ 18.751 (Taking only positive value, $T_H > T_L$).

or, $T_L = T_H - 18.751 = 293 - 18.751 = 274.249$ K = 1.249°C

4. **4 kg of water at 25° C is mixed with 1 kg of ice at 0° C in an isolated system. Calculate the change in entropy due to mixing process. [Take latent heat of ice L=336kJ/kgK and specific heat of water c=4.18kJ/kgK]**

Solution:

Given, Mass of water (m_w) = 4 kg

Initial temperature of water (T_{w1}) = 25°C = 25 + 273 = 298 K

Mass of ice (m_i) = 1 kg

Initial temperature of ice (T_{i1}) = 0°C = 0 + 273 = 273 K

Let T_2 be the final equilibrium temperature, then the heat lost by water is absorbed by the ice i.e.

$m_w c (T_{w1} - T_2) = m_i L + m_i c (T_2 - T_{i1})$

or, $4 \times 4.18 \times (298 - T_2) = 1 \times 336 + 1 \times 4.18 \times (T_2 - 273)$

or, $4892.56 - 16.72\ T_2 = 336 + 4.18\ T_2 - 1141.14$

or, $20.9T_2 = 5787.7$

$\therefore T_2 = 276.9234$ K

Then change in entropy of the water is given by

$$(\Delta S)_w = m_w c \ln\left(\frac{T_2}{T_{wi}}\right) = 4 \times 4.18 \times \ln\left(\frac{276.9234}{298}\right) = -1.22645 \text{ kJ/K}$$

Then change in entropy of the ice is given by the summation of change in entropy of the ice during melting of ice and the change entropy of water when its temperature increase from 273 K to T_2 i.e.,

$$(\Delta S)_i = \frac{m_i L}{273} + m_i c \ln\left(\frac{T_2}{273}\right)$$

$$= \frac{1 \times 336}{273} + 4 \times 4.18 \times \ln\left(\frac{276.9234}{273}\right) = 1.25189 \text{ kJ/K}$$

Net change in entropy due to the mixing process is then given by

$$\Delta S = (\Delta S)_w + (\Delta S)_i = -1.22645 + 1.25189 = 0.02544 \text{ kJ/K}$$

5. **Steam at 1 MPa and 300° C is flowing with a velocity of 50 m/s reversibly and adiabatically through a nozzle and leaves the nozzle at 150 kPa. Determine the exit velocity of steam from the nozzle.**

Solution:

Given, Properties of steam at inlet: $P_1 = 100$ kPa, $T_1 = 300°C$, $\overline{V_1} = 50$ m/s

Properties of steam at outlet: $P_2 = 150$ kPa

Process: Reversible and adiabatic (isentropic)

For the other properties of steam at inlet, referring to the Table A2.1, T_{sat} (1000 kPa) = 179.92°C. Here $T > T_{sat}$, hence it is a superheated steam. Now, referring to the Table A2.4,

$h_1 = 3050.6$ kJ/kg, $s_1 = 7.1219$ kJ/kgK

Since, entropy remains constant during isentropic process, entropy at the nozzle exit is $s_2 = 7.1219$ kJ/kg

Referring to the Table A2.1, s_f (150 kPa) = 1.4338 kJ/kgK, s_{fg} (150 kPa) = 5.7894kJ/kgK, s_g (150 kPa) = 7.2232 kJ/kgK, h_f (150 kPa) = 467.18 kJ/kg, h_{fg} (150kPa) = 2226.2 kJ/ kg. Here, $s_f < s_2 < s_g$, hence it is a two phase mixture. Quality at the nozzle exit is given by

$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{7.1219 - 1.4338}{5.7894} = 0.9823$$

Therefore, specific enthalpy of steam at the nozzle exits is given by

$$h_2 = h_f + x_2 h_{fg} = 467.18 + 0.9823 \times 2226.2 = 2654.427 \text{ kJ/kg}$$

Now, applying energy equation for an adiabatic nozzle,

$$h_1 + \frac{1}{2}\overline{V_1^2} = h_2 + \frac{1}{2}\overline{V_2^2}$$

$$\therefore \overline{V_2} = \sqrt{2(h_1 - h_2) + \overline{V_1^2}}$$

$$= \sqrt{2000\,(3050.6 - 2654.427) + 50^2} = 891.54 \text{ m/s}$$

6. **Steam enters into a turbine at 2 MPa, 400° C and with a velocity of 200 m/s and saturated vapor exits from the turbine at 100 kPa with a velocity of 80 m/s. The power output of the turbine is 800 kW when the mass flow rate of steam is 1.5 kg/s. Turbine rejects heat to the surroundings at 300 K. Determine the rate at which the entropy is generated within the turbine.**

Solution:

Given, Properties of steam at inlet, $P_1 = 2$ MPa = 2000 kPa, $T_1 = 400°C$

$\overline{V_1} = 200$ m/s

Properties of steam at outlet: $P_2 = 100$ kPa, saturated vapor, $\overline{V_2} = 80$ m/s

Mass flow rate of steam ($\dot{m}$) = 1.5 kg/s

Power output of the turbine ($\dot{W}_{CV}$) = 800 kW

Temperature of the surrounding (T_{surr}) = 300 K

For the other properties of steam at inlet, referring to Table A2.1, T_{sat} (2000 kPa) = 212.42°C. Here $T > T_{sat}$, hence it is a superheated steam. Now, referring to the Table A2.4,

$h_1 = 3247.5$ kJ/kg and $s_1 = 7.1269$ kJ/kgK

Similarly, for the other properties of steam at outlet, referring to Table A2.1, $h_2 = h_g$ (100 kPa) = 2675.1 kJ/kg and $s_2 = s_g$(100 kPa) = 7.3589 kJ/kgK

Now, applying energy equation for the turbine,

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}\left(\overline{V_2^2} - \overline{V_1^2}\right) + g(z_2 - z_1)\right]$$

$$\therefore \dot{Q}_{CV} = \dot{W}_{CV} + \dot{m}\left[(h_2 - h_1) + \frac{1}{2}\left(\overline{V_2^2} - \overline{V_1^2}\right) + g(z_2 - z_1)\right]$$

$$= 800 + 1.5\left[(2675.1 - 3247.5) + \frac{1}{2000}(80^2 - 200^2) + 0\right] = -83.8 \text{ kW}$$

Then the rate of entropy generation during the steady operation of any control volume is given by

$$\dot{S}_{gen} = (\dot{S}_{out} - \dot{S}_{in}) - \Sigma\left(\frac{\dot{Q}_i}{T_{surr}}\right)_{CV}$$

$$= \dot{m}(s_2 - s_1) - \frac{\dot{Q}_{CV}}{T_{surr}} = 1.5\,(7.3589 - 7.1269) - \frac{(-83.8)}{300} = 0.6273 \text{ kW/K}$$

7. **Steam enters into a turbine at a rate of 2 kg/s with $P_1 = 2$ MPa, $T_1 = 750°$ C and exits at $P_2 = 10$ kPa.**

   (a) **If the turbine is isentropic, what is the power output of the turbine?**

   (b) **If the isentropic efficiency of the turbine is 80 %, what is the power output?**

   (c) **What is the outlet enthalpy of the steam from the real turbine?**

Solution:

Given, Properties of steam at inlet: $P_1 = 2$ MPa = 2000 kPa, $T_1 = 750°C$

Properties of steam at outlet: $P_2 = 10$ kPa

a) For the other properties of steam at inlet, referring to Table A2.1,

T_{sat} (2000 kPa) = 212.42°C. Here $T > T_{sat}$, hence it is a superheated steam. Now, referring to the Table A2.4,

h_1 = 4033.5 kJ/kg, and s_1 = 8.0651 kJ/kgK

Since entropy remains constant during isentropic process, entropy at the turbine exit, s_2 = 8.0651 kJ/kgK

Referring to the Table A2.1, s_f (10kPa) = 0.6493 kJ/kgK, s_{fg} (10 kPa) = 7.4989 kJ/kgK, s_g (10kPa) = 8.1482 kJ/kgK and h_f (10kPa) = 191.83 kJ/kg, h_{fg} (10kPa) = 2392.0 kJ/kg. Here, $s_f < s_2 < s_g$, hence it is a two phase mixture.

Therefore, quality of the steam at exit is given by

$$x_2 = \frac{s_2 - s_f}{s_{fg}} = \frac{8.0651 - 0.6493}{7.4989} = 0.9889$$

Therefore, specific enthalpy of steam at exit is given by

$$h_2 = h_f + x_2 h_{fg} = 191.83 + 0.9889 \times 2392.0 = 2557.323 \text{ kJ/kg}$$

Now, applying energy equation for an isentropic turbine,

$$\dot{W}_{isen} = \dot{m}\,(h_1 - h_2) = 2 \times (4033.5 - 2557.323) = 2214.266 \text{ kW}$$

b) Isentropic efficiency of the turbine is given by

$$\eta_{isT} = \frac{\dot{W}_{real}}{\dot{W}_{isen}}$$

Therefore, power output from the real turbine is given by

$$\dot{W}_{real} = \eta_{isT} \times \dot{W}_{isen} = 0.8 \times 2214.266 = 1771.413 \text{ kW}$$

c) Power output from the real turbine is given as

$$\dot{W}_{real} = \dot{m}\,(h_1 - h_{2a})$$

Therefore, outlet specific enthalpy from the real turbine is given by

$$h_{2a} = h_1 - \frac{\dot{W}_{real}}{\dot{m}} = 4033.5 - \frac{1771.413}{2} = 2852.56 \text{ kJ/kg}$$

Chapter 6 Thermodynamic Cycles

6.1 Numerical problems

1. **Air at 100 kPa and 25 °C enters into a compressor of an ideal Brayton cycle and exits at 1000 kPa The maximum temperature during the cycle is 1127 °C. Determine**

   (a) **the pressure and temperature at each states of the cycle.**

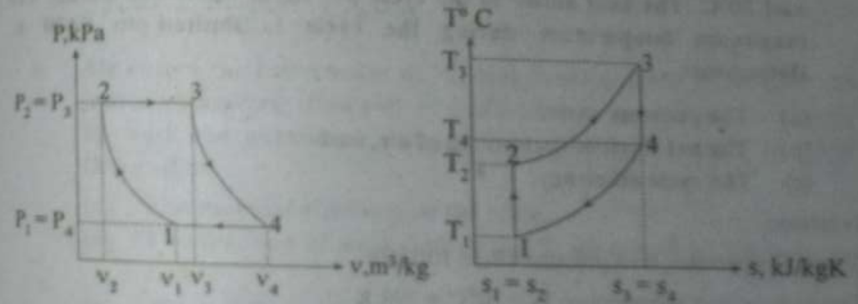
   (b) **the compressor work, turbine work and net work per kg of air, and**

   (c) **the cycle efficiency.**

Solution:

Given, Properties at state 1: P_1 = 100 kPa, T_1 = 25°C = 298 K

Pressure at compressor exit (P_2) = 1000 kPa $P_2 = P_3$



Maximum temperature during the cycle (T_{max}) = 1127° C = 1127 + 273 = 1400 K

State 2: P_2 = 1000 kPa

a) Applying P - T relation for an isentropic compression 1 - 2, temperature at state 2,

$$T_2 = T_1\left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}} = 298\left(\frac{100}{1000}\right)^{\frac{1-1.4}{1.4}} = 575.348 \text{ K}$$

State 3: P_3 = 1000 kPa, T_3 = 1400 K

State 4: $P_4 = 100$ kPa

Applying P - T relation for an insentropic expansion 3 - 4, temperature at state 4,.

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = 1400\left(\frac{100}{1000}\right)^{\frac{1.4-1}{1.4}} = 725.126 \text{ K}$$

b) Work consumed by the compressor per kg of air is then given by

$w_C = c_p (T_2 - T_1) = 1.00 \times (575.348 - 298) = 278.735$ kJ/kg

Work produced by the turbine per kg of air is then given by

$w_T = c_p (T_3 - T_4) = 1.005 \times (1400 - 725.126) = 678.248$ kJ/kg

Net work produced by the cycle per kg of air is then given by

$w_{net} = w_T - w_C = 678.248 - 278.735 = 399.51$ kJ/kg

c) Heat supplied per kg of air is then given by

$q_H = c_p (T_3 - T_2) = 1.005 \times (1400 - 575.348) = 824.652$ kJ/kg

$\therefore$ Efficiency of the cycle is then given by

$$\eta = \frac{w_{net}}{q_H} = \frac{399.51}{824.652} = 48.45\%$$

2. **Air at the compressor inlet of an ideal gas turbine cycle is at 100kPa and 20°C. The heat added to the cycle per kg of air is 800 kJ/kg. The maximum temperature during the cycle is limited to 1400 K. Determine:**

(a) The pressure ratio,

(b) The net work output per kg of air, and

(c) The cycle efficiency.

Solution:

Given, Compressor inlet pressure $(P_1) = 100$ kPa

Compressor inlet temperature $T_1 = 20^\circ C = 293$ K

Heat added to the cycle per kg of air $(q_H) = 800$ kJ/kg

Maximum temperature during the cycle $(T_{max}) = T_3 = 1400$ K

a) Heat added to the cycle per kg of air is then given by

$q_H = c_p (T_3 - T_2)$

or, $800 = 1.005 (1400 - T_2)$

$\therefore T_2 = 603.98$ K

Now, applying P - T relation for an isentropic compression 1-2, pressure at state 2,

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = 100\left(\frac{60.398}{293}\right)^{\frac{1.4}{1.4-1}} = 1257.6 \text{ kPa}$$

$\therefore$ Pressure ratio : $r_P = \frac{P_2}{P_1} = \frac{1257.6}{100} = 12.576$

b) State 3: $P_3 = P_2 = 1257.6$ kPa, $T_3 = 1400$ K

State 4: $P_4 = P_1 = 100$ kPa

Applying P - T relation for an isentropic expansion 3 -4, temperature at state 4,

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = 1400\left(\frac{100}{1257.6}\right)^{\frac{1.4-1}{1.4}} = 679.16 \text{ K}$$

Work produced by the turbine per kg of air is then given by

$w_T = c_p (T_3 - T_4) = 1.005 (1400 - 679.16) = 724.44$ kJ/kg

Work consumed by the compressor per kg of air is then given by

$W_C = c_p (T_2 - T_1) = 1.005 (603.98 - 293) = 312.535$ kJ/kg

Net work produced by the cycle per kg of air is then given by

$w_{net} = w_T - w_C = 724.44 - 312.535 = 411.905$ kJ/kg

c) Efficiency of the cycle is then given by

$$\eta = \frac{w_{net}}{q_H} = \frac{411.905}{800} = 51.488\%$$

3. **Air enters the compressor of an ideal Brayton cycle at 100 kPa , 290 K with a volumetric flow rate of 4 m³/s . the pressure ratio for the cycle is 10 and the maximum temperature during the cycle is 1500 K. Determine:**

(a) The thermal efficiency of the cycle,

(b) The fraction of work output that is consumed by the compressor, and

(c) The net power output

Solution:

Given,

Properties at state 1: $P_1 = 100$ kPa, $T_1 = 290$ K

Pressure ratio $(r_p) = \frac{P_2}{P_1} = 10$

Maximum temperature during the cycle $(T_{max}) = T_3 = 1500$ K

Volumetric flow rate at inlet of compressor $(\dot{V}_1) = 4$ m³/s

Temperature at the compressor exit is given by

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = T_1\,(r_p)^{\frac{\gamma-1}{\gamma}} = 290 \times (10)^{\frac{1.4-1}{1.4}} = 559.902 \text{ K}$$

Temperature at the turbine exist is given by

$$T_4 = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} T_3 = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} T_3 = \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} T_3 = \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} 1500$$

$= 776.921$ K

Work produced by the turbine per kg of air is then given by

$w_T = c_p\,(T_3\ T_4) = 1.005 \times (1500 - 776.921) = 726.694$ kJ/kg

Work consumed by the compressor per kg of air is the given by

$w_C = c_p\,(T_2 - T_1) = 1.005 \times (559.902 - 290) = 271.252$ kJ/kg

Net work produced by the cycle per kg of air is then given by

$w_{net} = w_T - w_C = 726.694 - 271.252 = 455.442$ kJ/kg

a) The thermal efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 48.205\ \%$$

b) The fraction of work output that is consumed by the compressor

$$= \frac{w_C}{w_T} \times 100\% = \frac{271.252}{726.694} \times 100\% = 37.33\%$$

c) Specific volume at the inlet of compressor is given by

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 290}{100} = 0.8323$$

Net power output is given as

$$\dot{W}_{net} = \frac{w_{net} \times \dot{V}_1}{v_1} = \frac{455.442 \times 4}{0.8323} = 2188.84 \text{ kW}$$

4. An ideal Brayton cycle has a pressure ratio of 12. The pressure and temperature at the compressor inlet are 100 kPa and 27°C respectively. The maximum temperature during the cycle is 12000C. If the mass flow rate of air is 8 kg/s, determine the power output and efficiency of the cycle.

Solution:

Given, Pressure ratio $(r_p) = \frac{P_2}{P_1} = 12$

Properties at state 1: $P_1 = 100$ kPa, $T_1 = 27 + 273 = 300$ K

Maximum temperature during the cycle $(T_{max}) = T_3 = 1200^0\text{C} = 1473$ K

Mass flow rate of air $(\dot{m}) = 8$ kg/s

Temperature at the compressor exits is given by

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = T_1\,(r_p)^{\frac{\gamma-1}{\gamma}} = 300 \times (12)^{\frac{1.4-1}{1.4}} = 610.181 \text{ K}$$

Temperature at the turbine exit is given by

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = T_3\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = T_3\left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1473\left(\frac{1}{12}\right)^{\frac{1.4-1}{1.4}} = 724.211 \text{ K}$$

work produced by the turbine per kg of air is given by

$w_T = c_p\,(T_3 - T_4) = 1.005 \times (1473 - 724.211) = 752.533$ kJ/kg

Work consumed by the compressor per kg of air is given by

$W_C = c_p\,(T_2 - T_1) = 1.005 \times (610.181 - 300) = 311.732$ kJ/kg

∴ Net work produced by the cycle per kg of air is then given by

$w_{net} = w_T - w_C = 752.533 - 311.732 = 440.801$ kJ/kg

Power output is then given by

$\dot{W} = w_{net} \times \dot{m} = 440.801 \times 8 = 3526.408$ kW

Efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{12}\right)^{\frac{1.4-1}{1.4}} = 50.83\%$$

5. A power plant operating on an ideal Brayton cycle delivers a power output of 80 MW. The minimum and maximum temperature during the cycle are 300 K and 1500 K respectively. The pressure at the compressor inlet and exit are 100 kPa and 1400 kPa respectively.

(a) Determine the thermal efficiency of the cycle

(b) Determine the power output from the turbine.

(c) What fraction of the turbine power output is required to drive the compressor? *(IOE 2068 Magh)*

Solution:

Given, Power output ($\dot{W}$) = 80 MW

Minimum temperature of the cycle (T_{min}) = T_1 = 300 K

Maxim temperature of the cycle (T_{max}) = T_3 = 1500 K

Pressure at the compressor inlet (P_1) = 100 kPa

Pressure at the compressor exit (P_2) = 1400 kPa

Pressure ratio is given by

$$r_p = \frac{P_2}{P_1} = \frac{1400}{100} = 14$$

Temperature at the compressor exit is given by

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1400}{100}\right)^{\frac{1.4-1}{1.4}} \times 300 = 637.656\ K$$

Temperature at the turbine exit is then given by

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = T_3\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = T_3\left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1500 \times \left(\frac{1}{14}\right)^{\frac{1.4-1}{1.4}} = 705.709\ K$$

Work produced by the turbine per kg of air is then given by

$w_T = c_p (T_3 - T_4) = 1.005 \times (1500 - 705.709) = 798.262$ kJ/kg

Work consumed by the compressor per kg of air is then given by

$W_C = c_p (T_2 - T_1) = 1.005 \times (637.656 - 300) = 339.344$ kJ/kg

Net work output in the cycle per kg of air is given by

$w_{net} = w_T - w_C = 798.262 - 339.344 = 458.918$ kJ/kg

Efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{14}\right)^{\frac{1.4-1}{1.4}} = 52.953\%$$

Mass flow rate of air is given by

$$\dot{m} = \frac{\dot{W}}{w_{net}} = \frac{80 \times 10^3}{458.918} = 174.323\ kg/s$$

Now, power output from the turbine ($\dot{W}_T$) = $w_T \times \dot{m}$

= 798.262 × 174.323 = 139.156 MW

Power consumed by the compressor ($\dot{W}_C$) = $w_C \times \dot{m}$

= 339.344 × 174.323 = 59.155 MW

Fraction of the turbine power output required to drive the compressor is given by

$$\% = \frac{59.155}{139.156} = 42.51\%$$

6. **An ideal gas turbine cycle produces 15 MW of power output. The properties of the air at the compressor inlet are 100 kPa and 17 ° C. The pressure ratio for the cycle is 15 and the heat added per kg of air per cycle is 900 kJ/kg. Determine**

(a) **The efficiency of the cycle**

(b) **The maximum temperature in the cycle**

(c) **The mass flow rate of air *(IOE 2070 Ashad)***

Solution:

Given, Power output in a cycle ($\dot{W}_{net}$) = 15 MW = 15×10^3 kW

Properties of air at compressor inlet (state 1):

P_1 = 100 kPa, $T_1 = 17^\circ C = 17 + 273 = 290$ K

Pressure ratio (r_p) = $\frac{P_2}{P_1} = 15$

Heat added per kg of air per cycle (q_H) = 900 kJ/kg

a) The efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{15}\right)^{\frac{1.4-1}{1.4}} = 53.87\ \%$$

b) Temperature at the compressor exit is then given by

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 290 \times (15)^{\frac{1.4-1}{1.4}} = 628.672\ K$$

Heat added per kg of air per cycle is then given by

$q_H = c_p (T_3 - T_2)$

$$\therefore T_3 = \frac{q_H}{c_p} + T_2 = \frac{900}{1.005} + 628.672 = 1524.19\ K$$

Hence, maximum temperature in the cycle (T_3) = 1524.19 K

Efficiency of the cycle is then given by

$$\eta = \frac{w_{net}}{q_H}$$

Therefore, net work output per kg of air per cycle is given as $w_{net} = \eta \times q_H$ = 0.5387 × 900 = 484.83 kJ/kg

Now, power output in a cycle is given by

$$\dot{W}_{net} = w_{net} \times \dot{m}$$

Therefore, mass flow rate of air is given as

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{15 \times 10^3}{484.83} = 30.938 \text{ kg/s}$$

7. **In an ideal Brayton cycle, air enters the compressor at 100 kPa and 300 K and the turbine at 1000 kPa and 1200 K. Heat is transferred to the air at arate of 30 MW. Determine the efficiency and the power output of the plant.**

Solution:

Given, Properties of air at compressor inlet (state 1):

$P_1 = 100$ kPa, $T_1 = 300$ K

Properties of air at turbine inlet (state 3):

$P_3 = 1000$ kPa $T_3 = 1200$ K

Rate of heat added in a cycle ($\dot{Q}_H$) = 30 MW = 30 × 10^3 kW

State 2: $P_3 = P_2 = 1000$ kPa

Temperature at the compressor exit is given by

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times \left(\frac{1000}{100}\right)^{\frac{1.4-1}{1.4}} = 579.21 \text{ K}$$

State 4: $P_1 = P_4 = 100$ kPa

Temperature at the turbine exits is then given by

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = 1200 \times \left(\frac{100}{1000}\right)^{\frac{1.4-1}{1.4}} = 624.537 \text{ K}$$

Heat added per kg of air per cycle is then given by

$q_H = c_p (T_3 - T_2) = 1.005 (1200 - 579.21) = 623.894$ kJ/kg

Work produced by the turbine per kg of air is then given by

$w_T = c_p (T_3 - T_4) = 1.005 \times (1200 - 621.537) = 581.355$ kJ/kg

Work consumed by the compressor per kg of air is then given by

$w_C = c_p (T_2 - T_1) = 1.005 \times 579.21 - 300) = 280.606$ kJ/kg

Net work produced per kg of air in a cycle is given by

$w_{net} = w_T - w_C = 581.355 - 280.606 = 300.749$ kJ/kg

Mass flow rate of air is given by

$$\dot{m} = \frac{\dot{Q}_H}{q_H} = \frac{30 \times 10^3}{623.894} = 48.085 \text{ kg/s}$$

Efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{10}{100}\right)^{\frac{1.4-1}{1.4}} = 48.205 \%$$

Power output of the cycle is given by

$$\dot{W}_{net} = w_{net} \times \dot{m} = 300.749 \times 48.085 = 14461.5 \text{ kW} = 14.462 \text{ MW}$$

8. **The minimum and the maximum temperature during an ideal Brayton cycle are 300 K and 1200 K respectively.. The pressure ratio is such that the net work developed is maximized. Determine:**

   (a) **The compressor and turbine work per unit mass of air, and**

   (b) **The thermal efficiency of the cycle.**

Solution:

Given, Maximum Temperature in a cycle (T_{max}) = T_3 = 1200 K

Minimum Temperature in a cycle (T_{min}) = T_1 = 300 K

Pressure ratio for maximum net work developed is then given by

$$r_p = \frac{P_2}{P_1} = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}} = \left(\frac{1200}{300}\right)^{\frac{1.4}{2(1.4-1)}} = 11.314$$

a) Applying P - T relation for an isentropic compression 1 -2, temperature at state 2,

$$T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \times T_1 = (r_p)^{\frac{\gamma-1}{\gamma}} \times T_1 = (11.314)^{\frac{1.4-1}{1.4}} \times 300 = 600.0044 \text{ K}$$

Temperature at the turbine exit is given by

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = T_3 \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = T_3 \times \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1200 \times \left(\frac{1}{11.314}\right)^{\frac{1.4-1}{1.4}} = 599.996 \text{ K}$$

Compressor work per unit mass of air is then given by

$w_C = c_p (T_2 - T_1) = 1.005 \times (600.0044 - 300) = 301.5044$ kJ/kg

Turbine work per unit mass of air is then given by

$w_T = c_p (T_3 - T_4) = 1.005 \times (1200 - 599.996) = 603.004$ kJ/kg

b) The thermal efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{11.314}\right)^{\frac{1.4-1}{1.4}} = 50\ \%$$

9. The compressor and turbine of an ideal gas turbine each have isentropic efficiencies of 80 %. The pressure ratio is 10. The minimum and maximum temperatures are 300 K and 1200 K respectively. Determine:

(a) The net work per kg of air,

(b) The thermal efficiency of the cycle, and

(c) Compare both of these for a cycle with ideal compressor and turbine.

(d) Determine the efficiency of an ideal Rankine cycle operating between the boiler pressure of 1.5 MPa and a condenser pressure of 8 kPa. The steam leaves the boiler as saturated vapor.

Solution:

Given, Pressure ratio $(r_p) = \frac{P_2}{P_1} = 10$

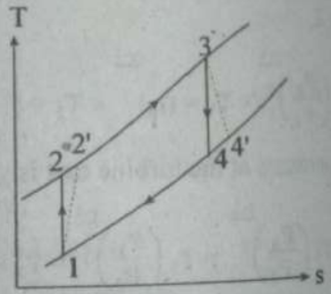
Maximum temperature $(T_{max}) = T_3 = 1200$ K

Minimum temperature $(T_{min}) = T_1 = 300$ K

Turbine efficiency $(\eta_{turbine}) = 80\ \%$

Compressor efficiency $(\eta_{compressor}) = 80\ \%$

Temperature at the compressor exit is given by



$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times (10)^{\frac{1.4-1}{1.4}}$$

$= 579.209$ K

Temperature at the turbine exit is given by

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = T_3 \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = T_3 \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} =$$

$$1200 \times \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 621.537 \text{ K}$$

Turbine isentropic efficiency is given by

$$\eta_{turbine} = \frac{c_p (T_3 - T_4')}{c_p (T_3 - T_4)} = \frac{T_3 - T_4'}{T_3 - T_4}$$

$$\text{or, } 0.8 = \frac{1200 - T_4'}{1200 - 621.537}$$

$T_4' = 737.229$ K

Similarly compressor isentropic efficiency is given by

$$\eta_{compressor} = \frac{c_p (T_2 - T_1)}{c_p (T_2' - T_1)} = \frac{T_2 - T_1}{T_2' - T_1}$$

$$\text{or, } 0.8 = \frac{579.209 - 300}{T_2' - 300}$$

$T_2' = 679.011$ K

Work consumed by compressor per kg of air in a cycle is given by

$w_C = c_p (T_2' - T_1) = 1.005\ (649.011 - 300) = 350.756$ kJ/kg

Work produced by turbine per kg of air in a cycle is given by

$w_T = c_p (T_3 - T_4') = 1.005\ (1200 - 737.229) = 465.085$ kJ/kg

a) Net work output per kg of air in a cycle is given by

$w_{net} = w_T - w_C = 465.085 - 350.756 = 114.329$ kJ/kg

Heat supplied per kg of air in a cycle is given by

$q_H = c_p (T_3 - T_2') = 1.005 \times (1200 - 649.011) = 553.744$ kJ/kg

b) Thermal efficiency of cycle is given by

$$\eta = \frac{w_{net}}{q_H} = \frac{114.329}{553.744} = 20.65\%$$

For a cycle with ideal compressor and turbine,

Work consumed by compressor per kg of air in a cycle is given by

$w_C = c_p (T_2 - T_1) = 1.005\ (579.209 - 300) = 280.605$ kJ/kg

Work produced by the turbine per kg of air in a cycle is given by

$w_T = c_p (T_3 - T_4) = 1.005\ (1200 - 621.537) = 581.955$ kJ/kg

Heat added per kg of air in a cycle is then given by

$q_H = c_p (T_3 - T_2) = 1.005 \times (1200 - 579.209) = 623.895$ kJ/kg

Net work output per kg of air in a cycle is given by

$w_{net} = w_T - w_C = 581.355 - 280.605 = 300.75$ kJ/kg

And efficiency of a cycle is given by

$$\eta = \frac{w_{net}}{q_H} = \frac{300.75}{623.895} = 48.205\%$$

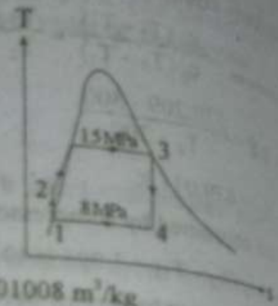
10. Determine the efficiency of an ideal Rankine cycle operating between the boiler pressure of 1.5 MPa and a condenser pressure of 8 kPa. The steam leaves the boiler as saturated vapor.

Solution:

Given, Boiler pressure (P_2) = 1.5 MPa = 1500 kPa

Condenser pressure (P_1) = 8 kPa

With reference to T-s diagram of the cycle shown in figure, properties of steam at each state are evaluated as follows:



State 1: P_1 = 8 kPa, saturated liquid

Referring to Table A2.1,

$h_1 = h_f$ (8 kPa) = 173.85 kJ/kg , $v_1 = v_f$ (8 kPa) = 0.001008 m^3/kg

State 2: P_2 = 1500 kPa, compressed liquid

Applying isentropic relation for an in compressible substance,

$h_2 - h_1 = v_1 (P_2 - P_1)$

$\therefore$ $h_2 = h_1 + v_1 (P_2 - P_1) = 173.85 + 0.001008 \times (1500 - 8)$

= 175.3539 kJ/kg

State 3: P_3 = 1500 kPa, saturated vapor

Referring to Table A2.1, $T_3 = T_{sat}$ (1500 kPa) = 198.33° C, $h_3 = h_g$ (1500 kPa) = 2791.5 kJ/kg, $s_3 = s_g$ (1500 kPa) = 6.4438 kJ/kgK

State 4: P_4 = 8 kPa

For isentropic expansion process 3 - 4, $s_4 = s_3$ = 6.4438 kJ/kgK

Referring to Table A2.1, s_f (8 kPa) = 0.5925 kJ/kgK, s_{fg} (8 kPa) = 7.6342 kJ/kgK, s_g (8 kPa) = 8.2267 kJ/kgK, h_f (8 kPa) = 173.85 kJ/kg, h_{fg} (8 kPa) = 2402.3 kJ/kg

Here, $s_f < s_4 < s_g$, hence it is a two phase mixture. Therefore, quality of steam at state 4,

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.4438 - 0.5925}{7.6342} = 0.7665$$

Specific enthalpy of steam at state 4 is then given by

$h_4 = h_f + x_4 h_{fg}$ = 173.85 + 0.7665 × 2402.3 = 2015.213 kJ/kg

Work produced by the turbine per kg of steam is given by

$w_T = w_{34} = h_3 - h_4$ = 2791.5 - 2015.213 = 776.287 kJ/kg

Work consumed by pump per kg of steam is given by

$w_P = w_{12} = h_2 - h_1$ = 175.3539 - 173.85 = 1.5039 kJ/kg

The net work delivered to the surrounding is given by

$w_{net} = w_T - w_P$ = 776.287 - 1.5039 = 774.7831 kJ/kg

Heat supplied to the steam in the boiler is given by

$q_H = q_{23} = h_3 - h_2$ = 2791.5 - 175.3539 = 2616.1461 kJ/kg

$\therefore$ Efficiency of the Rankine cycle is then given by

$$\eta = \frac{w_{net}}{q_H} = \frac{774.7831}{2616.1461} = 29.62\%$$

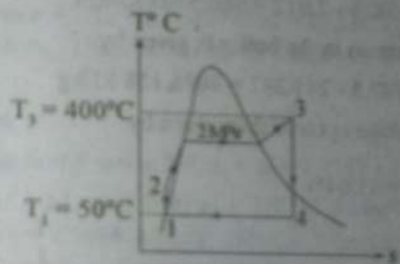
11. A Rankine cycle has a boiler working at a pressure of 2 MPa. The maximum and minimum temperatures during the cycle are 400 °C and 50 respectively. Determine the efficiency of the cycle and compare it with that of the Carnot cycle operating between the same temperature limits.

Solution:

Given, Boiler pressure (P_2) = 2 MPa = 2000 kPa

Maximum temperature (T_{max}) = T_3 = 400° C

Minimum temperature (T_{min}) = T_1 = 50° C



Properties of steam at each state are evaluated as follows:

State 3: P_3 = 2000 kPa, T_3 = 400° C,

Referring to Table A2.1, T_{sat} (2000 kPa) = 212.42° C. Hence, it is a superheated vapor. Then, referring to Table A2.4, h_3 = 3247.5 kJ/kg, s_3 = 7.1269 kJ/kgK

State 4: T_4 = 50° C

For isentropic expansion process 3 - 4, $s_4 = s_3$ = 7.1269 kJ/kgK

Referring to Table A 2.2, s_f (50° C) = 0.7037 kJ/kgK, s_g (50° C) = 8.0745 kJ/kgK, s_{fg}(50° C) = 7.3708 kJ/kgK, h_f(50° C) = 209.33 kJ/kg, h_{fg} (50° C) = 2381.9 kJ/kg.

Here, $s_f < s_4 < s_g$, hence it is a two phase mixture. Therefore quality of mixture at state 4,

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{7.1269 - 0.7037}{7.3708} = 0.8714$$

Specific enthalpy of steam at state 4 is then given by

$h_4 = h_f + x_4 h_{fg} = 209.33 + 0.8714 \times 2381.9 = 2284.9177$ kJ/kg

State 1: $T_1 = 50^0$ C, saturated liquid

Referring to Table A2.1, $P_1 = P_{sat}$ $(50^0$ C$) = 12.344$ kPa, $h_1 = h_f$ $(50^0$C$) = 209.33$ kJ/kg, $v_1 = v_f$ $(50^0$ C$) = 0.001012$ m^3/kg

State 2: $P_2 = 2000$ kPa, compressed liquid

Applying isentropic relation for an incompressible substance,

$h_2 - h_1 = v_1 (P_2 - P_1)$

$\therefore$ $h_2 = h_1 + v_1 (P_2 - P_1) = 209.33 + 0.001012 (2000 - 12.344)$

$= 211.342$ kJ/kg

Work produced by the turbine per kg of steam is given by

$w_T = w_{34} = h_3 - h_4 = 3247.5 - 2284.9177 = 962.5823$ kJ/kg

Work consumed by the pump per kg of steam is given by

$w_P = w_{12} = h_2 - h_1 = 211.342 - 209.33 = 2.012$ kJ/kg

The net work delivered to the surroundings is given by

$w_{net} = w_T - w_P = 962.5823 - 2.012 = 960.5703$ kJ/kg

Heat supplied to the steam in the boiler is given by

$q_H = q_{23} = h_3 - h_2 = 3247.5 - 211.342 = 3036.158$ kJ/kg

Efficiency of the Rankine cycle is then given by

$$\eta = \frac{w_{net}}{q_H} = \frac{960.5703}{3036.158} = 31.64\%$$

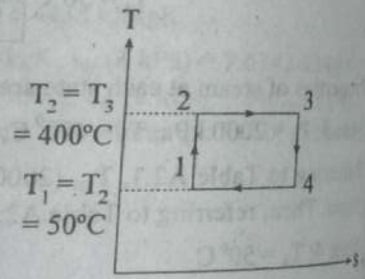
For Carnot cycle:

Carnot cycle efficiency is given by

$$\eta_{carnot} = 1 - \frac{T_1}{T_2} \quad \text{Figure}$$

$$= 1 - \frac{50 + 273}{400 + 273}$$

$= 52.01\%$



12. A steam power plant operates on a simple Rankine cycle between the pressure limits of 2 MPa and 20 kPa. The temperature of the steam at the turbine inlet is 4000C, and the mass flow rate of steam is 50 kg/s. Determine:

(a) the thermal efficiency of the cycle, and

(b) the net power output of the plant. ***(IOE 2068 Bhadra)***

Solution:

Given, Boiler pressure $(P_2) = 2$ MPa $= 2000$ kPa

Condenser pressure $(P_1) = 20$ kPa

Temperature of the steam at the turbine inlet $(T_3) = 400^0$ C

Mass flow rate of steam $(\dot{m}) = 50$ kg/s

Properties of steam at each is evaluated as follow:

State 1: $P_1 = 20$ kPa, saturated liquid

Referring to Table A2.1, $h_1 = h_f$ (20 kPa) $= 251.46$ kJ/kg,

$v_1 = v_f$ (20 kPa) $= 0.001017$ kJ/kg

State 2: $P_2 = 2000$ kPa, compressed liquid

Applying isentropic relation for an incompressible substance,

$h_2 - h_1 = v_1 (P_2 - P_1)$

$\therefore$ $h_2 = h_1 + v_1 (P_2 - P_1) = 251.46 + 0.001017 (2000 - 20)$

$= 253.4737$ kJ/kg

State 3: $P_3 = 2000$ kPa, $T_3 = 400^0$C

Referring to the Table A2.1, T_{sat} (2000 kPa) $= 212.92^0$C. Here, $T > T_{sat}$, hence it is a superheated vapor. Then, referring to Table A2.4, $h_3 = 3247.5$ kJ/kg, $s_3 = 7.1269$ kJ/kgK

State 4: $P_4 = 20$ kPa

For isentropic expansion process 3-4, $s_4 = s_3 = 7.1269$ kJ/kgK

Referring to Table A2.1, s_f (20 kPa) $= 0.8321$ kJ/kgK, s_{fg} (20 kPa) $= 7.0747$ kJ/kgK, s_g (20 kPa) $= 7.9068$ kJ/kgK. Here, $s_f < s_4 < s_g$, hence it is a two phase mixture. Therefore, quality of mixture at state 4,

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{7.1269 - 0.8321}{7.0747} = 0.8898$$

Specific enthalpy of steam at state 4 is then given by

$h_4 = h_f + x_4 h_{fg} = 251.46 + 0.8898 \times 2357.4$ kJ/kg

Work produced by the turbine per kg of steam is given by

$w_T = w_{34} = h_3 - h_4 = 3247.5 - 2349.075 = 898.425$ kJ/kg

Work consumed by the pump per kg of steam is given by

$w_C = w_{12} = h_2 - h_1 = 253.4737 - 251.46 = 2.0137$ kJ/kg

The net work delivered to the surrounding is given by

$w_{net} = w_T - w_P = 898.425 - 2.0137 = 896.4113$ kJ/kg

Heat supplied to the steam in the boiler is given by

$q_H = q_{23} = h_3 - h_2 = 3247.5 - 253\text{-}4737 = 2994.0263$ kJ/kg

a) Efficiency of the Rankine cycle is given by

$$\eta = \frac{w_{net}}{q_H} = \frac{896.4113}{2994.0263} = 29.94\ \%$$

b) The net power output of the plant is given by

$$\dot{W} = \dot{W}_{net\ t} \times \dot{m} = 896.4113 \times 50 = 44.82\ \text{MW}$$

13. An ideal Rankine cycle operates between a boiler pressure of 4 MPa and a condenser pressure of 10 kPa. The exit steam from the turbine should have a quality of 96 % and the power output of the turbine should be 80 MW. Determine

(a) the minimum boiler exit temperature,

(b) the efficiency of the cycle, and

(c) the mass flow rate of steam

Solution:

Given, Boiler pressure (P_2) = 4 MPa = 400 kPa

Condenser pressure (P_1) = 10 kPa

Quality of steam at turbine exit (x_4) = 96% = 0.96

Power output of the turbine ($\dot{W}_T$) = 80 MW = 80×10^3 kW

Properties of steam at each state is evaluated as follows

State 1: P_1 : P_1 = 10 kPa, saturated liquid

Referring to Table A2.1, $h_1 = h_f$ (10 kPa) = 191.83 kJ/kg, $v_1 = v_f$ (10 kPa) = 0.00101 m^3/kg

State 2: P_2 = 4000 kPa, compressed liquid

Applying isentropic relation for an incompressible substance,

$$h_2 - h_1 = v_1 (P_2 - P_1)$$

$$\therefore\ h_2 = h_1 + v_1 (P_2 - P_1) = 191.83 + 0.00101 \times (4000 - 10)$$

= 195.86 kJ/kg

State 4: P_4 = 10 kPa, x_4 = 0.96

Referring to the Table A2.1, h_f (10 kPa) = 191.83 kJ/kg, h_{fg} (10 kPa) = 2392.0 kJ/kg, s_f (10 kPa) = 0.6493 kJ/kgK, s_{fg} (10 kPa) = 7.4989 kJ/kgK, $T_4 = T_{sat}$ (10 kPa) = 45.817°C

Specific enthalpy of steam at state 4 is then given by

$$h_4 = h_f + x_4 h_{fg} = 191.83 + 0.96 \times 2392.0 = 2488.15\ \text{kJ/kg}$$

$s_4 = s_f + x_4 s_{fg} = 0.6493 + 0.96 \times 7.4989 = 7.8482$ kJ/kg

State 3 : P_3 = 4000 kPa

For isentropic expansion process 3-4, $s_3 = s_4 = 7.8482$ kJ/kgK

Referring to Table A2.1, $s_3 > s_g$, hence it is a superheated steam.

Now, referring to Table A 2.4, specific enthalpy and temperature of superheated steam with specific entropy 7.8482 kJ/kgK can be listed as:

s_g, kJ/kgK	T°C	h, kJ/kg	
7.7371	750°C	4023.0	(a)
7.8503	800°C	4141.7	(b)

Now, applying linear interpolation for temperature abd specific enthalpy,

$$T_3 = T_a + \frac{T_b - T_a}{s_b - s_a}(s_3 - s_a)$$

$$= 750 + \frac{800 - 750}{7.8503 - 7.7371}(7.8482 - 7.7371)$$

= 799.07°C

$$h_3 = h_a + \frac{h_b - h_a}{s_b - s_a}(s_3 - s_a)$$

$$= 4023.0 + \frac{4141.7 - 4023.0}{7.8503 - 77371} \times (7.8482 - 77571)$$

= 4118.526 kJ/kg

a) The minimum boiler exit temperature (T_3) = 799.07°C

b) Work produced by the turbine per kg of steam is given by

$w_T = w_{34} = h_3 - h_4 = 4118.526 - 2488.15 = 1630.376$ kJ/kg

Work consumed by the pump per kg of steam is given by

$w_P = w_{12} = h_2 - h_1 = 195.86 - 191.83 = 4.03$ kJ/kg

Net work delivered to the surrounding is given by

$W_{net} = w_T - w_P = 1630.376 - 4.03 = 1626.346$ kJ/kg

Heat supplied to the steam in the boiler is given by

$q_H = q_{23} = h_3 - h_2 = 4118.\Omega6 - 195.86 = 3922.67$ kJ/kg

∴ Efficiency of the cycle is given by

$$\eta = \frac{W_{net}}{q_H} = \frac{1626.346}{3922.67} = 41.46\%$$

c) Power output of the turbine is given

$\dot{W}_T = w_T \times \dot{m}$

Therefore, mass flow rate of steam is given by

$$\dot{m} = \frac{\dot{W}_T}{w_T} = \frac{80 \times 10^3}{1630.376} = 49.068 \text{ kg/s}$$

14. Saturated vapor enters into a turbine of an ideal Rankine cycle at 10 MPa and saturated liquid exits the condenser at 10 kPa. The power output of the cycle is 120 MW. Determine:

- **(a) the thermal efficiency of the cycle,**
- **(b) the back work ratio,**
- **(c) the mass flow rate of steam,**
- **(d) the rate at which heat is supplied to the boiler,**
- **(e) the rate at which heat is rejected from the condenser, and**
- **(f) the mass flow rate of condenser cooling water, if the cooling water enters at 20 °C and exits at 35 °C. [Take specific heat of water as 4.18 kJ/kgK].**

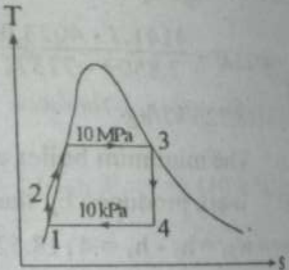
Solution:

Given, Turbine inlet pressure (P_3) = 10 MPa = 10000 kPa

Condenser pressure (P_1) = 10 kPa

Power output of the cycle ($\dot{W}$) = 120 MW

$= 120 \times 10^3$ kW



With reference to T-s diagram of the cycle, properties of steam at each state are evaluated as follows:

State 1: P_1 = 10kPa, saturated liquid

Referring to Table A2.1, $h_1 = h_f$ (10 kPa) = 191.83 kJ/kg

$v_1 = v_f$ (10kPa) = 0.00101 m³/kg

State 2: P_2 = 10000 kPa, compressed liquid

Applying isentropic relations for incompressible substance

$h_2 = h_1 = v_1 (P_2 - P_1)$

$\therefore\ h_2 = h_1 + v_1 (P_2 - P_1) = 191.83 + 0.00101 \times (10000 - 10)$

$= 201.92$ kJ/kg

State 3: P_3 = 10000 kPa, saturated vapor

Referring to Table A2.1, $h_3 = h_g$ (10000 kPa) = 2724.5 kJ/kg

$s_3 = s_g$ (10000 kPa) = 5.6139 kJ/kgK

State 4: P_4 = 10 kPa

For isentropic expansion process 3 - 4, $s_4 = s_3$ = 5.6139 kJ/kgK

Referring to Table A2.1, h_f (10 kPa) = 191.83 kJ/kg, h_{fg} (10 kPa) = 2392.0 kJ/kg, s_f (10 kPa) = 0.6493 kJ/kgK, s_{fg} (10 kPa) = 7.4989 kJ/kgK, $T_4 = T_{sat}$ (10 kPa) = 45.817°C. Here, $s_f < s_4 < s_g$, hence it is a two phase mixture. Therefore, quality of mixture is given by

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{5.6139 - 0.6493}{7.4989} = 0.66204$$

Specific enthalpy of the steam at state 4 is then given by

$h_4 = h_f + x_4 h_{fg} = 191.83 + 0.66204 \times 2392.0 = 1775.4297$ kJ/kg

Work produced by the turbine per kg of steam is then give by

$w_T = w_{34} = h_3 - h_4 = 2724.5 - 1775.4297 = 949.07$ kJ/kg

Work consumed by the pump per kg of steam is then given by

$w_P = w_{12} = h_2 - h_1 = 201.92 - 191.83 = 10.09$ kJ/kg

Net work delivered to the surrounding per kg of steam is given by

$w_{net} = w_T - w_P = 494.07 - 10.09 = 938.98$ kJ/kg

Heat supplied to the steam in the boiler is given by

$q_H = q_{23} = h_3 - h_2 = 2724.5 - 201.92 = 2522.58$ kJ/kg

a) Thermal efficiency of the cycle is given by

$$\eta = \frac{w_{net}}{q_H} = \frac{938.98}{2529.58} = 37.22\%$$

b) Back work ratio is given by

$$1 - \frac{w_T - w_P}{w_T}$$

$$= 1 - \frac{w_{net}}{w_T} = 1 - \frac{938.98}{949.07} = 1.063\%$$

c) Mass flow rate of steam is given by

$$\dot{m} = \frac{\dot{W}}{w_{net}} = \frac{120 \times 10^3}{938.98} = 127.9 \text{ kg/s}$$

d) Rate at which heat is supplied to the boiler is given by

$$\dot{Q}_H = q_H \times \dot{m} = 2529.38 \times 127.9 = 322.39 \text{ MW}$$

e) Heat rejected from the condenser is given by

$q_R = q_{14} = h_4 - h_1 = 1775.4297 - 191.83 = 1583.598$ kJ/kg

∴ Rate of heat rejection from the condenser is given by

$\dot{Q}_L = q_R \times \dot{m} = 1583.598 \times 127.8 = 202.38$ MW

f) Heat taken by condenser cooling water is given by

$q_{cooling\ water} = c_p (T_{in} - T_{out})$

$= 4.18 (35 - 20) = 62.7$ kJ/kg

Therefore, mass flow rate of condenser cooling water is given by

$$\dot{m}_{cooling\ tower} = \frac{\dot{Q}_L}{q_{cooling\ water}} = \frac{202.38 \times 10^3}{62.7} = 3227.8 \text{ kg/s}$$

15. Superheated steam at 8 MPa, 500 °C enters into turbine of a steam power plant working on a Rankine cycle. The steam leaves the condenser as saturated liquid at 8 kPa. The turbine and pump have isentropic efficiencies of 90 % and 80 % respectively. For the cycle, determine:

(a) the net work per kg of steam,

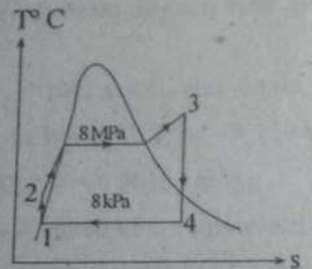
(b) the heat supplied into the boiler per kg of steam, and

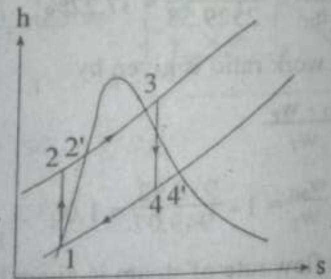
(c) the thermal efficiency.

Solution:

Given, Properties of steam at turbine inlet: $P_3 = 8$ MPa = 8000 kPa, $T_3 = 500$°C

Properties of steam at condenser outlet: $P_1 = 8$ kPa, saturated liquid





With reference to T-s diagram of the cycle shown in figure, properties of steam at each state are evaluated as follows:

State 1: $P_1 = 8$ kPa, saturated liquid

Referring to Table A 2.1, $h_1 = h_f (8\text{kPa}) = 173.85$ kJ/kg

$v_1 = v_f (8 \text{ kPa}) = 0.001008$ m³/kg

State 2: $P_2 = 8000$ kPa, compressed liquid

Applying isentropic relation for an incompressible substance

$h_2 - h_1 = v_1 (P_2 - P_1)$

$h_2 = h_1 + v_1 (P_2 - P_1) = 173.85 + 0.001008 (8000 - 8) = 181.906$ kJ/kg

State 3: $P_3 = 8000$ kPa, $T_3 = 500$°C, superheated vapor

Referring to Table A2.4, $h_3 = 3398.5$ kJ/kg, $s_3 = 6.7243$ kJ/kgK

State 4: $P_4 = 8$ kPa

For isentropic expansion process 3 - 4, $s_4 = s_3 = 6.7243$ kJ/kgK

Referring to Table A2.1 $s_f < s_4 < s_g$, hence it is a two phase mixture. Therefore quality of steam at state 4.

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.7243 - 0.5925}{7.6342} = 0.8032$$

Specific enthalpy of steam at state 4 is then given by

$h_4 = h_f + x_4 h_{fg} = 173.85 + 0.8032 \times 2402.3 = 2103.38$ kJ/kg

Efficiency of turbine is given by

$$\eta_T = \frac{h_3 - h_{4'}}{h_3 - h_4}$$

∴ $h_3 - h_{4'} = 0.9 (3398.5 - 2103.38)$

Hence, work produced by the turbine per kg of steam is given by

$w_T = w_{34'} = h_3 - h_{4'} = 1165.608$ kJ/kg

Efficiency of the pump is given by

$$\eta_P = \frac{h_2 - h_1}{h_{2'} - h_1}$$

$$\therefore h_{2'} - h_1 = \frac{(181.906 - 173.85)}{0.8} = 10.07 \text{ kJ/kg}$$

And $h_{2'} = 173.85 + 10.07 = 183.92$ kJ/kg

Hence, work consumed by pump per kg of steam is given by

$w_P = w_{12'} = h_{2'} - h_1 = 10.07$ kJ/kg

a) Net work delivered to the surrounding per kg of steam is given by

$w_{net} = w_T - w_P = 1165.608 - 10.07 = 1155.54$ kJ/kg

b) Heat supplied into the boiler per kg of steam is given by

$q_H = q_{2'3} = h_3 - h_{2'} = 3398.5 - 183.92 = 3214.58$ kJ/kg

c) The thermal efficiency of a cycle is given by

$$\eta = \frac{w_{net}}{q_H} = \frac{1155.54}{3214.58} = 35.95\%$$

16. An air standard Otto cycle has a compression ratio of 10. At the beginning of the compression stroke, the pressure and temperature are 100 kPa and 20°C respectively. The peak temperature during the cycle is 2000 K. Determine.

(a) The pressure and temperature at the end of each process of the cycle,

(b) The thermal efficiency , and

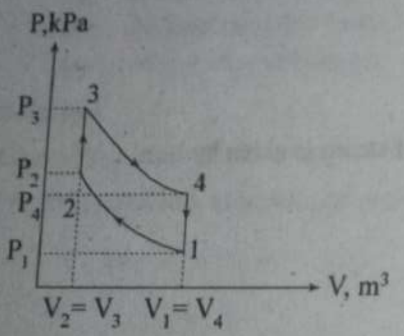
(c) The mean effective pressure.

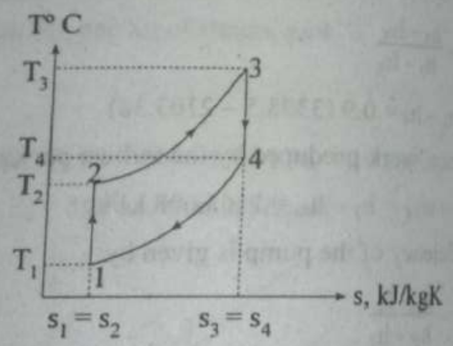
Solution:

Given, Compression ratio $(r) = \frac{V_1}{V_2} = 10$

Properties at state 1: $P_1 = 100$ kPa, $T_1 = 20^0 C = 20 + 273 = 293$ K

Peak temperature during the cycle $(T_{peak}) = T_3 = 2000$ K





a) Applying P - V relation for an isentropic compression 1 - 2, pressure at state 2,

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = P_1 (r)^{\gamma} = 100\ (10)^{1.4} = 2511.87 \text{ kPa}$$

Similarly, applying T - V relation for an isentropic compression 1 - 2, temperature at state 2.

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 (r)^{\gamma-1} = 293 \times (10)^{1.4-1} = 735.98 \text{ K}$$

Temperature at state 3, $T_3 = 2000$ K

Applying P-T relation for an isochoric heat addition process 2-3, pressure at state 3,

$$P_3 = \frac{T_3}{T_2} \times P_2 = \frac{2000}{735.98} \times 2511.87 = 6825.92 \text{ kPa}$$

Similarly, applying P - V and T- V relation for isentropic expansion 3-4, pressure and temperature at state 4,

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\gamma} = P_3 \times \left(\frac{V_2}{V_1}\right)^{\gamma} = P_3 \times \left(\frac{1}{r}\right)^{\gamma} = 6825.92\left(\frac{1}{10}\right)^{1.4}$$

$$= 271.75 \text{ kPa}$$

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3\left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_3\left(\frac{1}{r}\right)^{\gamma-1} = 2000\left(\frac{1}{10}\right)^{1.4-1}$$

$$= 796.21 \text{ K}$$

b) The thermal efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1\left(\frac{1}{10}\right)^{1.4-1} = 60.19\%$$

c) Specific volume of air at state 1.

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 293}{100} = 0.841 \text{ m}^3/\text{kg}$$

Specific volume of air at state 2

$$v_2 = \frac{v_1}{r} = \frac{0.841}{10} = 0.0841 \text{ m}^3/\text{kg}$$

Alternatively,

$$v_2 = \frac{RT_2}{P_2} = \frac{0.287 \times 735.98}{2511.87} = 0.0841 \text{ m}^3/\text{kg}$$

Heat added during the cycle is given by

$$q_H = q_{23} = c_v (T_3 - T_2) = 0.718 \times (2000 - 735.98) = 907.57 \text{ kJ/kg}$$

Heat rejected during the cycle is given by

$$q_L = q_{41} = c_v (T_4 - T_1) = 0.718 \times (796.21 - 293) = 361.305 \text{ kJ/kg}$$

Work output per kg of air per cycle is given by

$$w = q_H - q_L = 907.57 - 361.305 = 546.265 \text{ kJ/kg}$$

∴ Mean effective pressure of the cycle is given by

$$P_{MEP} = \frac{w}{v_1 - v_2} = \frac{546.265}{0.841 - 0.0841} = 721.71 \text{ kPa}$$

17. An air standard Otto cycle has a compression ratio of 8. Before compression stroke has air at 100 kPa and 300 K. The peak pressure during the cycle is 6000 kPa. Determine

(a) The peak temperaturein the cycle,

(b) The temperature at the end of expansion stroke, and

(c) The cycle efficiency.

Solution:

Given, Compression ratio (r) $= \frac{V_1}{V_2} = 8$

Properties at state 1: $P_1 = 100$ kPa, $T_1 = 300$ K

Peak pressure during the cycle $(P_{Peak}) = P_3 = 6000$ kPa

a) Applying T - V and P - V relation for an isentropic compression 1-2, temperature and pressure at state 2,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1\,(r)^{\gamma-1} = 300 \times (8)^{1.4-1} = 689.22 \text{ K}$$

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = P_1\,(r)^{\gamma} = 100 \times (8)^{1.4} = 1837.92 \text{ kPa}$$

Applying P - T relation for an isochoric heat addition process 2-3, temperature at state 3,

$$T_3 = T_2\left(\frac{P_3}{P_2}\right) = 689.22 \times \left(\frac{6000}{1837.92}\right) = 2250 \text{ K}$$

$\therefore$ The peak temperature in the cycle $(T_{peak}) = T_3 = 2250$ K

b) Applying T - V relations for an isentropic expansion 3 - 4, temperature at state 4 is given by

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3\left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_3\left(\frac{1}{r}\right)^{\gamma-1}$$

$$= 2250 \times \left(\frac{1}{8}\right)^{1.4-1} = 979.369 \text{ K}$$

c) The cycle efficiency is given by

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \left(\frac{1}{8}\right)^{1.4-1} = 56.472\%$$

18. An ideal Otto cycle has a compression ratio of 8. The minimum and maximum temperatures during the cycle are 300 K and 1500 K respectively. Determine:

(a) the heat added per kg of air,

(b) the thermal efficiency, and

(c) the efficiency of a Carnot cycle operating between the same temperature limits.

Solution:

Given, Compression ratio (r) $= \frac{V_1}{V_2} = 8$

Maximum temperature during the cycle $(T_3) = 1500$ K

Minimum temperature during the cycle $(T_1) = 300$ K

a) Applying T - V relation for an isentropic compression process 1-2, temperature at state 2,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1\,(r)^{\gamma-1} = 300 \times (8)^{1.4-1} = 689.219 \text{ K}$$

$\therefore$ The heat added per kg of air in a cycle is given by

$$q_H = q_{23} = c_v\,(T_3 - T_2) = 0.718 \times (1500 - 689.219) = 582.14 \text{ kJ/kg}$$

b) The thermal efficiency of a cycle is given by

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \left(\frac{1}{8}\right)^{1.4-1} = 56.47\%$$

c) Efficiency of a Carnot cycle operating between the same temperature limits i.e. 1500 K and 300 K is given bys.

$$\eta_{carnot} = 1 - \frac{T_1}{T_3} = 1 - \frac{300}{1500} = 80\%$$

19. The compression ratio of an ideal Otto cycle is 8.5. At the beginning of the compression stroke, air is at 100 kPa and 27 °C. The pressure is doubled during the constant volume heat addition process. Determine:

(a) the heat added per kg of air,

(b) the net work output per kg of air,

(c) the thermal efficiency, and

(d) the mean effective pressure

Solution:

Given, Compression ratio (r) $= \frac{V_2}{V_1} = 8.5$

Properties at state 1: $P_1 = 100$kPa, $T = 27^0 C = 27 + 273 = 300$ K

Pressure at state 3: $P_3 = 2P_2$

Applying P - V and T - V relation for an isentropic compression process 1-2, pressure and temperature at state 2,

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$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = P_1\,(r)^{\gamma} = 100 \times (8.5)^{1.4} = 2000.721 \text{ kPa}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1\,(r)^{\gamma-1} = 300 \times (8.5)^{1.4-1} = 706.137 \text{ K}$$

$$P_3 = 2P_2 = 2 \times 2000.721 = 4001.442 \text{ kPa}$$

Applying P - T relation for an isochoric heat addition process 2 -3, temperature at state 3,

$$T_3 = \frac{P_3}{P_2} \times T_2 = 2 \times 706\,.137 = 1412.274 \text{ K}$$

Similarly, applying P- V and T - v relations for an isentropic expansion process 3 - 4, pressure and temperature at state 4 are given by

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\gamma} = P_3\left(\frac{V_2}{V_1}\right)^{\gamma} = P_3\left(\frac{1}{r}\right)^{\gamma} = 4001.442 \times \left(\frac{1}{8.5}\right)^{1.4}$$

$= 200$ kPa

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_3\left(\frac{1}{r}\right)^{\gamma-1} = 1412.274\left(\frac{1}{8.5}\right)^{1.4}$$

$= 600$ K

a) Heat added per kg of air is given by

$$q_H = q_{23} = c_v\,(T_3 - T_2) = 0.718\,(1412.274 - 706.317)$$
$$= 507.01 \text{ kJ/kg}$$

b) Heat rejected per kg of air during the cycle is given by

$$q_L = q_{41} = c_v\,(T_4 - T_1) = 0.718\,(600 - 300) = 215.4 \text{ kJ/kg}$$

∴ The net work output per kg of air is given by

$$w = q_H - q_L = 507.01 - 215.4 = 291.61 \text{ kJ/kg}$$

c) The thermal efficiency of the cycle is given by

$$\eta = \frac{w}{q_H} = \frac{291.91}{507.01} = 57.52\%$$

Alternatively,

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \left(\frac{1}{8.5}\right)^{1.4-1} = 57.52\%$$

d) Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 300}{100} = 0.861 \text{ m}^3\text{/kg}$$

Specific volume of air at state 2,

$$v_2 = \frac{v_1}{r} = \frac{0.861}{8.5} = 0.1013 \text{ m}^3\text{/kg}$$

∴ Mean effective pressure of the cycle is given by

$$P_{MEP} = \frac{w}{v_1 - v_2} = \frac{291.61}{0.861 - 0.1013} = 383.85 \text{ kPa}$$

10. Air at the beginning of the compression stroke in an air standard Otto cycle is at 100 kPa and 300 K. The temperature of the air before and after the expansion stroke is 1500 K and 650 K respectively. If the air circulation rate is 3 kg/min, determine the compression ratio, air standard efficiency and the power output.

Solution:

Given, Properties at state 1: $P_1 = 100$ kPa, $T_1 = 300$ K

Temperature of air before entering the expansion stroke $(T_3) = 1550$ K

Temperature of air after entering the expansion stroke $(T_4) = 650$ K

Air circulation rate $(\dot{m}) = 3 \text{ kg/min} = \frac{3}{60} = 0.05$ kg/s

Applying T - V relation for an isentropic expansion process 3 - 4,

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1}$$

$$\text{or, } \frac{V_4}{V_3} = \left(\frac{T_3}{T_4}\right)^{1/\gamma-1}$$

$$\text{or, } \frac{V_1}{V_2} = \left(\frac{T_3}{t_4}\right)^{1/\gamma-1} = \left(\frac{1550}{650}\right)^{\frac{1}{1.4-1}} = 8.78$$

$$\therefore r = \frac{V_1}{V_2} = 8.78$$

Air standard efficiency is given by

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \left(\frac{1}{8.78}\right)^{1.4-1} = 58.063\%$$

Applying T - V relation for an isentropic compression process 1 - 2, temperature at state 2,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 \times (8.78)^{1.4-1} = 714.372 \text{ K}$$

Heat added per kg of air during the cycle is given by

$$q_H = q_{23} = c_v\,(T_3 - T_2) = 0.718\,(1550 - 714.372)$$

$= 599.981$ kJ/kg

Work output per kg of air per cycle is given by

$w = \eta q_H = 0.58063 \times 599.981 = 348.367$ kJ/kg

∴ Power output per cycle is given by

$W = \dot{m} \times w$

$= 0.05 \times 348.367 = 17.42$ kW

21. The properties of air at the beginning of an air standard Otto cycle are $P_1 = 100$ kPa, $T_1 = 300$ K and $V_1 = 0.5 \times 10^{-3}$ m³. The maximum temperature during the cycle is 2400 K and the compression ratio is 8. Determine

(a) the heat added during the cycle,

(b) the net work output,

(c) the thermal efficiency, and

(d) the mean effective pressure.

Solution:

Given, Properties at state 1:- $P_1 = 100$ kPa, $T_1 = 300$ K, $V_1 = 0.5 \times 10^{-3}$ m³

Maximum temperature during the cycle $(T_{max}) = T_3 = 2400$ K

Compression ratio $(r) = \frac{V_1}{V_2} = 8$

Applying P - V and T - V relations for an isentropic compression process 1 - 2, pressure and temperature at state 2,

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = P_1 (r)^{\gamma} = 100 \times (8)^{1.4} = 1837.927 \text{ kPa}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 (r)^{\gamma-1} = 300\,(8)^{1.4-1} = 689.219 \text{ K}$$

Applying T - V relation for an isentropic expansion process 3 - 4, temperature at state 4,

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3\left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_3\left(\frac{1}{r}\right)^{\gamma-1} = 2400 \times \left(\frac{1}{8}\right)^{1.4-1}$$

$= 1044.661$ K

a) Heat added per kg of air during the cycle is given by

$q_H = q_{23} = c_v (T_3 - T_2) = 0.718\,(2400 - 689.219) = 1228.34$ kJ/kg

Specific volume at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 300}{100} = 0.861 \text{ m}^3\text{/kg}$$

Mass of air $(m) = \frac{V_1}{v_1} = \frac{0.5 \times 10^{-3}}{0.861} = 5.81 \times 10^{-4}$ kg

∴ Heat added during the cycle is given by

$Q_H = mq_H = 5.81 \times 10^{-4} \times 1228.34 = 0.7137$ kJ

b) Heat rejected per kg of air during cycle is given by

$q_L = q_{41} = c_v (T_4 - T_1) = 0.718\,(1044.661 - 300) = 534.667$ kJ/kg

∴ Net work output is given by

$W = m\,(q_H - q_L) = 5.81 \times 10^{-4}\,(1228.34 - 534.667) = 0.4030$ kJ

c) The thermal efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \left(\frac{1}{8}\right)^{1.4-1} = 56.47\%$$

Alternatively,

$$\eta = \frac{W}{Q_H} = \frac{0.4030}{0.7137} = 56.47\%$$

22. The following data are obtained for a four stroke petrol engine:

Cylinder bore = 14 cm

Stroke length = 15 cm

Clearance volume = 231cm³

Determine:

(a) the ratio of clearance volume and swept volume,

(b) the compression ratio, and

(c) the thermal efficiency.

Solution:

Given, Cylinder bore $(D_p) = 14$ cm

Stroke length $(L_S) = 15$ cm

Clearance volume $(V_C) = 231$ cm³

Swept volume is given by

$$V_S = \frac{\pi}{4} D_p^2 . L_S = \frac{\pi}{4} \times (14)^2 \times 15 = 2309.071 \text{ cm}^3$$

a) The ratio of clearance volume and swept volume,

$$\frac{V_C}{V_S} = \frac{231}{2309.071} = 0.1$$

b) The compression ratio is then given by

$$r = 1 + \frac{V_s}{V_C} = 1 + \frac{1}{C} = 1 + \frac{1}{0.1} = 11$$

c) The thermal efficiency is given by

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \left(\frac{1}{11}\right)^{1.4-1} = 61.68\ \%$$

23. In an ideal Otto cycle, heat added to the system due to combustion is twice the heat rejected through the exhaust gas. Determine the thermal efficiency and compression ratio of the engine.

Solution:

Given, Heat added to the system = 2 × (Heat rejected through the exhaust gas)

i.e. $Q_H = 2 \times Q_L$

Thermal efficiency of the cycle is given by

$$\eta = 1 - \frac{Q_L}{Q_H} = 1 - \frac{Q_L}{2Q_L} = 1 - \frac{1}{2} = 50\%$$

Also, $\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$

or, $0.5 = 1 - \left(\frac{1}{r}\right)^{\gamma-1}$

or, $\frac{1}{r^{0.4}} = 0.5$

$$\therefore\ r = \left(\frac{1}{0.5}\right)^{\frac{1}{0.4}} = 5.657$$

24. At the beginning of the compression stroke of an air standard diesel cycle having a compression ratio of 16, the temperature is 300 K and the pressure is 100 kPa. If the cut off ratio for the cycle is 2, determine:

(a) the pressure and temperature at the end of each process of the cycle, the thermal efficiency, and

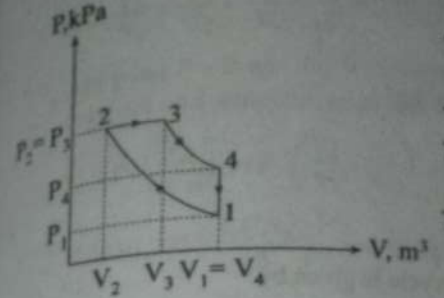
(b) the mean effective pressure.

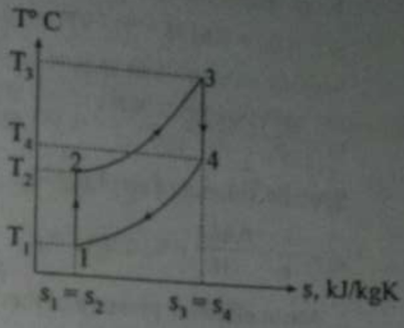
Solution:

Given, Compression ratio (r) $= \frac{V_1}{V_2} = 16$

Properties at state 1: $P_1 = 100$ kPa, $T_1 = 300$ K

Cut off ratio $(\alpha) = \frac{V_3}{V_2} = 2$





a) Applying P - V and T - V relations for an isentropic compression process 1 - 2, pressure and temperature at state 2.

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 100\,(16)^{1.4} = 4850.293 \text{ kPa}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 \times (16)^{1.4-1} = 909.43 \text{ K}$$

Applying T - V relation for an isobaric heat addition process 2 - 3, temperature at state 3,

$$T_3 = T_2\left(\frac{V_3}{V_2}\right) = 909.43 \times 2 = 1818.86 \text{ K}$$

$P_3 = P_2 = 4850.293$ kPa

Applying P - V and T - V relations for an isentropic expansion Process 3-4, pressure and temperature at state 4,

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\gamma} = P_3\left(\frac{V_3}{V_2}\frac{V_2}{V_4}\right)^{\gamma} = P_3\left(\frac{V_3}{V_2}\frac{V_2}{V_1}\right)^{\gamma} = P_3\left(\frac{\alpha}{r}\right)^{\gamma}$$

$$= 4850.293\left(\frac{2}{16}\right)^{1.4} = 263.901 \text{ kPa}$$

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} - T_3\left(\frac{\alpha}{r}\right)^{\gamma-1} = 1818.86\left(\frac{2}{16}\right)^{1.4-1} = 791.71\text{K}$$

b) The thermal efficiency is given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^{\gamma}-1}{\alpha-1}\right] = 1 - \frac{1}{1.4}\left(\frac{1}{16}\right)^{1.4-1}\left[\frac{2^{1.4}-1}{2-1}\right] = 61.38\%$$

c) Heat added per kg of air during the cycle is given by

$$q_H = q_{23} = c_P\,(T_3 - T_2) = 1.005 \times (1818.86 - 909.43) = 913.977 \text{ kJ/kg}$$

Work output per kg of air per cycle is given by

$$w = \eta\, q_H = 0.6138 \times 913.977 = 561 \text{ kJ/kg}$$

Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 300}{100} = 0.861 \text{ m}^3\text{/kg}$$

Specific volume of air at state 2,

$$v_2 = \frac{v_1}{r} = \frac{0.861}{16} = 0.0538 \text{ m}^3\text{/kg}$$

∴ Mean effective pressure of thecycle is given by

$$P_{MEP} = \frac{w}{v_1 - v_2} = \frac{561}{0.861 - 0.0538} = 695 \text{ kPa}$$

25. An air standard Diesel cycle has a compression ratio of 16. At the beginning of the compression stroke, the pressure and temperature are 100 kPa and 27°C respectively. The heat added per kg of air during the cycle is 2000 kj/kg. Determine

(a) the pressure and the temperature at the end of each process of the cycle,

(b) The thermal efficiency and

(c) the mean effective pressure.

Solution:

Given, Compression ratio$(r) = \frac{V_1}{V_2} = 16$

Properties at state 1: $P_1 = 100$ kPa $T_1 = 27^0C = 27 + 273 = 300$ K

Heat added per kg of air during the cycle $(q_H) = 2000$ kJ/kg

a) Applying P - V and T - V relations for an isentropic compression process 1 - 2, pressure and temperature at state 1,

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 100 \times (16)^{1.4} = 4850.293 \text{ kPa}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 \times (16)^{1.4-1} = 909.43 \text{ K}$$

Heat added per kg of air during the cycle is given by

$$q_H = q_{23} = c_p(T_3 - T_2)$$

$$\therefore T_3 = \frac{q_H}{c_p} + T_2 = \frac{2000}{1.005} + 909.43 = 2899.48 \text{ K}$$

$$P_3 = P_2 = 4850.293 \text{ kPa}$$

Cut off ratio for the cycle is given by

$$\alpha = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2899.48}{909.43} = 3.1882$$

Applying P - V and T - V relations for an isentropic expansion process 3 -4, pressure and temperature at state 4,

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\gamma} = P_3\left(\frac{V_3}{V_2}\cdot\frac{V_2}{V_4}\right)^{\gamma} = P_3\left(\frac{V_3}{V_2}\cdot\frac{V_2}{V_1}\right)^{\gamma} = P_3\left(\frac{\alpha}{r}\right)^{\gamma}$$

$$= 4850.293\left(\frac{3.1882}{16}\right)^{1.4} = 506.96 \text{ kPa}$$

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3\left(\frac{\alpha}{r}\right)^{\gamma-1} = 2899.48\left(\frac{3.1882}{10}\right)^{1.4-1} = 1520.86 \text{ K}$$

b) The thermal efficiency of the cycle is given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^{\gamma} - 1}{\alpha - 1}\right]$$

$$= 1 - \frac{1}{1.4}\left(\frac{1}{16}\right)^{1.4-1}\left[\frac{3.1882^{1.4} - 1}{3.1882 - 1}\right] = 56.18\ \%$$

c) Work output per kg of air per cycle is given by

$$w = \eta q_H = 0.5618 \times 2000 = 1123.6 \text{ kJ/kg}$$

Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 300}{100} = 0.861 \text{ m}^3\text{/kg}$$

specific volume of air at state 2,

$$v_2 = \frac{v_1}{r} = \frac{0.861}{16} = 0.0538 \text{ m}^3\text{/kg}$$

∴ Mean effective pressure of the cycle is given by

$$P_{MEP} = \frac{w}{v_1 - v_2} = \frac{1123.6}{0.861 - 0.0538} = 1391.97 \text{ kPa}$$

26. An air standard diesel cycle has a compression ratio of 22 and expansion ratio of 11. Determine its cut off ratio and the efficiency.

Solution:

Given, Compression ratio $(r) = \frac{V_1}{V_2} = 22$

Expansion ratio $(r_e) = \frac{V_4}{V_3} = 11$

Expansion ratio is given by

$$r_e = \frac{V_4}{V_3} = \frac{V_2}{V_3} \times \frac{V_4}{V_2} = \frac{V_2}{V_3} \times \frac{V_1}{V_2} = \frac{r}{\alpha}$$

or, $11 = \frac{22}{\alpha}$

$\therefore \alpha = 2$

Now, efficiency of the cycle is given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^{\gamma}-1}{\alpha-1}\right]$$

$$= 1 - \frac{1}{1.4}\left(\frac{1}{22}\right)^{1.4-1}\left[\frac{2^{1.4}-1}{2-1}\right]$$

$= 65.99\ \%$

27. The pressure and temperature at the beginning of the compression stroke of an air standard Diesel cycle are 100 kPa and 300 K. The peak pressure and temperature during the cycle are 8000 kPa and 3000 K respectively. Determine the compression ratio and the cycle efficiency.

Solution:

Given, Properties at state 1 : $P_1 = 100$ kPa, $T_1 = 300$ K

Peak pressure during the cycle (P_{peak}) = P_3 = 8000 kPa

Peak temperature during the cycle (T_{peak}) = T_3 = 3000 K

Since, heat addition is at constant pressure, $P_2 = P_3 = 8000$ kPa

Applying P - V relation for an isentropic compression process 1 - 2,

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\therefore \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/\gamma} = \left(\frac{8000}{100}\right)^{1/1.4} = 22.8744$$

Compression ratio is given by

$$r = \frac{V_1}{V_2} = 22.8744$$

Applying T - V relation for an isentropic compression process 1 - 2, temperature at state 2,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\ (22.8744)^{1.4-1} = 1049.21\ \text{K}$$

Cut off ratio for the cycle is given by

$$\alpha = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{3000}{1049.21} = 2.8593$$

$\therefore$ The cycle efficiency is given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^{\gamma}-1}{\alpha-1}\right]$$

$$= 1 - \frac{1}{1.4}\left(\frac{1}{22.85744}\right)^{1.4-1}\left[\frac{2.8593^{1.4}-1}{2.8593-1}\right]$$

$= 63.17\ \%$

28. A diesel cycle has a compression ratio of 20. The air at the beginning of the compression stroke is at P_1=100 kPa , T_1 = 290 K and V_1=0.5*10^{-3} m^3. The maximum temperature during the cycle is 2000 K. Determine

(a) the maximum pressure during the cycle,

(b) the cycle efficiency, and

(c) the work output

Solution:

Given, Compression ratio (r) = $\frac{V_1}{V_2} = 20$

Properties at state 1: $P_1 = 100$ kPa, $T_1 = 290$ K, $V_1 = 0.5 \times 10^{-3}$ m^3

Maximum temperature during the cycle (T_{max}) = T_3 = 2000 K

a) Applying P - V and T - V relations for an isentropic compression process 1 - 2, pressure and temperature at state 2,

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 100 \times (20)^{1.4} = 6628.91\ \text{kPa}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 290 \times (20)^{1.4-1} = 961.19\ \text{K}$$

$\therefore$ Maximum pressure during the cycle (P_{max}) = $P_3 = P_2$ = 6628.91 kPa

b) Cut off ratio for the cycle is give by

$$\alpha = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2000}{961.19} = 2.0808$$

$\therefore$ The cycle efficiency is given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left(\frac{\alpha^{\gamma}-1}{\alpha-1}\right)$$

$$= 1 - \frac{1}{1.4}\left(\frac{1}{20}\right)^{1.4-1}\left[\frac{2.0808^{1.4}-1}{2.0808-1}\right]$$

$= 64.32\%$

c) Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 290}{100} = 0.8323 \text{ m}^3/\text{kg}$$

Mass of air is given by

$$m = \frac{V_1}{v_1} = \frac{0.5 \times 10^{-3}}{0.8323} = 600.745 \times 10^{-6} \text{ kg}$$

Heat added during the cycle is given by

$$Q_H = Q_{23} = mc_p (T_3 - T_2)$$

$$= 600.745 \times 10^{-6} \times 1.005 \times (2000 - 961.19) = 0.6272 \text{ kJ}$$

$\therefore$ The work output in a cycle is given by

$$W = \eta\, Q_H = 0.6432 \times 0.6272 = 0.4034 \text{ kJ}$$

29. The properties of air at the beginning of compression stroke in an air standard Diesel cycle are 100 kPa and 300 K. The air at the beginning of the expansion stroke is at 6500 kPa and 2000 K. Determine:

(a) the compression ratio,
(b) the thermal efficiency, and
(c) the mean effective pressure.

Solution:

Given, Properties at state 1: $P_1 = 100$ kPa, $T_1 = 300$ K

Properties at state 3: $P_3 = 6500$ kPa, $T_3 = 2000$ K

Since, heat addition in a cycle is at constant pressure

$\therefore P_2 = P_3 = 6500$ kPa

Applying P - V and T - V relations for an isentropic compression process 1-2, temperature at state 2,

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$\therefore \frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/\gamma} = \left(\frac{6500}{100}\right)^{1/1.4} = 19.7214$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\,(19.7214)^{1.4-1} = 988.7724 \text{ K}$$

a) The compression ratio is given by

$$r = \frac{V_1}{V_2} = 19.7214$$

b) The cut off ratio for a cycle is given by

$$\alpha = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2000}{988.7724} = 2.0227$$

$\therefore$ The thermal efficiency of a cycle is given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^{\gamma} - 1}{\alpha - 1}\right]$$

$$= 1 - \frac{1}{1.4}\left(\frac{1}{19.7214}\right)^{1.4-1}\left[\frac{2.0227^{1.4} - 1}{2.0227 - 1}\right]$$

$= 64.337\%$

c) Heat added per kg of air per cycle is given by

$$q_H = q_{23} = c_p (T_3 - T_2) = 1.005\,(2000 - 988.7724)$$

$= 1016.2837$ kJ/kg

Work output per kg of air per cycle is given by

$$w = \eta\, q_H = 0.64337 \times 1016.2837 = 653.6465 \text{ kJ/kg}$$

Specific volume of air at state 1

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 300}{100} = 0.861 \text{ m}^3/\text{kg}$$

Specific volume of air at state 2,

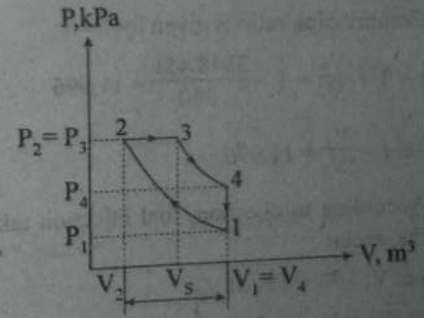
$$v_2 = \frac{v_1}{r} = \frac{0.861}{19.7214} = 0.04366 \text{ m}^3/\text{kg}$$

$\therefore$ The mean effective pressure of the cycle is given by

$$P_{MEP} = \frac{w}{v_1 - v_2} = \frac{653.6465}{0.861 - 0.04366} = 799.724 \text{ kPa}$$

30. An engine working on a diesel cycle has a compression ratio of 16 and the cut off takes place at 8 % of the stroke. Determine its air standard efficiency.

Solution:



Given, Compression ratio $(r) = \frac{V_1}{V_2} = 16$

According to the question, cut off takes place at 8% of stroke i.e.

$V_3 - V_2 = 8\%$ of V_S

or, $V_3 - V_2 = 0.08\ V_S$

or, $V_3 - V_2 = 0.08\ (V_1 - V_2)$

or, $V_3 - V_2 = 0.08\ (16V_2 - V_2)$

or, $V_3 = V_2 + 1.2\ V_2 = 2.2V_2$

$\therefore \frac{V_3}{V_2} = 2.2$

Cut off ratio is given by

$\alpha = \frac{V_3}{V_2} = 2.2$

Efficiency of the cycle is given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^\gamma - 1}{\alpha - 1}\right]$$

$$= 1 - \frac{1}{1.4}\left(\frac{1}{16}\right)^{1.4-1}\left[\frac{2.2^{1.4} - 1}{2.2 - 1}\right]$$

$= 60.42\%$

31. The following data are given for a four stroke diesel engine:

Cylinder bore = 14 cm
Stroke length = 25 cm
Clearance volume = 350 cm³
Determine the air standard efficiency, if fuel injection takes place at constant pressure for 5 % of the stroke.

Solution:

Given, Cylinder bore $(D_P) = 14$ cm

Stroke length $(L_S) = 25$ cm

Clearance volume $(V_C) = 350\ cm^3$

Swept volume is given by

$$V_S = \frac{\pi}{4}(D_P)^2.L_S = \frac{\pi}{4} \times 14^2 \times 25 = 3848.451\ cm^3$$

Compression ratio is given by

$$r = 1 + \frac{V_S}{V_C} = 1 + \frac{3848.451}{350} = 11.996$$

i.e. $r = \frac{V_1}{V_2} = 11.996$

According to question, fuel injection takes place at constant pressure for 5% of the stroke

i.e. $V_3 - V_2 = 5\%$ of V_S

or, $V_3 = V_2 + 0.05\ (V_1 - V_2) = V_2 + 0.05\ (11.996\ V_2 - V_2)$

or, $V_3 = 1.5498\ V_2$

$\frac{V_3}{V_2} = 1.5498$

Cut off ratio is given by

$\alpha = \frac{V_3}{V_2} = 1.5498$

The air standard efficiency of the cycle is given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^\gamma - 1}{\alpha - 1}\right] = 1 - \frac{1}{1.4}\left(\frac{1}{11.996}\right)^{1.4-1}\left[\frac{1.5498^{1.4} - 1}{1.5498 - 1}\right]$$

$= 59.285\%$

32. Air at the beginning of compression stroke in an ideal Diesel cycle is at 100 kPa and 295 K and the compression ratio is 20. Determine the maximum temperature during the cycle to have an efficiency of 65 %.

Solution:

Given, Properties at state 1: $P_1 = 100$ kPa, $T_1 = 295$ K

Compression ratio $(r) = \frac{V_1}{V_2} = 20$

Cycle efficiency $(\eta) = 65\% = 0.65$

Applying T - V relation for an isentropic compression process 1 - 2, temperature at state 2,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 295 \times (20)^{1.4-1} = 977.764\ K$$

Efficiency of the cycle is given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^\gamma - 1}{\alpha - 1}\right]$$

$$\text{or, } 0.65 = 1 - \frac{1}{1.4}\left(\frac{1}{20}\right)^{1.4-1}\left[\frac{\alpha^{1.4} - 1}{\alpha - 1}\right]$$

Solving for α,

$\alpha = 1.9289$

Cut off ratio is given by

$\alpha = \frac{V_3}{V_2} = \frac{T_3}{T_2}$

$\therefore$ Temperature at state 3 $(T_3) = \alpha \cdot T_2 = 1.9289 \times 977.764 = 1886$ K

33. An engine with bore of 8 cm and stroke of 12 cm has a compression ratio of 6. To increase the compression ratio 1.5 mm is machined off the cylinder head face. Determine the new compression ratio.

Solution:

Given, Cylinder bore (D_P) = 8 cm

Stroke length (L_S) = 12 cm

Compression ratio (r) = $\frac{V_1}{V_2} = 6$

Swept volume is given by

$$V_S = \frac{\pi}{4} D_P^2 . L = \frac{\pi}{4} \times 8^2 \times 12 = 603.186 \text{ cm}^3$$

Compression ratio is then given by

$$r = 1 + \frac{V_S}{V_C}$$

$$\therefore \text{ Clearance volume, } V_c = \frac{V_S}{r - 1} = \frac{603.186}{6-1} = 120637 \text{ cm}^3$$

Now according to the question, 1.5 mm of cylindrical head face is machined off then,

$$\therefore V_C' = V_C - \frac{\pi}{4} \times 8^2 \times 0.15 = 120.637 - \frac{\pi}{4} \times 8^2 \times 0.15 = 113.097 \text{ m}^3$$

∴ New compression ratio is given by

$$r' = 1 + \frac{V_S}{V_C'} = 1 + \frac{603.186}{113.097} = 6.33$$

6.2 IOE Solution

1. In an air standard Brayton cycle the air enters the compressor at 0.18 MPa, 34° C. The pressure leaving the compressor is 2.3 MPa, and the maximum temperature in the cycle is 2350° C. Determine:

(a) The pressure and temperature at each point cycle

(b) The compressor work, turbine work, and cycle efficiency [Take c_p = 1005 J/kgK, γ = 1.4] *(IOE 2070 Bhadra)*

Solution:

Given, Properties at state 1: P_1 = 0.18 MPa = 180 kPa, T_1 = 34°C = 34 + 273 = 307 K

Compressor exit pressure (P_2) = 2.3 MPa = 2300 kPa

Maximum temperature in the cycle (T_{max}) = T_3 = 2350°C = 2350 + 273 = 2623K

a) State 2: P_2 = 2300 kPa

Applying P - T relation for an isentropic compression 1- 2, temperature at state 2,

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 307 \times \left(\frac{2300}{180}\right)^{\frac{1.4-1}{1.4}} = 635.724 \text{ K}$$

State 3: $P_3 = P_2$ = 2300 kPa, T_3 = 2623 K

State 4: $P_4 = P_1$ = 180 kPa

Applying P - T relation for an isentropic expansion 3 - 4, temperature at state 4,

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = 2623 \left(\frac{180}{2300}\right)^{\frac{1.4-1}{1.4}} = 1266.684 \text{ K}$$

b) Work consumed by the compressor per kg of air is given by

$$w_C = w_{12} = c_p (T_2 - T_1) = 1.005 (635.724 - 307) = 330.368 \text{ kJ/kg}$$

Work produced by the turbine per kg of air is then given by

$$w_T = w_{34} = c_p (T_3 - T_4) = 1.005 (2623 - 1266.684) = 1363.098 \text{ kJ/kg}$$

Pressure ratio is given by

$$r_p = \frac{P_2}{P_1} = \frac{2300}{180} = 12.778$$

∴ Efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{12.778}\right)^{\frac{1.4-1}{1.4}}$$

$$= 0.51709 = 51.709 \%$$

2. Air is used as the working fluid in a simple ideal Brayton cycle that has a pressure ratio of 12, a compressor inlet temperature of 300 K, and a turbine inlet temperature of 1000 K. Determine the required mass flow rate of air for a net power output of 90 MW also calculate thermal efficiency of the cycle. *(IOE 2069 Bhadra)*

Solution:

Given, Pressure ratio (r_p) = $\frac{P_2}{P_1}$ = 12

Compressor inlet temperature (T_1) = 300 K

Turbine inlet temperature (T_3) = 1000 K

Power output of the cycle ($\dot{W}$) = 90 MW

Temperature at the compressor exit is given by

$$T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} T_1 = (r_p)^{\frac{\gamma-1}{\gamma}} T_1 = (12)^{\frac{1.4-1}{1.4}} \times 300 = 610.181 \text{ K}$$

Temperature at the turbine exit is given by

$$T_4 = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} T_3 = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \times T_3 = \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} T_3 = \left(\frac{1}{12}\right)^{\frac{1.4-1}{1.4}} \times 1000$$

= 491.657 K

Heat supplied per kg of air is then given by

$q_H = q_{23} = c_p (T_3 - T_2) = 1.005 (1000 - 610.181) = 391.768$ kJ/kg

Efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{12}\right)^{\frac{1.4-1}{1.4}} = 0.50834 = 50.834\%$$

Also, net work produced by the cycle per kg of air is given by

$w_{net} = \eta q_H = 0.50834 \times 391.768 = 199.1524$ kJ/kg

∴ Mass flow rate of air is then given by

$$\dot{m} = \frac{\dot{W}}{w_{net}} = \frac{90 \times 10^3}{199.1524} = 451.915 \text{ kg/s}$$

3. The following data relate to an air-standard Diesel cycle. The pressure and temperature at the end of suction stroke are 1 bar and 30° C respectively. Maximum temperature during the cycle if 1500° C and compression ratio is 16. Determine:

(a) The percentage of stroke at cut-off takes place,

(b) The temperature at the end of expansion stroke, and

(c) Theoretical efficiency [Take R= 287 J/kg, γ= 1.4] ***(IOE 2069 Poush)***

Solution:

Given, Compression ratio (r) = $\frac{V_1}{V_2} = 16$

Properties at state 1: P_1 = 1 bar = 100 kPa, T_1 = 30°C = 30 + 273 = 303 K

Maximum temperature during the cycle (T_{max}) = T_3 = 1500°C

= 1500 + 273 =1773 K

Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 303}{100} = 0.86961 \text{ m}^3\text{/kg}$$

Specific volume of air at state 2,

$$v_2 = \frac{v_1}{r} = \frac{0.86961}{16} = 0.054351 \text{ m}^3\text{/kg}$$

Applying T- V relation for an isentropic compression 1 - 2, temperature at state 2,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 (r)^{\gamma-1} = 303 \times (16)^{1.4-1} = 918.524 \text{ K}$$

Applying T- v relation for an isobaric heat addition process 2 - 3, temperature at state 3,

$$v_3 = \frac{T_3}{T_2} \times v_2 = \frac{1773}{918.524} \times 0.054351 = 0.10491 \text{ m}^3\text{/kg}$$

∴ Percentage of stroke at which cut off takes place is given by

$$\frac{v_3 - v_2}{v_1 - v_2} = \frac{0.10491 - 0.054351}{0.86961 - 0.054351} = 0.06202 = 6.202\%$$

b) Cut off ratio for the cycle is given by

$$\alpha = \frac{v_3}{v_4} = \frac{T_3}{T_2} = \frac{1773}{918.524} = 1.9303$$

Applying t - V relation for an isentropic expansion 3 - 4, Temperature at (the end of expansion stroke) state 4,

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3\left(\frac{V_3}{V_2}\cdot\frac{V_2}{V_4}\right)^{\gamma-1} = T_3\left(\frac{V_3}{V_2}\cdot\frac{V_2}{V_1}\right)^{\gamma-1} = T_3\left(\frac{\alpha}{r}\right)^{\gamma-1}$$

$$= 1773 \times \left(\frac{1.9303}{16}\right)^{1.4-1} = 760.866 \text{ K}$$

c. Efficiency of the cycle is then given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^\gamma - 1}{\alpha - 1}\right] = 1 - \frac{1}{1.4}\left(\frac{1}{16}\right)^{1.4-1}\left[\frac{1.9303^{1.4} - 1}{1.9303 - 1}\right] = 61.725\%$$

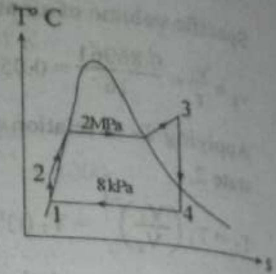
4. Steam at 2 MPa, 350° C is expanded in a steam turbine working on a Rankine to 8 kPa. Determine the net work per kg of steam and the cycle efficiency assuming ideal process. What will be the difference in the efficiency if the pump work is neglected? ***(IOE 2069 Chaitra)***

Solution:

Given, Properties of steam at turbine inlet: $P_3 = 2$ MPa = 2000 kPa, $T_3 = 350°C$

Properties of steam at turbine exit: $P_4 = 8$ kPa

With reference to T - s diagram of cycle shown in figure, properties of steam at each state are evaluated as follows.



State 1: $P_1 = 8$ kPa, saturated liquid

Referring to Table A2.1,

$h_1 = h_f$ (8 kPa) = 173.85 kJ/kg,

$v_1 = v_f$ (8 kPa) = 0.001008 m^3/kg

State 2: $P_2 = P_3 = 2000$ kPa, compressed liquid

Applying isentropic relation for an incompressible substance,

$h_2 - h_1 = v_1 (P_2 - P_1)$

$\therefore\ h_2 = h_1 + v_1 (P_2 - P_1) = 173.85 + 0.001008 \times (2000 - 8) = 175.878$ kJ/kg

State 3: $P_3 = 2000$ kPa, $T_3 = 350°C$, superheated vapor

Referring to Table A2.4, $h_3 = 3136.6$ kJ/kg, $s_3 = 6.9556$ kJ/kgK

State 4: $P_4 = 8$ kPa

For isentropic expansion process 3 - 4, $s_4 = s_3 = 6.9556$ kJ/kgK

Referring to Table A2.1, s_f (8 kPa) = 0.5925 kJ/kgK , s_{fg} (8 kPa) = 7.6342 kJ/kgK, s_g (8 kPa) = 8.2267 kJ/kgK, h_f (8 kPa) = 173.85 kJ/kg, h_{fg} (8 kPa) = 2402.3kJ/kg

Here, $s_f < s_4 < s_g$. Hence it is a two phase mixture. Therefore quality of steam at state 4,

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.9556 - 0.5925}{7.6342} = 0.83349 \text{ kJ/kgK}$$

Specific enthalpy of steam at state 4 is given by

$h_4 = h_f + x_4 h_{fg} = 173.85 + 0.83349 \times 2402.3 = 2176.143$ kJ/kg

Work produced by the turbine per kg of steam is given by

$w_T = w_{34} = h_3 - h_4 = 3136.6 - 2176.143 = 960.457$ kJ/kg

Work consumed by the pump per kg of steam is given by

$w_P = w_{12} = h_2 - h_1 = 175.878 - 173.85 = 2.028$ kJ/kg

The net work delivered to the surroundings is given by

$w_{net} = w_T - w_P = 960.457 - 2.028 = 958.429$ kJ/kg

Heat supplied to the steam in the boiler is given by

$q_H = q_{23} = h_3 - h_2 = 3136.6 - 175.878 = 2960.722$ kJ/kg

Efficiency of the Rankine cycle is then given by

$$\eta = \frac{w_{net}}{q_H} = \frac{958.429}{2960.722} = 32.37\%$$

If pump work is neglected,

$w_{net} = w_T = 960.457$ kJ/kg

Efficiency of the cycle is given by

$$\eta = \frac{w_{net}}{q_H} = \frac{960.457}{2960.722} = 32.44\ \%$$

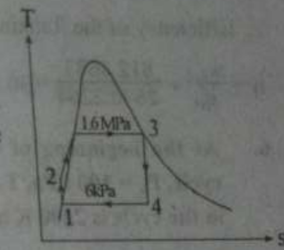
5. **Determine the efficiency of an ideal Rankine cycle operating between boiler pressure of 1.6 MPa and condenser pressure of 6 kPa. Steam leaves the boiler as saturated vapor. *(IOE 2069 Ashad)***

Solution:

Given, Boiler pressure: $P_3 = P_2 = 1600$ kPa

Condenser pressure: $P_1 = P_4 = 6$ kPa

With reference to T - s diagram of the cycle shown in figure, properties of steam at each state are evaluated as follows.



State 1: $P_1 = P_4 = 6$ kPa, saturated liquid

Referring to Table A2.1,

$h_1 = h_f$ (6 kPa) = 151.47 kJ/kg, $v_1 = v_f$(6kPa) = 0.001006 kJ/kg

State 2: $P_2 = P_3 = 1600$ kPa, compressed liquid

Applying isentropic relation for an incompressible substance,

$h_2 - h_1 = v_1 (P_2 - P_1)$

$\therefore\ h_2 = h_1 + v_1 (P_2 - P_1) = 151.47 + 0.001006 (1600 - 6) = 153.0736$ kJ/kg

State 3: $P_3 = 1600$ kPa, saturated vapor

Referring to Table A 2.1, $h_3 = 2793.3$ kJ/kg, $s_3 = 6.4207$ kJ/kg K

State 4: $P_4 = 6$ kPa

For isentropic expansion process 3 - 4, $s_4 = s_3 = 6.4207$ kJ/kgK

Referring to Table A2.1, $s_f < s_4 < s_g$, hence it is a two phase mixture. Therefore quality of steam at state 4,

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.4207 - 0.5208}{7.8075} = 0.75667$$

Specific enthalpy of steam at state 4 is given by

$$h_4 = h_f + x_4 h_{fg} = 151.47 + 0.75667 \times 2415.0 = 1978.8281 \text{ kJ/kg}$$

Work produced by the turbine per kg of steam is given by

$$w_T = w_{34} = h_3 - h_4 = 2793.3 - 1978.8281 = 814.4719 \text{ kJ/kg}$$

Work consumed by the pump per kg of steam is given by

$$w_P = w_{12} = h_2 - h_1 = 153.0736 - 151.47 = 1.6036 \text{ kJ/kg}$$

The net work delivered to the surrounding is given by

$$w_{net} = w_T - w_P = 814.4719 - 1.6036 = 812.8683 \text{ kJ/kg}$$

Heat supplied to the steam in the boiler is given by

$$q_H = q_{23} = h_3 - h_2 = 2793.3 - 153.0736 = 2640.2264 \text{ kJ/kg}$$

∴ Efficiency of the Rankine cycle is then given by

$$\eta = \frac{w_{net}}{q_H} = \frac{812.8683}{2640.2264} = 30.788\%$$

6. At the beginning of the compression process of an air standard Otto cycle, $P_1 = 100$ kPa, $T_1 = 290$ K, $V_1 = 400$ cm^2. The maximum temperature in the cycle is 2200 K and the compression ratio is 8. Determine:

(a) The heat addition, in kJ

(b) The net work, in kJ

(c) The thermal efficiency

(d) The mean effective pressure. [Take R = 287 J/kgK, c_v = 718 J/kgK] ***(IOE 2068 Chaitra)***

Solution:

Given, Compression ratio, $r = \frac{V_1}{V_2} = 8$

Properties at state 1: $P_1 = 100$ kPa, $T_1 = 290$ K, $V_1 = 400$ cm^3 = 400×10^{-6} m^3

Maximum temperature in the cycle $(T_{max}) = T_3 = 2200$ K

Applying T - V relation for an isentropic compression 1 - 2, temperature at state 2,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 290\,(8)^{1.4-1} = 666.245 \text{ K}$$

Similarly, applying T- V relation for an isentropic expansion 3 - 4, temperature at state 4 is given by

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3\left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_3\left(\frac{1}{r}\right)^{\gamma-1} = 2200\left(\frac{1}{8}\right)^{1.4-1} = 957.606 \text{ K}$$

Specific volume at state 1 is given by

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 290}{100} = 0.8323 \text{ m}^3\text{/kg}$$

Specific volume at state 2 is given by

$$v_2 = \frac{v_1}{r} = \frac{0.8323}{8} = 0.10404 \text{ m}^3\text{/kg}$$

Mass of air is given by

$$m = \frac{V_1}{v_1} = \frac{400 \times 10^{-6}}{0.8323} = 480.596 \times 10^{-6} \text{ kg}$$

a) Heat addition during the cycle is given by

$Q_H = Q_{23} = mc_v\,(T_3 - T_2) = 480.596 \times 10^{-6} \times 0.718 \times (2200 - 666.245) = 0.52925$ kJ

b) Net work output is given by

$$W_{net} = \eta Q_H = 0.5647 \times 0.52925 = 0.2989 \text{ kJ}$$

c) Efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \left(\frac{1}{8}\right)^{1.4-1} = 56.47\%$$

d) Mean effective pressure of the cycle is given by

$$P_{MEP} = \frac{W_{net}}{m\,(v_1 - v_2)} = \frac{0.2989}{480.596 \times 10^{-6}\,(0.8323 - 0.10404)} = 854 \text{ kPa}$$

7. Air enters the compressor of an ideal air standard Brayton cycle at 100 kPa, 300 K, with a volumetric flow of 5 m^3/s. The compressor pressure ratio is 10. The turbine inlet temperature is 1400 K. Determine:

(a) The thermal efficiency of the cycle.

(b) The net power developed in kW. ***(IOE 2068 Shrawan)***

Solution:

Given, Properties at state 1: $P_1 = 100$ kPa, $T_1 = 300$ K

Pressure ratio $(r_p) = \frac{P_2}{P_1} = 10$

Turbine inlet temperature $(T_3) = 1400$ K

Volumetric flow rate of air at inlet of compressor ($\dot{V}_1$) = 5 m^3/s

Temperature at the compressor exit is given by

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times (10)^{\frac{1.4-1}{1.4}} = 579.209 \text{ K}$$

Temperature at the turbine exit is given by

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = T_3\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = T_3\left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1400 \times \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 725.1265 \text{ K}$$

a) The thermal efficiency of the cycle is then given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 48.205\%$$

b) Heat added per kg of air in a cycle is given by

$$q_H = q_{23} = c_p (T_3 - T_2) = 1.005 (1400 - 579.209) = 824.895 \text{ kJ/kg}$$

∴ Net work output per kg of air in a cycle is then given by

$$w_{net} = \eta q_H = 0.48205 \times 824.895 = 397.641 \text{ kJ/kg}$$

Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 300}{10} = 0.861 \text{ m}^3/\text{kg}$$

Mass flow rate is then given by

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{5}{0.861} = 5.8072 \text{ kg/s}$$

∴ The net power developed is given by

$$\dot{W}_{net} = \dot{m} \times w_{net} = 5.8072 \times 397.641 = 2309.18 \text{ kW}$$

8. **The compression ratio in an air standard Otto cycle is 8. At the beginning of the compression stroke, the pressure is 0.1 MPa and the temperature is 15^0 C. The heat transfer to the air per cycle is 1800 kJ/kg of air. Determine:**

(a) **The pressure and temperature at the end of each process of the cycle.**

(b) **The thermal efficiency.**

(c) **The mean effective pressure. [R= 2871 J/kgK, c_v= 718 J/kgK] *(IOE 2068 Baishak)***

Solution:

Given, Compression ratio $(r) = \frac{V_1}{V_2} = 8$

Properties at state 1: $P_1 = 0.1$ MPa = 100 kPa, $T_1 = 15°C = 15 + 273 = 288$ K

Heat transfer to the air per cycle $(q_H) = 1800$ kJ/kg

a) Applying P - V and T - V relations for an isentropic compression 1 - 2, pressure and temperature at state 2 are given by

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 100\ (8)^{1.4} = 1837.917 \text{ kPa}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 288\ (8)^{1.4-1} = 661.65 \text{ K}$$

Heat added during the cycle is given by

$$q_H = q_{23} = c_v (T_3 - T_2)$$

∴ Temperature at state 3 is given as

$$T_3 = \frac{q_H}{c_v} + T_2 = \frac{1800}{0.718} + 661.65 = 3168.614 \text{ K}$$

Applying P - T relation for an isochoric heat addition process 2 - 3, pressure at state 3,

$$P_3 = \frac{T_3}{T_2} P_2 = \left(\frac{3168.614}{661.65}\right) \times 1837.917 = 8801.707 \text{ kPa}$$

Similarly, applying P - V and T - V relations for an isentropic expansion 3 - 4, pressure and temperature at state 4 are given by

$$P_4 = P_3\left(\frac{V_3}{V_4}\right)^{\gamma} = P_3\left(\frac{V_2}{V_1}\right)^{\gamma} = P_3\left(\frac{1}{r}\right)^{\gamma} = 8801.707\left(\frac{1}{8}\right)^{1.4} = 478.896 \text{ kPa}$$

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3\left(\frac{V_2}{V_1}\right)^{\gamma-1} = T_3\left(\frac{1}{r}\right)^{1.4-1} = 1379.2193 \text{ K}$$

b) Efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \left(\frac{1}{8}\right)^{1.4-1} = 56.472\%$$

c) Work output per kg of air per cycle is given by

$$w = \eta\, q_H = 0.56472 \times 1800 = 1016.496 \text{ kJ/kg}$$

Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.887 \times 288}{100} = 0.82656 \text{ m}^3\text{/kg}$$

Specific volume of air at state 2,

$$v_2 = \frac{v_1}{r} = \frac{0.82656}{8} = 0.10332 \text{ m}^3\text{/kg}$$

∴ Mean effective pressure of the cycle.

$$P_{MEP} = \frac{w}{v_1 - v_2} = \frac{1016.496}{0.82656 - 0.10332} = 1405.475 \text{ kPa}$$

9. **Calculate the efficiency and specific work output of a simple gas turbine working on the Brayton cycle. The maximum and minimum temperatures of the cycle are 1000 K and 288 K respectively, the pressure ratio is 6. [Take $\gamma = 1.4$, $c_p = 1005$ J/kgK]** ***(IOE 2067 Ashad)***

Solution:

Given, Pressure ratio $(r_p) = \frac{P_2}{P_1} = 6$

Maximum temperature of the cycle $(T_{max}) = T_3 = 1000$ K

Minimum temperature of the cycle $(T_{min}) = T_1 = 288$ K

Temperature at the compressor exit is given by

$$T_2 - T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 288\,(6)^{\frac{1.4-1}{1.4}} = 480.531 \text{ K}$$

Temperature at the turbine exit is given by

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = T_3 \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = T_3 \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1000 \left(\frac{1}{8}\right)^{\frac{1.4-1}{1.4}} = 599.337 \text{ K}$$

Heat added per kg of air per cycle is then given by

$$q_H = q_{23} = c_p (T_3 - T_2) = 1.005\,(1000 - 480.531) = 522.066 \text{ kJ/kg}$$

∴ Efficiency of the cycle is then given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{6}\right)^{\frac{1.4-1}{1.4}} = 40.066\%$$

Then, specific work output per cycle given by

$$w_{net} = \eta q_H = 0.40066 \times 522.066 = 209.171 \text{ kJ/kg}$$

10. **Calculate the efficiency and specific work output of a simple gas turbine working on the Brayton cycle. The maximum and minimum temperatures of the cycle are 1000 K and 300 K respectively, and the pressure ratio is 6. [Take $\gamma = 1.4$, $c_p = 1005$ J/kgK]** ***(IOE 2067 Chaitra)***

Solution:

Given, Pressure ratio $(r_p) = \frac{P_2}{P_1} = 6$

Maximum temperature of the cycle $(T_{max}) = T_3 = 1000$ K

Minimum temperature of the cycle $(T_{min}) = T_1 = 300$ K

Temperature at the compressor exit is given by

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 300\,(6)^{\frac{1.4-1}{1.4}} = 500.553 \text{ K}$$

Temperature at the turbine exit is given by

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = T_3 \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = T_3 \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1000 \times \left(\frac{1}{6}\right)^{\frac{1.4-1}{1.4}} = 599.337 \text{ K}$$

Heat added per kg of air per cycle is given by

$$q_H = q_{23} = c_p (T_3 - T_2) = 1.005\,(1000 - 599.337) = 402.67 \text{ kJ/kg}$$

∴ Efficiency of the cycle is given by

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{6}\right)^{\frac{1.4-1}{1.4}} = 40.066\%$$

Then, specific work output per cycle is given by

$$w_{net} = \eta q_H = 0.40066 \times 402.67 = 161.334 \text{ kJ/kg}$$

11. **An air standard Diesel cycle has a compression ratio of 18, and the heat transferred to the working fluid per cycle is 1800 kJ/kg. AT the beginning of the compression process, the pressure is 0.1 MPa and the temperature is 15°C. Determine:**

(a) Maximum pressure and temperature of the cycle.

(b) Thermal efficiency, and

(c) Mean effective pressure. ***(IOE 2067 Mangsir)***

Solution:

Given, Compression ratio (r) $= \frac{V_1}{V_2} = 18$

Properties at state 1: $P_1 = 0.1$ MPa = 1000 kPa, $T_1 = 15°C = 15 + 273 = 288$ K

Heat added per kg of air during the cycle (q_h) = 1800 kJ/kg

a) Applying P - V and T - V relations for an isentropic compression 1 - 2, pressure and temperature at state 2.

$$P_2 = P_1\left(\frac{V_1}{V_1}\right)^{\gamma} = 100 \times (18)^{1.4-1} = 915.1694 \text{ K}$$

Heat added per kg of air during the cycle is given by

$$q_H = q_{23} = c_p (T_3 - T_2)$$

∴ Temperature at state 3,

$$T_3 = \frac{q_H}{c_p} + T_2 = \frac{1800}{1.005} = 915.1694 = 2706.214 \text{ K}$$

∴ Maximum pressure of the cycle, $P_{max} = P_3 = P_2 = 5719.809$ kPa

And, maximum temperature of the cycle, $T_3 = 2706.214$ K

b) Cut off ratio for the cycle is given by

$$\alpha = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{2706.214}{915.1694} = 2.9571$$

∴ The thermal efficiency of the cycle is given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^{\gamma} - 1}{\alpha - 1}\right] = 1 - \frac{1}{1.4}\left(\frac{1}{18}\right)^{1.4-1}\left[\frac{2.9571^{1.4} - 1}{2.9571 - 1}\right] = 59.082\%$$

c) Work output per kg of air per cycle is given by

$$w = \eta q_H = 0.59071 \times 1800 = 1063.476 \text{ kJ/kg}$$

Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 288}{100} = 0.82656 \text{ m}^3/\text{kg}$$

Specific volume of air at state 2,

$$v_2 = \frac{v_1}{r} = \frac{0.82656}{18} = 0.04592 \text{ m}^3/\text{kg}$$

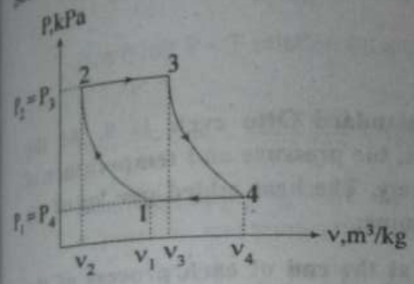
∴ Mean effective pressure of the cycle is given by

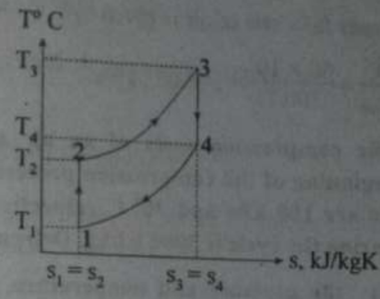
$$P_{MEP} = \frac{w}{v_1 - v_2} = \frac{1063.476}{0.82656 - 0.04592} = 1362.313 \text{ kPa}$$

6.3 Some Important Extra Questions

1. **An ideal Brayton cycle has pressure ratio of 10. The temperature of air at compressor and turbine inlets are 300 K and 1200 K respectively. Determine its thermal efficiency and the mass flow rate of air required to produce net power output of 80MW.**

Solution:





Given, Pressure ratio (r_p) $= \frac{P_2}{P_1} = 10$

Compressor inlet temperature (T_1) = 300 K

Turbine inlet temperature (T_3) = 1200 K

Power output of the cycle ($\dot{W}$) = 80 MW

Temperature at the compressor exit is given by

$$T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \times T_1 = (10)^{\frac{1.4-1}{1.4}} \times 300 = 579.209 \text{ K}$$

Temperature at the turbine exit is given by

$$T_4 = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \times T_3 = \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} \times T_3 = \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} \times 1200 = 621.537 \text{ K}$$

Work produced by the turbine per kg of air is then given by

$$w_T = c_p (T_3 - T_4) = 1.005 \times (1200 - 621.537) = 581.355 \text{ kJ/kg}$$

Work consumed by the compressor per kg of air is then given by

$$w_C = c_p (T_2 - T_1) = 1.005 \times (579.209 - 300) = 280.605 \text{ kJ/kg}$$

Heat supplied per kg of air is then given by

$$q_H = c_p (T_3 - T_2) = 1.005 \times (1200 - 579.209) = 623.895 \text{ kJ/kg}$$

Net work produced by the cycle per kg of air is then given by

$$w_{net} = w_T - w_C = 581.355 - 280.605 = 300.75 \text{ kJ/kg}$$

Efficiency of the cycle is then given by

$$\eta = \frac{w_{net}}{q_H} = \frac{300.75}{623.895} = 48.205\%$$

Alternatively,

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 48.205\%$$

Also, mass flow rate of air is given by

$$\dot{m} = \frac{\dot{W}}{w_{net}} = \frac{80 \times 10^3}{300.75} = 266.002 \text{ kg/s}$$

2. The compression ratio of an air standard Otto cycle is 8. At the beginning of the compression process, the pressure and temperature of air are 100 kPa and 20^0 C respectively. The heat added per kg of air during the cycle is 2000 kJ/kg. Determine:

(a) the pressure and temperature at the end of each process of the cycle,

(b) the thermal efficiency, and

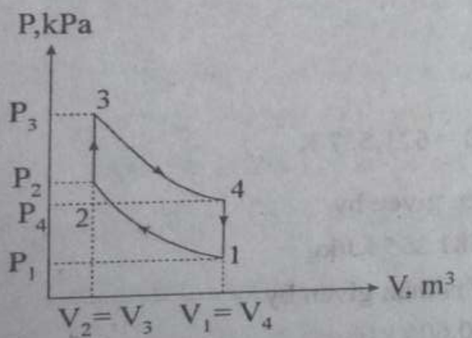
(c) the mean effective pressure. ***(IOE 2070 Chaitra), (IOE 2070 Magh)***

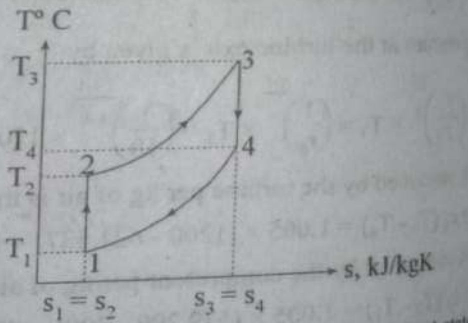
Solution:

Given, Compression ratio (r) $= \frac{V_1}{V_2} = 8$

Properties at state 1: $P_1 = 100$ kPa, $T_1 = 293$ K

Heat added per kg of air during cycle (q_H) = 2000 kJ





a) Applying P - V relation for an isentropic compression 1 - 2, pressure at state 2,

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = P_1 (r)^{\gamma} = 100 \times (8)^{1.4} = 1837.917 \text{ kPa}$$

Similarly, applying T - V relation for an isentropic compression 1 - 2, temperature at state 2,

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 293 \times (8)^{1.4-1} = 673.137 \text{ K}$$

Heat added during the cycle is given by $q_H = q_{23} = c_v (T_3 - T_2)$

∴ Temperature at state 3,

$$T_3 = \frac{q_H}{c_v} + T_2 = \frac{2000}{0.718} + 673.137 = 3458.652 \text{ K}$$

Applying P - T relation for an isochoric heat addition process 2 - 3, pressure at state 3,

$$P_3 = \frac{T_3}{T_2} \times P_2 = \frac{3458.652}{673.137} \times 1837.917 = 9443.419 \text{ kPa}$$

Similarly, applying P - V and T - V relations for an isentropic expansion 3 - 4, pressure and temperature at state 4 are given by,

$$P_4 = P_3 \left(\frac{V_3}{V_4}\right)^{\gamma} = P_3 \left(\frac{V_2}{V_1}\right)^{\gamma} = P_3 \left(\frac{1}{r}\right)^{\gamma} = 9443.419 \times \left(\frac{1}{8}\right)^{1.4} = 513.8109 \text{ kPa}$$

$$T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3 \left(\frac{1}{r}\right)^{\gamma-1} = 3458.652 \times \left(\frac{1}{8}\right)^{1.4-1} = 1505.465\text{K}$$

b) Heat rejected during the cycle.

$$q_L = q_{41} = c_v (T_4 - T_1) = 0.718 (1505.465 - 293) = 870.55 \text{ kJ/ kg}$$

∴ Efficiency of the cycle is given by

$$\eta = 1 - \frac{q_L}{q_H} = 1 - \frac{870.55}{2000} = 56.472 \%$$

Alternatively,

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 = \left(\frac{1}{8}\right)^{1.4-1} = 56.472\%$$

c) Work output per kg of air per cycle,

$$w = q_H - q_L = 2000 - 870.55 = 1129.45 \text{ kJ/kg}$$

Alternatively,

$$w = \eta q_H = 0.56772 \times 2000 = 1129.45 \text{ kJ/kg}$$

Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 293}{100} = 0.84091 \text{ m}^3\text{/kg}$$

Specific volume of air at state 2,

$$v_2 = \frac{v_1}{r} = \frac{0.84091}{8} = 0.10511\ m^3/kg$$

Alternatively,

$$v_2 = \frac{RT_2}{P_2} = \frac{0287 \times 673.137}{1837.917} = 0.10511\ m^3/kg$$

∴ Mean effective pressure of the cycle.

$$P_{MEP} = \frac{w}{v_1 - v_2} = \frac{1129.45}{0.84091 - 0.10511} = 1535.003\ kPa$$

3. The pressure and temperature at the end of suction stroke are 100 kPa and 27° C respectively. Maximum temperature during the cycle is 1600° C and the compression ratio is 16. Determine:

(a) the percentage of stroke at which cut-off takes places,

(b) the temperature at the end of the expansion stroke, and

(c) the thermal efficiency.

Solution:

Given, Compression ratio $(r) = \frac{V_1}{V_2} = 16$

Properties at state 1: $P_1 = 100$ kPa, $T_1 = 300$ K

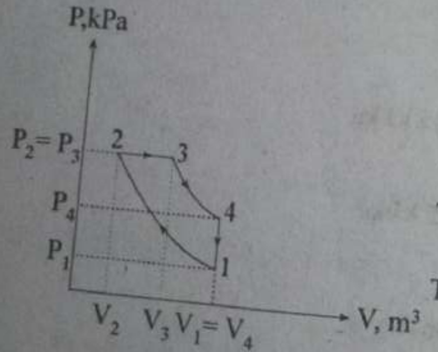
Maximum temperature during the cycle, $T_{max} = T_3 = 1600 + 273 = 1873$ K

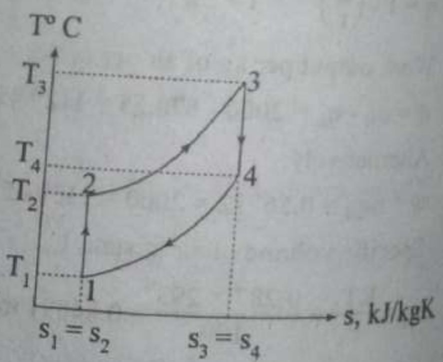
a) Specific volume of air at state 1,

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \times 300}{100} = 0.861 m^3/kg$$

Specific volume of air at state 2,

$$v_2 = \frac{v_1}{r} = \frac{0.861}{16} = 0.0538125\ m^3/kg$$





Applying T - V relation for an isentropic compression 1 -2, temperature at state 2,

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 (r)^{\gamma-1} = 300 \times (16)^{1.4-1} = 909.429\ K$$

Applying T-V relation for an isentropic heat addition process 2-3, volume at state 3,

$$v_3 = \frac{T_3}{T_2} \times v_2 = \frac{1873}{909.429} \times 0.0538125 = 0.110829\ m^3/kg$$

Percentage of stroke at which cut off takes place;

$$\frac{v_3 - v_2}{v_1 - v_2} = \frac{0.110829 - 0.0538125}{0.861 - 0.0538125} = 0.706355 = 7.06355\%$$

b) Cut off ratio for the cycle is given by

$$\alpha = \frac{v_3}{v_2}\left(= \frac{T_3}{T_2}\right) = \frac{0.110829}{0.0538125} = 2.0595$$

Applying T - V relations for an isentropic expansion 3-4, temperature at (the end of expansion stroke) state 4 is given by

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = T_3\left(\frac{V_3}{V_2}\frac{V_2}{V_4}\right)^{\gamma-1} = T_3\left(\frac{V_3}{V_2}\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$= T_3\left(\frac{\alpha}{r}\right)^{\gamma-1} = 1873\left(\frac{2.0595}{10}\right)^{1.4-1} = 824.887\ K$$

c) Efficiency of the cycle is then given by

$$\eta = 1 - \frac{1}{\gamma}\left(\frac{1}{r}\right)^{\gamma-1}\left[\frac{\alpha^{\gamma} - 1}{\alpha - 1}\right] = 1 - \frac{1}{1.4}\left(\frac{1}{16}\right)^{1.4-1}\left[\frac{2.0595^{1.4} - 1}{2.0595 - 1}\right] = 61.09\%$$

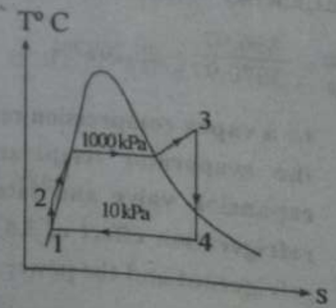
4. Steam at 1 MPa and 400° C is expanded on a steam turbine working on a Rankine cycle to 10 kPa. Determine the net work per kg of steam and the cycle efficiency.

Solution:

Given, Properties of steam at turbine inlet: $P_3 = 1000$ kPa, $T_3 = 400$°C

Properties of steam at turbine exit: $P_4 = 100$ kPa

With reference to T -S diagram of the cycle shown in figure, properties of steam at each state are evaluated as follows.



State 1: $P_1 = 10$ kPa, saturated liquid

Referring to Table A2.1, $h_1 = h_f(10kPa) = 191.83$ kJ/kg,

$v_1 = v_f(10kPa) = 0.00101\ m^3/kg$

State 2: $P_2 = 1000kPa$, compressed liquid

Applying isentropic relation for an incompressible substance,

$h_2 - h_1 = v_1(P_2 - P_1)$

$\therefore\ h_2 = h_1 + v_1(P_2 - P_1) = 191.83 + 0.00101\ (1000 - 10) = 192.8299$ kJ/kg

State 3: $P_3 = 1000$ kPa, $T_3 = 400°C$, superheated vapor

Referring to Table A2.3, $h_3 = 3263.8$ kJ/kg, $s_3 = 7.4648$ kJ/kgK

State 4: $P_4 = 10$ kPa

For isentropic expansion process 3 - 4, $S_3 = S_4 = 7.4648$ kJ/kgK.

Referring to Table A4.1, $s_f < s_4 < s_g$, hence it is a two phase mixture. Therefore quality of steam at state 4,

$$x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{7.4648 - 0.6493}{7.4989} = 0.90887$$

$\therefore\ h_4 = h_f + x_4 h_{fg} = 191.83 + 0.90887 \times 2392.0 = 2365.8389$ kJ/kg

Work produced by the turbine per kg of steam is given by

$w_T = w_{34} = h_3 - h_4 = 3263.8 - 2365.8389 = 897.961$ kJ/kg

Work consumed by the pump per kg of steam is given by $W_p = W_{12} = h_2 - h_1 =$ $192.8299 - 191.83 = 0.9999$ kJ/kg

The net work delivered to the surrounding is given by

$w_{net} = w_T - w_P = 897.961 - 0.9999 = 896.96$ kJ/kg

Heat supplied to the steam in boiler is given by

$q_H = q_{23} = h_3 - h_2 = 3263.8 - 192.8299 = 3070.97$ kJ/kg

$\therefore$ Efficiency of the Rankine cycle is then given by $\eta =$

$$\frac{w_{net}}{q_H} = \frac{896.97}{3070.97} = 29.2077\%$$

5. **In a vapor compression refrigeration system, the condenser is 20°C and the evaporator temperature is -10°C. Saturated liquid enters the expansion valve and saturated vapor enters the compressor. For a refrigeration effect of 3.5 kW, Determine COP, mass flow rate of the refrigerant and the power input if the refrigerant is ammonia.**

Solution:

Given, Temperature of refrigerant at condenser $(T_3) = 20°C$

Temperature of refrigerant at evaporator $(T_1) = -10°C$

Refrigerant effect $(\dot{Q}_L) = 3.5$ kW

With reference to T - S diagram of the cycle shown in figure, properties of refrigerant at each states are evaluated as follows.

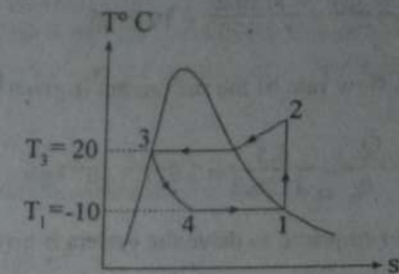
State 1: $T_1 = -10°C$, saturated vapor

Referring to Table A4.2,

$h_1 = h_g(-10°C) = 1450.5$ kJ/kg

$s_1 = s_g = (-10°C) = 5.7564$ kJ/kgK

State 2: $T_2 = 20°C$, superheated vapor.



For an isentropic compression process 1 - 2, $s_2 = s_1 = 5.7564$ kJ/kgK

Referring to Table A4.3, other properties of superheated ammonia at 20°C can be listed as

P, kPa	h kJ/kg	s, kJ/kgK	
450	1511.8	5.7749	(a)
500	1508.1	5.7138	(b)

Now, applying linear interpolation for specific enthalpy

$$h_2 = h_a + \frac{h_b - h_a}{s_b - s_a}(s_2 - s_a)$$

$$= 15118 + \frac{1508.1 - 1511.8}{5.7138 - 5.7749}(5.7564 - 5.7749) = 1510.6797 \text{ kJ/kg}$$

State 3: $T_3 = 20°C$, saturated liquid

Referring to Table A4.1, $h_3 = h_f(20°C) = 294.3$ kJ/kg

State 4: $T_4 = -10°C$, Two phase mixture

For throttling process 3 - 4, $h_3 = h_4 = 294.3$ kJ/kg

Heat removed per kg of refrigerant from the desired space

$q_L = h_1 - h_4 = 1450.5 - 294.3 = 1156.2$ kJ/kg

Work required per kg of refrigerant is given as

$w_{in} = 1510.6797 - 1450.5 = 60.1797$ kJ/kg

Therefore, COP of the system is given by

$$COP = \frac{q_L}{w_{in}} = \frac{1156.2}{60.1797} = 19.2125$$

Mass flow rate of the refrigerant is given by

$$\dot{m} = \frac{\dot{Q}_L}{q_L} = \frac{3.5}{1156.2} = 3.027 \times 10^{-3} \text{ kg/s}$$

Power required to drive the system is given by

$$W = \dot{m}\,(w_{in}) = 3.027 \times 10^{-3} \times 60.1797 = 0.18217 \text{ kW}$$

Chapter 7 Introduction to Heat Transfer

7.1 Numerical Problems

1. **An insulating material having a thermal conductivity of 0.08 W/mK is used to limit the heat transfer of 80 W/m² for a temperature difference of 150° C across the opposite faces. Determine the required thickness of the material.**

Solution:

Given, Thermal conductivity of an insulating material (k) = 0.08 W/mK

Heat transfer rate per unit area $\left(\frac{\dot{Q}}{A}\right) = 80$ W/m²

Temperature difference (ΔT) = 150° C

Heat transfer per unit area is given by

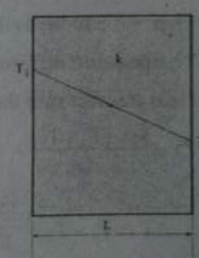
$$\frac{\dot{Q}}{A} = \frac{k\Delta T}{L} = \frac{0.8 \times 150}{L}$$

$$\text{or, } 80 = \frac{0.8 \times 150}{L}$$

$\therefore$ L = 15 m = 15 cm

2. **A brick wall 12 cm thick and 5 m² surface area is exposed to 250° C at one face and 50° C to another face. If the thermal conductivity of the material is 1.5 W/mK, determine the heat transfer rate.**

Solution:



Given, Thickness of brick wall (L) = 12 cm = 0.12 m

Surface area of brick wall (A) = 5 m²

Temperature of one face (T_1) = 250° C

Temperature of another face (T_2) = 50° C

Thermal conductivity of the material (k) = 1.5 W/mK

Heat transfer rate through a plane wall is given by

$$\dot{Q} = \frac{kA(T_1 - T_2)}{L} = \frac{1.5 \times 5 \times (250 - 50)}{20.12} = 12.5\ kW$$

3. **The heat transfer rate through a wooden board of 3 cm thick for a temperature difference of 24° C between the inner and outer surface is 80 W/m². Determine the thermal conductivity of the board.**

Solution:

Given, Heat transfer rate per unit area through a wooden board $\left(\frac{\dot{Q}}{A}\right) = 80\ W/m^2$

Thickness of a wooden board (L) = 3 cm = 0.03 m

Temperature difference between inner and outer surface (ΔT) = 24° C

Thermal conductivity of a board (k) = ?

Heat transfer rate through a plane wall is given by

$$\dot{Q} = \frac{kA\Delta T}{L}$$

$$\text{or, } \frac{\dot{Q}}{A} = \frac{k}{L}\Delta T$$

$$\therefore k = \frac{\dot{Q}}{A} \times \frac{L}{\Delta T} = \frac{80 \times 0.03}{24} = 0.1\ W/mk$$

4. **Magnitude of conduction heat transfer through an insulating layer of 0.8 m² surface area, 5 cm thick and having a thermal conductivity of 0.25 W/mK is found to be 1600 W. Determine the temperature difference existing across the materialSolution:**

Given, Heat transfer rate through an insulating layer ($\dot{Q}$) = 1600 W

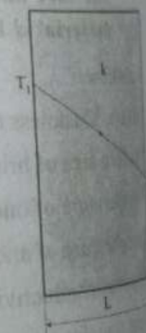
Surface area of insulating layer (A) = 0.8 m²

Thickness of insulating layer (L) = 5 cm = 0.05 m

Thermal conductivity of insulating layer (k) = 0.25 W/mk

Temperature difference existing across the material (ΔT)) = ?

Heat transfer rate through an insulating layer is given by



$$\dot{Q} = \frac{kA(T_1 - T_2)}{L}$$

$$\therefore \Delta T = T_1 - T_2 = \frac{\dot{Q}L}{kA} = \frac{1600 \times 0.05}{0.25 \times 0.8} = 400^0 C$$

5. **Determine the rate of heat loss from a brick wall (k=0.7 W/mK) of length 5 m, height 4 m and 0.25 m thick. The temperature of the inner surface is 120 °C and that of outer surface is 30 °C. Also calculate the distance from the inner surface at which temperature is 90 °C.**

Solution:

Given, Thermal conductivity of a brick wall (k) = 0.7 W/mK

Area of a brick wall (A) = 5 m × 4 m = 20 m²

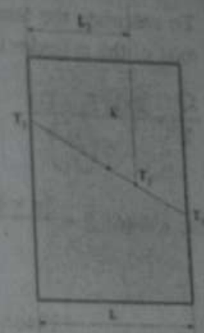
Thickness of brick wall (L) = 0.25 m

Temperature of the inner surface (T_1) = 120° C

Temperature of the outer surface (T_3) = 30° C

Rate of heat loss from a brick wall ($\dot{Q}$) = ?

Rate of heat loss from a brick wall is given by



$$\dot{Q} = \frac{kA(T_1 - T_3)}{L} = \frac{0.7 \times 20 \times (120 - 30)}{0.25}$$

$$= 5040\ W = 5.04\ kW$$

Now, for calculating the distance from the inner surface at which temperature is 90° C i.e., T_2 = 90° C

$$\dot{Q} = \frac{kA(T_1 - T_2)}{L_1}$$

$$\therefore L_1 = \frac{kA(T_1 - T_2)}{\dot{Q}} = \frac{0.7 \times 20 \times (120 - 90)}{5040} = 8.33\ cm$$

6. **A hollow cylinder with inner and outer diameter of 8 cm and 12cm respectively has an inner surface temperature of 200° C and outer surface temperature of 50° C. if the thermal conductivity of the cylinder material is 60 W/mK, determine the heat transfer from the unit length of the pipe. Also determine the temperature at the surface at a radial distance of 5 cm from the axis of the cylinder.**

Solution:

Given, Inner radius of hollow cylinder (r_1) = 4 cm

Outer radius of hollow cylinder (r_2) = 6 cm

Inner surface temperature (T_1) = 200° C

Outer surface temperature (T_2) = 50° C

Thermal conductivity of the cylinder material (k) = 60 W/mK

Heat transfer per unit length of pipe $\left(\frac{\dot{Q}}{L}\right) = ?$

Heat transfer per unit length for the hollow cylinder is given by

$$\frac{\dot{Q}}{L} = \frac{2\pi(T_1 - T_2)\,k}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi(200 - 50) \times 60}{\ln\left(\frac{6}{4}\right)} = 139466.17 = 139.47 \text{ kW}$$

To calculate the temperature at the surface at a radial distance of 5 cm from the axis of the cylinder i.e, $r_3 = 5$ cm

$$\frac{\dot{Q}}{L} = \frac{2\pi k(T_1 - T_3)}{\ln\left(\frac{r_3}{r_1}\right)}$$

$$\text{or, } 139466.2 = \frac{2\pi \times 60 \times (200 - T_3)}{\ln\left(\frac{5}{4}\right)}$$

$$\text{or, } 200 - T_3 = \frac{139466.2 \times \ln\left(\frac{5}{4}\right)}{2\pi \times 60}$$

$$\therefore T_3 = 200 - 82.55 = 117.45^0\text{C}$$

7. **The inside and outside surface temperature of a window are at 25^{0}C and 0^{0}C, respectively. If the window is 80 cm by 50 cm and 1.6 cm thick and has a thermal conductivity of 0.8 W/mK, determine the heat loss through the glass in 1 hour.**

Solution:

Given, Inside surface temperature of a window (T_1) = 25^{0}C

Outside surface temperature of a window (T_2) = 0^{0}C

Area of a window, A = 80 cm × 50 cm = 4000 cm^2 = 0.4 m^2

Thickness of a window (L) = 1.6 cm = 0.016 m

Thermal conductivity of a window (k) = 0.8 W/mk

Heat loss through the gas per sec is given by

$$\dot{Q} = \frac{kA}{L}(T_2 - T_1) = \frac{0.8 \times 0.4}{0.016} \times (25 - 0) = 500 \text{ W}$$

∴ Heat loss through the glass in 1 hour (Q) = 500 × 3600 kJ = 1800 kJ

8. **The roof of an electrically heated home is 10 m long, 8 m wide, and 0.25 m thick, and is made of a flat layer of concrete whose thermal conductivity is k=0.8 W/mK. The temperatures of the inner and the outer surfaces of the roof one night are measured to be 18^{0}C and 5^{0}C, respectively, for a period of 12 hours. Determine:**
   - **(a) the rate of heat loss through the roof that night and**
   - **(b) the cost of that heat loss to the home owner if the cost of electricity is Rs 10/kWh.**

Solution:

Given, Area of roof (A) = 10 m × 8 m = 80 m^2

Thickness of roof (L) = 0.25 m

Thermal conductivity of concrete (k) = 0.8 w/mK

Inner surface temperature (T_1) = 18^{0}C

Outside surface temperature (T_2) = 5^{0}C

a) Rate of heat loss through the roof is given by

$$\dot{Q} = \frac{kA}{L}(T_1 - T_2) = \frac{0.8 \times 80}{0.25} \times (18 - 5) = 3.328 \text{ kW}$$

b) Cost of heat loss = rate of heat loss × rate of cost

= 3.328 × 12 × 10 = Rs 399.36

9. **A plate having a surface area of 4m^2 and temperature of 80^0 C is exposed to air at 25^0 C. if the heat transfer coefficient between the surface and air is 20 W/m^{2}K, determine the heat transfer rate from the plate to the air.**

Solution:

Given, Surface area of plate (A) = 4 m^2

Heat transfer coefficient (h) = 20 W/m^{2}K

Temperature of plate (T_A) = 80^{0}C

Temperature of air (T_B) = 25^{0}C

Heat transfer rate from plate to the air is given by

$$\dot{Q} = h_A(T_A - T_B) = 20 \times 4 \times (80 - 25) = 4.4 \text{ kW}$$

10. **A 1.2 m long tube with outer diameter of 4cm having outside temperature of 120^0 C is exposed to the ambient air at 20^0 C. if the heat transfer coefficient between the tube surface and the air is 20W/m^{2}K, determine the heat transfer rate from the tube to the air.**

Solution:

Given, Length of a tube (L) = 1.2 m

Outer radius of a tube (R) = 2 cm = 0.02 m

Outside temperature of a tube $(T_A) = 120^0 C$

Temperature of ambient air $(T_B) = 20^0 C$

Heat transfer coefficient (h) = $20 W/m^2K$

Heat transfer rate from tube to air is given by

$\dot{Q} = h_A (T_A - T_B) = h(2\pi RL)(T_A - T_B)$

$= 20 \times (2 \times \pi \times 0.02 \times 1.2) \times (120 - 20) = 301.6$ W

11. An electric current is passed through a wire 2 mm in diameter and 8 cm long. The wire is submerged in the liquid water. During the boiling of water temperature of water is 100 ^{0}C and convection heat transfer coefficient is 4500 W/m^2 K. Determine the power supplied to the wire to maintain the wire surface temperature at 120 ^{0}C.

Solution:

Given, Diameter of a wire (D) = 2 mm = 0.002 m

Length of a wire (L) = 8 cm = 0.08 m

Temperature of water during boiling $(T_B) = 100^0 C$

Wire surface temperature $(T_A) = 120^0 C$

Heat transfer coefficient (h) = $4500 W/m^2K$

Heat transfer rate from wire to water is given by

$\dot{Q} = h_A (T_A - T_B) = h(\pi DL)(T_A - T_B)$

$= 4500 \times (\pi \times 0.002 \times 0.08) \times (120 - 100) = 45.239$ W

Hence, power supplied to the wire to maintain the wire surface temperature at $120^0 C = 45.239$ W

12. The heat flux at the surface of an electrical heater is 3500 W/m^2. the heater surface temperature is 120^0 C when it is cooled by air at 50^{0}C. what is the average convective heat transfer coefficient? What will the heater temperature be if the power is reduced so that heat flux is 2500 W/m^2?

Solution:

Given, Heat flux at the surface of an electrical heater $\left(\frac{\dot{Q}}{A}\right) = 3500 W/m^2$

Heater surface temperature $(T_A) = 120^0 C$

Temperature of air $(T_B) = 50^0 C$

Heat transfer coefficient (h) = ?

Heat transfer rate from heater to air is given by

$\dot{Q} = h_A (T_A - T_B)$

or, $h = \left(\frac{\dot{Q}}{A}\right) \times \frac{1}{T_A - T_B}$

$\therefore h = 3500 \times \frac{1}{(120 - 50)} = 50 W/m^2K$

Again,

Heat transfer rate from heater to air is given by

$\dot{Q} = hA (T_A - T_B)$

$\therefore$ Heater surface temperature when heat flux is $2500 W/m^2K$ is given as

$T_A = T_B + \left(\frac{\dot{Q}}{A}\right)\frac{1}{h}$

$\therefore T_A = 50 + 2500 \times \frac{1}{50} = 100^0 C$

13. A 2 m long, 0.35 cm diameter electrical wire extends across a room at 20^0 C. heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 150^0 C in steady operation. Also the voltage drop and electric current through the wire are measured to be 50 V and 2 A respectively. Neglecting the effect heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.

Solution:

Given, Length of a wire (L) = 2 m

Diameter of wire (D) = 0.35 cm

Room temperature (T_B) =

Surface temperature of the wire $(T_A) = 150^0 C$

Voltage drop (V) = 50 V

Electric current through the wire (I) = 2 A

Convection heat transfer coefficient (h) = ?

Here, electric power developed in the wire = heat loss rate from the electric wire

$\dot{Q} = VI = hA(T_A - T_B)$

$\therefore h = \frac{VI}{A(T_A - T_B)} = \frac{VI}{\pi DL(T_A - T_B)}$

$= \frac{50 \times 2}{\pi \times 0.35 \times 10^{-3} \times 2 \times (150 - 20)} = 34.979\ W/m^2K$

14. Two very large plates are maintained at 1200° C and 400°C respectively. Calculate the heat transfer rate due to radiation per unit area. Assume black body properties.

Solution:

Given, Temperature of first plate $(T_1) = 1200^0\ C = 1473\ K$

Temperature of second plate $(T_2) = 400^0\ C = 673\ K$

Heat transfer rate due to radiation per unit area $\left(\frac{\dot{Q}}{A}\right) = ?$

From Stefan-Boltzmann law for black body,

$\frac{\dot{Q}}{A} = \sigma\,(T_1^4 - T_2^4) = 5.67 \times 10^{-8} \times (1473^4 - 673^4) = 255.296\ kW/m^2$

15. The inner surfaces of the walls, floors, and the ceiling of the house are observed to be at an average temperature of 5 °C in winter and 26 °C in summer. Determine the rate of radiation heat transfer between a person inside the house and the surrounding surfaces if the exposed surface area, the average outer surface temperature and the surface emissivity of the person are 1.4 m² and 30 °C and 0.95, respectively.

Solution:

Given, Surface area $(A) = 1.4\ m^2$

Average outer surface temperature $(T_1) = 30^0\ C = 303\ K$

Surface emissivity of the person $(\in) = 0.95$

In winter, average inner surface temperature $(T_2) = 5^0 C = 278\ K$

In summer, average inner surface temperature $(T_3) = 26^0 C = 299\ K$

Then, rate of radiation heat transfer between a person inside the house and the surrounding surface in winter is given by

$\dot{Q} = \sigma \in A(T_1^4 - T_2^4) = 5.67 \times 10^{-8} \times 0.95 \times 1.4 \times (303^4 - 278^4) = 185.215\ W$

Again, rate of radiation heat transfer between a person inside the house and the surrounding surface in summer is given by

$\dot{Q} = \sigma \in A(T_1^4 - T_3^4) = 5.67 \times 10^{-8} \times 0.95 \times 1.4 \times (303^4 - 299^4) = 32.906\ W$

16. A room is maintained at 22° C by an air conditioning unit. Determine the total rate of heat transfer from the person standing in the room if the exposed surface area and the average outer surface temperature of the person are 1.5 m² and 30° C, respectively, and the convection heat transfer coefficient is 10 W/m²K. Take surface emissivity as 0.95.

Solution:

Given, Room temperature $(T_2) = 22^0 C = 295\ K$

Exposed surface area $(A) = 1.5\ m^2$

Average outer surface temperature $(T_1) = 30^0 C = 303\ K$

Convection heat transfer coefficient $(h) = 10\ W/m^2K$

Emissivity $(\in) = 0.95$

Total rate of heat transfer from person standing in the room $(\dot{Q}) = ?$

Here, total rate of heat transfer $(\dot{Q}) = \dot{Q}_{convection} + \dot{Q}_{radiation}$

$= hA(T_1 - T_2) + \sigma \in A(T_1^4 - T_2^4)$

$= 10 \times 1.5 \times (30 - 22) + 5.67 \times 10^{-8} \times 0.95 \times 1.5 \times (303^4 - 295^4) = 189.126\ W$

17. A steel pipe having an outer diameter of 4 cm is maintained at a temperature of 80° C in a room where the ambient temperature is 25° C. the emissivity of the surface and air is 10 W/m²K. Determine the total heat loss from the unit length of the pipe.

Solution:

Given, Outer diameter of a steel pipe $(D) = 4\ cm = 0.04\ m$

Surface temperature of steel pipe $(T_1) = 80^0 C = 353\ K$

Ambient air temperature $(T_B) = 25^0 C = 298\ K$

Emissivity $(\in) = 0.8$

Convection heat transfer coefficient $(h) = 10\ W/m^2K$

Total heat loss from the unit length of pipe $\left(\frac{\dot{Q}}{L}\right) = ?$

Here, total heat loss $(\dot{Q}) = \dot{Q}_{convection} + \dot{Q}_{radiation}$

$= hA(T_1 - T_B) + \sigma \in A(T_1^4 - T_B^4) = h(\pi\ DL)(T_1 - T_2) + \sigma \in (\pi\ DL)(T_1^4 - T_B^4)$

∴ Total heat loss from the unit length of pipe is given as,

$$\frac{\dot{Q}}{L} = h\pi D(T_1 - T_B) + \sigma\varepsilon\pi D(T_1^4 - T_B^4)$$

$$= 10 \times \pi \times 0.04 \times (80 - 25) + 5.67 \times 10^{-8} \times 0.8 \times \pi \times 0.04 \times (353^4 - 298^4) = 112.671 \text{ W/m}$$

18. A hot plate of length 80 cm, width 50cm and thickness 4 cm is placed in air stream at 20^0 C. it is estimated that a total of 300 W is lost from the plate surface by radiation when it has an outer surface temperature of 250^{0}C at steady state. If the convective heat transfer coefficient is 25 W/m^{2}K and the thermal conductivity of the plate is 50 W/mK, determine the inside surface temperature of the plate.

Solution:

Given, Area of hot plate (A) = 80 cm × 50 cm = 0.04 m^2

Thickness of hot plate (L) = 4 cm = 0.04 m

Temperature of air stream (T_3) = 20^{0}C

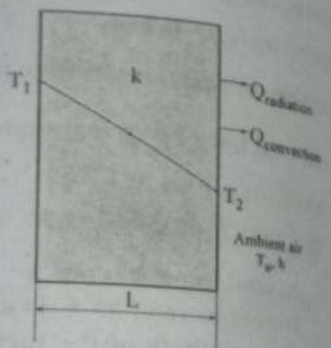
Heat lost by radiation ($\dot{Q}_{radiation}$) = 300 W

Outer surface temperature of plate (T_2) = 250^{0}C

Convective heat transfer coefficient (h) = 25 W/m^{2}K

Thermal conductivity of the plate (k) = 50 W/mK

Inside surface temperature of the plate (T_1) = ?



For steady state heat transfer,

$$\dot{Q}_{conduction} = \dot{Q}_{convection} + \dot{Q}_{radiation}\ \frac{kA}{L}(T_1 - T_2) = hA\,(T_2 - T_3) + 300$$

$$\text{or, } \frac{50 \times 0.4}{0.04}(T_1 - 250) = 25 \times 0.4 \times (250 - 20) + 300$$

$$\therefore T_1 = 250 + 5.2 = 255 \text{ K}$$

19. A flat plate solar collector is insulated at the back surface and exposed to solar radiation at the front surface. The front surface absorbs solar radiation at a rate of 800 W/m^2 and losses heat by convection to the ambient air at 25^{0}C. If the heat transfer coefficient between the plate and the air is 20 W/m^2 K, determine the surface temperature of the plate.

Solution:

Given, Rate of heat absorbed per unit area by radiation: $\left(\frac{\dot{Q}}{A}\right)_{radiation}$ = 800 W/m^2

Ambient air temperature (T_2) = 25^{0}C

Convective heat transfer coefficient (h) = 20 W/ m^{2}K

Surface temperature of the plate (T_1) = ?

Here,

Rate of heat loss per unit area by convection = Rate of heat absorbed per unit area by radiation

or, $h(T_1 - T_2) = 800$

$$\therefore T_1 = \frac{800}{h} + T_2 = \frac{800}{20} + 25 = 65^0\text{C}$$

Hence, the surface temperature of the plate (T_1) = 65^{0}C

20. A flat plate collector having collection efficiency of 80% is insulated at the back surface and exposed to solar radiation at the front surface. The front surface receives solar radiation at a rate of 850 W/m^2 and dissipates heat to the ambient air at 20^0 C both by convection and radiation. If the convection heat transfer coefficient between the plate and air is 16 W/m^{2}K, determine the surface temperature of plate.

Solution:

Given, Ambient air temperature (T_2) = 20^{0}C = 293 K

Convection heat transfer coefficient (h) = 16 W/ m^{2}K

Rate of solar radiation absorbed $\left(\frac{\dot{Q}}{A}\right)_{absorbed}$ = 850 W/m^2

Collection efficiency (η) = 80%

Surface temperature of the plate (T_1) = ?

Here,

Rate of solar radiation absorbed per unit area = Rate of heat dissipated per unit area by convection + rate of heat dissipated per unit area by radiation.

$$\eta \times \left(\frac{\dot{Q}}{A}\right)_{absorbed} = h(T_1 - T_2) + T(T_1^4 - T_2^4)$$

$$0.8 \times 850 = 16(T_1 - 293) + 5.67 \times 10^{-8} \times (T_1^4 - 293^4)$$

$$\therefore T_1 = 323.031 \text{ K}$$

21. A thin metal plate is insulated o the back and exposed to solar radiation at the front surface. The exposed surface of the plate has an emissivity of 0.7. if the solar radiation is incident on the plate at the rate of 750 W/m² and the surrounding air temperature is 20° C, determine the surface temperature of the plate. Assume the convection heat transfer coefficient to be 40 W/m²K.

Solution:

Given, Emissivity of exposed surface of plate $(\in) = 0.7$

Rate of solar radiation incident per unit area $\left(\frac{\dot{Q}}{A}\right)_{incident} = 750\ W/m^2$

Ambient air temperature $(T_B) = 20^0 C = 293$ K

Convection heat transfer coefficient (h) = 40 W/m²K

Outer surface temperature of the plate $(T_1) = ?$

Here, rate of heat incident per unit area by radiation = Rate of heat dissipated per unit area by convection + Rate of heat dissipated per unit area by radiation

or, $\left(\frac{\dot{Q}}{A}\right)_{incident} = h(T_1 - T_B) + \sigma \varepsilon (T_1^4 - T_B^4)$

or, $750 = 40(T_1 - 293) + 5.67 \times 10^{-8} \times 0.7 (T_1^4 - 293^4)$

$\therefore T_1 = 309.91$ K

22. The inner surface of a 2 cm thick 50 cm×50 cm plate (k=10 W/mK) is at400 °C. The outer surface dissipates heat by combined convection and radiation to the ambient air at 27 ° C. If the plate surface has an emissivity 0.85 of and the convection heat transfer coefficient between the outer plate surface and the ambient air is 20 W/m² K, determine the outer surface temperature of the plate.

Solution:

Given, Thickness, of plate (L) = 2 cm = 0.02 m

Area of plate (A) = 50 cm × 50 cm = 0.25 m²

Thermal conductivity of plate (k) = 10 W/mK

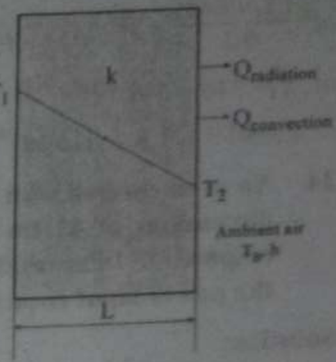
Inner surface temperature of plate $(T_1) = 400^0 C = 673$ K

Convective heat transfer coefficient (h) = 20 W/m²K

Ambient air temperature $(T_B) = 27^0 C = 300$ K

Emissivity of plate $(\in) = 0.85$

Outer surface temperature of plate $(T_2) = ?$



Here,

Rate of heat flow by conduction = Rate of heat dissipated by convection and radiation.

or, $\dot{Q}_{conduction} = \dot{Q}_{convection} + \dot{Q}_{radiation}$

or, $\frac{kA(T_1 - T_2)}{L} = hA(T_2 - T_B) + \sigma \in A(T_2^4 - T_B^4)$

or, $\frac{10 \times 0.25 \times (673 - T_2)}{0.02} = 20 \times 0.25 \times (T_2 - 300) + 5.67 \times 10^{-8} \times 0.85 \times 0.25 \times (T_2^4 - 300^4)$

or, $500(673 - T_2) = 20(T_2 - 300) + 5.67 \times 10^{-8} \times 0.85(T_2^4 - 300^4)$

Solving for T_2,

$T_2 = 643.511$ K

23. The inner surface of a 0.2 m thick wall (k=1 W/mK) is exposed to hot combustion gas and its outer surface is exposed to ambient air at 20° C. the emissivity of the wall surface is 0.8 and convection heat transfer coefficient for the wall surface and air is 25 W/m²K. Under steady state condition, temperature at the outer surface of the wall is found as 75° C. Determine the wall inner surface temperature of the inner surface of the wall.

Solution:

Given, Thickness of wall (L) = 0.2 m

Thermal conductivity of wall (k) = 1 W/mK

Ambient air temperature $(T_B) = 20^0 C = 293$ K

Outer surface temperature of wall $(T_2) = 75^0 C = 248$ K

Convection heat transfer coefficient (h) = 25 W/m²K

Emissivity of the wall surface $(\in) = 0.8$

Inner surface temperature of wall $(T_1) = ?$

Here, under steady state condition,

$\dot{Q}_{conduction} = \dot{Q}_{convection} + \dot{Q}_{radiation} \quad \frac{kA(T_1 - T_2)}{L} = hA(T_2 - T_B) + \sigma \in A(T_2^4 - T_B^4)$

or $\frac{1\,(T_1 - 348)}{0.2} = 25\,(348 - 293) + 5.6 \times 10^{-8} \times 0.8 \times (348^4 - 293^4)$

or, $T_1 - 348 = 341.191$

$\therefore T_1 = 689.191\ K = 416.191^0 C$

24. An oven covered with asbestors (k= 0.2 W/mK) has the inner and outer dimensions of 45 cm * 60 cm * 75 cm and 50 cm *65 cm * 80 cm respectively. The inside wall temperature is 35° C. determine the power the power input required to maintain the steady state conditions.

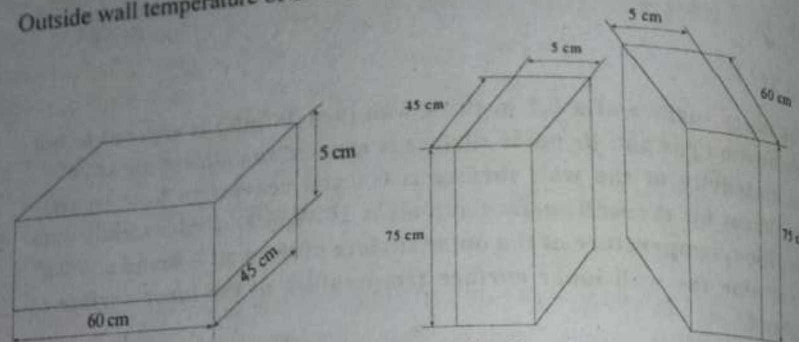
Solution:

Given, Inner dimensions of an oven = 45 cm × 60 cm × 75 cm

Outer dimensions of an oven = 50 cm × 65 cm × 80 cm

Inside wall temperature of oven $(T_1) = 250^0 C$

Outside wall temperature of oven $(T_2) = 35^0 C$



Thermal conductivity of asbestos (k) = 0.2 W/mK

For top and bottom faces:

Rate of heat transfer through wall is given by

$$\dot{Q}_1 = 2 \times \frac{k\,A_1\,(T_1 - T_2)}{4} = 2 \times \frac{0.2 \times 0.6 \times 0.45 \times (250 - 35)}{0.05}$$

= 464.4 W

For lateral faces:

Rate of heat transfer through wall of cross-sectional area 45 cm × 75 cm is given by

$$\dot{Q}_2 = 2 \times \frac{kA_2\,(T_1 - T_2)}{L_2} = 2 \times \frac{0.2 \times 0.45 \times 0.75 \times (250 - 35)}{0.05}$$

= 580.5 W

Rate of heat transfer through wall of cross-sectional area 60 cm × 75 cm is given by

$$\dot{Q}_3 = \frac{2 \times kA_3\,(T_1 - T_2)}{L_3}$$

$$= \frac{2 \times 0.2 \times 0.6 \times 0.75 \times (250 - 35)}{0.05} = 774\ W$$

$\therefore$ Total rate of heat transfer = power input required to maintain steady state condition

$= \dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 = 464.4 + 580.5 + 774 = 1818.9\ W$

25. A freezer compartment consists of a cubical cavity of 1m side. The bottom of the compartment is completely insulated. Determine the minimum thickness of the insulation (k= 0.025 W/mK) that must be applied to the top and side walls to ensure a heat load of less than 400W when the inner and outer surface are at - 5° C and 30° C respectively.

Solution:

Given, Thermal conductivity of insulation (k) = 0.025 W/mK

Inner surface temperature $(T_2) = -5^0 C$

Outer surface temperature $(T_1) = 30^0 C$

Area of cubical cavity (A) = 1 m^2

Thickness of the insulation of single face = L

Total heat load $(\dot{Q})$ = 400 W

Then, rate of heat transfer through one face of cubical cavity is given as,

$$\dot{Q} = \frac{kA\,(T_1 - T_2)}{L}$$

$$400 = \frac{0.025 \times 1 \times (30 + 5)}{L}$$

$\therefore$ L = 2.1875 mm

Since bottom of the compartment is completely insulated and remaining five faces of cube needs insulation, total thickness of the insulation to the top and side faces of cubical cavity

= 5L = 5 × 2.1875 = 10.9375 mm

26. The walls of a furnace 4 m ×3 m are constructed from an inner fire brick (k=0.4 W/mK) wall 30 cm thick, a layer of ceramic blanket insulation (k=0.2 W/mK) 10 cm thick and steel protective layer (k=50

W/mK) 4 mm thick. The inside temperature of the fire brick layer was measured as 500 °C and the temperature of the outside of the insulation as 50 °C. Determine

(a) the rate of heat loss through the wall,

(b) the temperature at the interface between fire brick layer and insulation layer, and

(c) the temperature at the outside surface of the steel layer.

Solution:

Given, Thickness of fire brick (L_1) = 30 cm = 0.3 m

Thermal conductivity of fire brick (k_1) = 0.4 W/mK

Thickness of ceramic blanket insulation (L_2) = 10 cm = 0.1 m

Thermal conductivity of ceramic blanket insulation (k_2) = 0.2 W/mK

Thickness of steel protective layer (L_3) = 4 mm = 0.604 m

Inside temperature of the fire brick layer $(T_1) = 500^0$ C

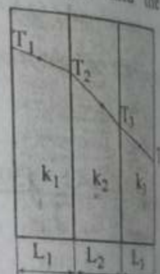
Temperature of the outside of insulation $(T_3) = 50^0$ C

Area of walls of furnace (A) = 4 m × 3 m = 12 m²

a) Rate of heat loss through the wall is given by

$$\dot{Q} = \frac{A\,(T_1 - T_3)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}} = \frac{12\,(500 - 50)}{\frac{0.3}{0.4} + \frac{0.1}{0.2}} = 4320 \text{ W}$$

b) For the temperature at the interface between fire brick layer and the insulation layer, T_2:



$$\dot{Q} = \frac{k_1 A\,(T_1 - T_2)}{L_1}$$

$$\text{or, } 4320 = \frac{0.4 \times 12(500 - T_2)}{0.3}$$

$$\therefore T_2 = 230^0 \text{C}$$

c) For the temperature at the outside surface of the steel layer, T_4:

$$\dot{Q} = \frac{k_3 A\,(T_3 - T_4)}{L_3}$$

$$\text{or, } 4320 = \frac{50 \times 12\,(50 - T_4)}{4 \times 10^{-3}}$$

$$\therefore T_4 = 49.9712^0 \text{C}$$

27. **A furnace is made of fireclay brick of thickness 0.3m and thermal conductivity of 1.2 W/mK. The outside surface is to be insulated by an insulating material with the thermal conductivity of 0.05 W/mK. Determine the thickness of the insulating layer in order to limit the heat loss per unit area of the furnace wall to 1200 W/m² when the inside surface of the wall is at 900° C and the outside surface is at 25° C.**

Solution:

Given, Thickness of fire clay brick (L_1) = 0.3 m

Thermal conductivity of fire clay brick (k_1) = 1.2 W/mK

Thermal conductivity of insulating material (k_2) = 0.05 W/mK

Rate of heat loss per unit area of the furnace wall $(\dot{q})$ = 1200 W/ m²

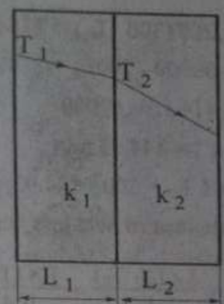
Inside surface temperature $(T_1) = 900^0$ C

Outside surface temperature $(T_3) = 25^0$ C

Thickness of insulating material (L_2) = ?

Here,

Rate of heat loss per unit area is given by



$$\dot{q} = \frac{(T_1 - T_3)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

$$\text{or, } 1200 = \frac{900 - 25}{\frac{0.3}{1.2} + \frac{L_2}{0.05}}$$

$$\text{or, } 0.25 + \frac{L_2}{0.05} = 0.7292$$

$$\therefore L_2 = 0.02396 \text{ m} = 0.396 \text{ cm}$$

28. **A furnace wall 300 mm thick is made up of an inner layer of fire brick (k = 1 W/mK) covered with a layer of insulation (k = 0.2 W/mK). The inner surface of the wall is at 1300° C and the outer surface is at 300° C. Under steady state condition, temperature at the interface is measured to be 1100°C. Determine:**

(a) heat loss per unit area of the wall, and

(b) the thickness of each layer. *(IOE 2070 Ashad)*

Solution:

Given, Thermal conductivity of inner layer of fire brick (k_1) = 1W/mK

Thermal conductivity of layer of insulation (k_2) = 0.2 W/mK

Inner surface temperature of wall (T_1) = 1300°C

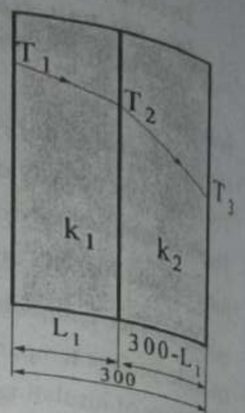
Outer surface temperature of wall (T_3) = 30°C

Temperature at the interface (T_2) = 1100°C

Total thickness of the wall (L) = 300 mm = 0.3 m

Thickness of fire brick (L_1) = ?

Thickness of layer of insulation (L_2) = ?



Here, for steady state, heat transfer heat flowing through each layer is same. So,

$$\dot{Q} = \frac{k_1 A (T_1 - T_2)}{L_1} \text{ ....(i)}$$

$$\dot{Q} = \frac{k_2 A (T_2 - T_3)}{L_2} = \frac{k_2 A (T_2 - T_3)}{300 - L_1} \text{ ...... (ii)}$$

From equation (i) and equation (ii)

$$\frac{k_1 A (T_1 - T_2)}{L_1} = \frac{k_2 A (T_2 - T_3)}{300 - L_1}$$

$$\text{or, } \frac{1 \times (1300 - 1100)}{L_1} = \frac{0.2 \times (1100 - 30)}{300 - L_1}$$

or, $200 (300 - L_1) = 214 L_1$

or, $60000 - 200L_1 = 214L_1$

or, $414 L_1 = 60000$

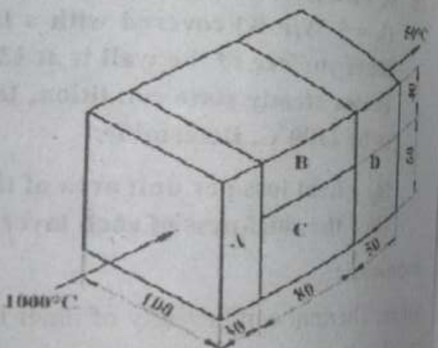
$\therefore L_1 = 144.93$ mm

And, $L_2 = 300 - 144.93 = 155.07$ mm

Rate heat of heat loss per unit area of the wall is given by

$$\frac{\dot{Q}}{A} = \frac{k_1(T_1 - T_2)}{L_1} = \frac{1 \times (1300 - 1100)}{144.93 \times 10^{-3}} = 1379.977 \text{ W/m}^2$$

29. Find the heat transfer through the composite wall as shown in Figure P7.8. Assume one dimensional flow. The thermal conductivities of wall materials are k_A = 150 W/mK, k_B = 30 W/mK, k_C = 65 W/mK and k_D = 50 W/mK. All dimensions are in cm.



Solution:

Given, Thermal conductivities of wall materials are:

$k_A = 150$ W/mK

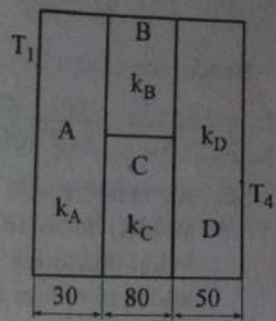
$k_B = 30$ W/mK

$k_C = 65$ W/mK

$k_D = 50$ W/mK

$T_1 = 1000$°C

$T_4 = 50$°C



For the section of material A,

Thickness (L_A) = 30 cm = 0.3 m

Area (A_A) = 100 × 100 cm²

$$\text{Thermal resistance: } (R_{th})_A = \frac{L_A}{A_A k_A} = \frac{0.3}{1 \times 1 \times 150} = 2 \times 10^{-3}$$

For the section of material B,

Thickness (L_B) = 80 cm = 0.8 m

Area (A_B) = 100 cm × 40cm = 0.4 m²

$$\therefore \text{ Thermal Resistance } (R_{th})_B = \frac{L_B}{A_B K_B} = \frac{0.8}{0.4 \times 30} = \frac{1}{15}$$

For the section of material C,

Thickness (L_C) = 80 cm = 0.8 m

Area (A_C) = 100cm × 60 cm = 0.6 m²

$$\therefore \text{ Thermal Resistance } (R_{th})_C = \frac{L_C}{A_C K_C} = \frac{0.8}{0.6 \times 65} = \frac{4}{195}$$

For the section of material D,

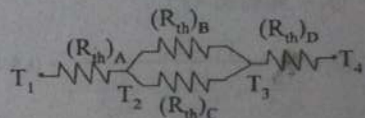
Thickness (L_D) = 50 cm = 0.5 m

Area (A_D) = 100 cm × 100 cm = 1 m²

$$\therefore \text{ Thermal Resistance, } (R_{th})_D = \frac{L_D}{A_D K_D} = \frac{0.5}{1 \times 50} = \frac{1}{100}$$

Using electric analogy of heat conduction, equivalent thermal resistance for the given circuit is given by

$$R_{eq} = (R_{th})_A + \frac{(R_{th})_B \times (R_{th})_C}{(R_{th})_B + (R_{th})_C} + (R_{th})_D$$



$$= 2 \times 10^{-3} + \frac{\frac{1}{15} \times \frac{4}{195}}{\frac{1}{15} + \frac{4}{195}} + 1 \times 10^{-2} = 0.027696$$

Hence, rate of heat transfer through the composite wall is given as

$$\dot{Q} = \frac{T_1 - T_4}{R_{eq}} = \frac{1000 - 50}{0.027696} = 34.313 \text{ kW}$$

30. An exterior wall of a house consists of 10 cm of common brick (k=0.8 W/mK) followed by a 4 cm layer of gypsum plaster (k=0.5 W/mK). What thickness of rock wool insulation (k=0.065 W/mK) should be added to reduce the heat transfer through the wall by 50 %? (IOE, 20[illegible] ***Bhadra)***

Solution:

Given, Thickness of common brick $(L_1) = 10$ cm $= 0.1$ m

Thermal conductivity of common brick $(k_1) = 0.8$ W/mK

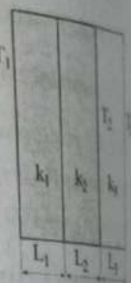
Thickness of gypsum plaster $(L_2) = 4$ cm $= 0.04$ m

Thermal conductivity of gypsum plaster $(k_2) = 0.5$ W/mK

Thermal conductivity of rock wool insulation $(k_3) = 0.065$ W/ mK

Thickness of rock wool insulation $(L_3) = ?$

Rate of heat flow through the wall without insulation is given by



$$\dot{Q}_1 = \frac{T_1 - T_2}{\frac{L_1}{k_1 A_1} + \frac{L_2}{k_2 A_2}}$$

Again, rate of heat flow through the wall after adding of insulation is given by

$$\dot{Q}_2 = \frac{T_1 - T_3}{\frac{L_1}{k_1 A_1} + \frac{L_2}{k_2 A_2} + \frac{L_2}{k_3 A_3}}$$

According to the questions.

$\dot{Q}_2 = 50\%$ of $\dot{Q}_1$

$$\text{or, } \frac{T_1 - T_3}{\frac{L_1}{k_1 A_1} + \frac{L_2}{k_2 A_2} + \frac{L_2}{k_3 A_3}} = 0.5 \times \frac{T_1 - T_3}{\frac{L_1}{k_1 A_1} + \frac{L_2}{k_2 A_2}}$$

$$\text{or, } \frac{L_1}{k_1} + \frac{L_2}{k_2} = 0.5\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)$$

$$\text{or, } 0.5\left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right) = 0.5\left(\frac{L_3}{k_3}\right)$$

$$\text{or, } \frac{0.1}{0.8} + \frac{0.04}{0.5} = \frac{L_3}{0.065}$$

$$L_3 = 0.205 \times 0.065 = 0.013325 = 0.3325 \text{ cm}$$

31. A composite wall consists of 12 cm thick layer of common brick of thermal conductivity 0.8 W/mK and 4 cm thick plaster of thermal conductivity 0.5 W/mK. An insulating material of thermal conductivity 0.1 W/mK is to be added to reduce the heat transfer through wall by 75%. Determine the required thickness of the insulating layer.

Solution:

Given, Thickness of common brick $(L_1) = 12$ cm $= 0.12$ m

Thermal conductivity of common brick $(k_1) = 0.8$ W/mK

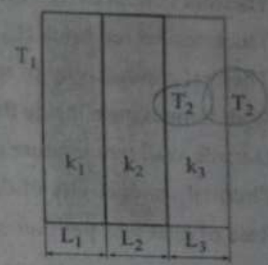
Thickness of plaster $(L_2) = 4$ cm $= 0.04$ m

Thermal conductivity of plaster $(k_2) = 0.5$ W/mK

Thermal conductivity of insulating material $(k_3) = 0.1$ W/mK

Thickness of insulating material $(L_3) = ?$

Rate of heat flow through the composite wall without adding insulation is given as,



$$\dot{Q}_1 = \frac{T_1 - T_2}{\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A}}$$

Also, rate of heat flow through the composite wall after adding insulation is given as,

$$\dot{Q}_2 = \frac{T_1 - T_3}{\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A}}$$

According to the question,

$\dot{Q}_2 = 75\%$ of $\dot{Q}_1$

$$\text{or, } \frac{T_1 - T_3}{\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A}} = 0.75 \times \frac{T_1 - T_2}{\frac{L_1}{k_1 A} + \frac{L_2}{k_2 A}}$$

$$\text{or, } \frac{L_1}{k_1} + \frac{L_2}{k_2} = 0.75\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)$$

or, $0.25\left(\frac{L_1}{k_1}+\frac{L_2}{k_2}\right) = 0.75\left(\frac{L_3}{k_3}\right)$

or, $0.25\times\left(\frac{0.12}{0.8}+\frac{0.04}{0.5}\right) = 0.75\times\frac{L_3}{0.1}$

$\therefore L_3 = 7.667 \text{ mm} = 0.7667 \text{ cm}$

32. A furnace wall is made of a layer of fire clay (k=0.5 W/mK) 12.5 cm thick and a layer of red brick (k=0.8 W/mK) 50 cm thick. If the wall temperature inside the furnace is 1200° C and that on the outside wall is 1000C. Determine the heat loss per unit area of the wall. If it is desired to reduce the thickness of the red brick layer by filling the space between the two layers by diatomite (k=0.1 W/mK) such that total thickness remains same. Determine the required thickness of the filling to ensure the same amount of heat transfer for the same temperature difference.

Solution:

Given, Thickness of fire clay (L_1) = 12.5 cm = 0.125 m

Thermal conductivity of fire clay (k_1) = 0.5 W/mK

Thickness of red brick (L_2) = 50 cm = 0.5 m

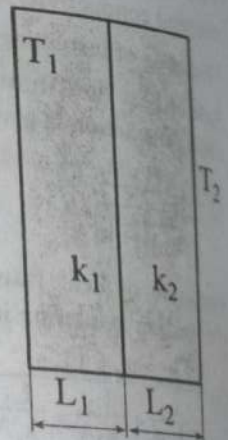
Thermal conductivity of red brick (k_2) = 0.8 W/mK

Wall temperature inside the furnace (T_1) = 1200° C

Outside wall temperature (T_2) = 100° C

Thermal conductivity of diatomite (K_3) = 0.1 W/mK

Rate of heat loss per unit area of the wall ($\dot{q}$) = ?



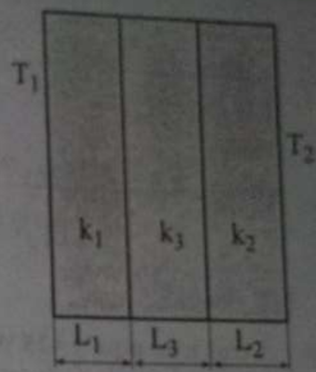
Case I: Rate of heat loss through the wall is given by

$$\dot{Q} = \frac{A(T_1 - T_2)}{\frac{L_1}{k_1}+\frac{L_2}{k_2}}$$

$$\text{or, } \frac{\dot{Q}}{A} = \dot{q} = \frac{T_1 - T_2}{\frac{L_1}{k_1}+\frac{L_2}{k_2}} = \frac{1200 - 100}{\frac{0.125}{0.5}+\frac{0.5}{0.8}} = 1257.14 \text{ W/m}^2$$

Case II:

Rate of heat loss through the wall is given as



$$\dot{Q} = \frac{A(T_1 - T_2)}{\frac{L_1}{k_1}+\frac{L_2}{k_2}+\frac{L_3}{k_3}}$$

$$\text{or, } \frac{\dot{Q}}{A} = \dot{q} = 1257.14 = \frac{1200 - 100}{\frac{0.125}{0.5}+\frac{0.5-L_3}{0.8}+\frac{L_3}{0.1}}$$

$$\text{or, } \frac{0.125}{0.5}+\frac{0.5-L_3}{0.8}+\frac{L_3}{0.1} = 1.143$$

$\therefore L_3 = 2.273$ cm

33. A pipe (k= 20 W/mK) with inner and outer diameter of 2 cm and 4 cm respectively is covered with 2 cm layer of insulation (k= 0.2 W/mK). If the inside and outside surface of the combination are at 500° C and 100°C respectively. Determine the heat loss from the unit length of the pipe. Also determine the pipe insulation interface temperature.

Solution:

Given, Inner radius of pipe (r_1) = 1 cm = 0.01 m

Outer radius of pipe (r_2) = 2 cm = 0.02 m

Outer radius of insulation (r_3) = 2 + 2 = 4 cm

= 0.04 m

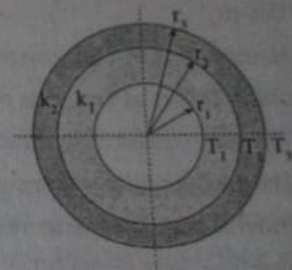
Thermal conductivity of pipe (k_1) = 20 W/mK

Thermal conductivity of layer of insulation

(k_2) = 0.2 W/mK

Inside surface temperature (T_1) = 500° C

Outside surface temperature (T_3) = 100° C



Rate of heat loss per the unit length of the pipe $\left(\frac{\dot{Q}}{L}\right)$ = ?

Interface temperature (T_2) =?

Rate of heat loss per unit length for the composite cylinder is given by

$$\frac{\dot{Q}}{L} = \frac{2\pi(T_1 - T_3)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1}+\frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2}} = \frac{2\pi(500 - 100)}{\frac{\ln\left(\frac{0.02}{0.01}\right)}{20}+\frac{\ln\left(\frac{0.04}{0.02}\right)}{0.2}}$$

Applying heat transfer equation for pipe only,

$$\frac{\dot{Q}}{L}=\frac{2\pi k_1(T_1-T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

$$\text{or, } 717.99=\frac{2\pi\times 20(500-T_2)}{\ln\left(\frac{0.02}{0.01}\right)}$$

$\therefore T_2=496.039^0C$

34. **A cast iron pipe (k=25 W/mK) with inner and outer diameters of 60 mm and 70 mm respectively is covered by an insulator (k=0.05 W/mK). Under steady state condition, temperature between the pipe and insulator interface is found to be250 ^{0} C. The allowable heat loss from the unit length of the pipe is 500 W and outer surface temperature of the insulator should not exceed50 ^{0} C. Determine:**

(a) **minimum thickness of the insulation required, and**

(b) **temperature at the inner surface of the pipe.**

Solution:

Given,

Inner radius of cast iron piper $(r_1)=30\text{ mm}=0.03\text{ m}$

Outer radius of cast iron pipe $(r_2)=35\text{ mm}=0.035\text{ m}$

Thermal conductivity of cast iron pipe $(k_1)=25\text{ W/mK}$

Thermal conductivity insulator $(k_2)=0.05\text{ W/mK}$

Outer surface temperature $(T_3)=50^0C$

Interface temperature $(T_2)=250^0C$

Heat loss per unit length $\left(\frac{\dot{Q}}{L}\right)=500\text{ W}$

Inner surface temperature $(T_1)=?$

Minimum thickness of insulation required $(x)=?$

Outer Radius of the insulation $(T_3)=r_2+x=0.035+x$

Applying heat transfer equation for pipe only,

$$\frac{\dot{Q}}{L}=\frac{2\pi k_1(T_1-T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

$$\text{or, } 500=\frac{2\pi\times 25\times(T_1-250)}{\ln\left(\frac{0.035}{0.03}\right)}$$

$\therefore T_1=250.491^0C$

Heat loss per unit length for a composite cylinder is given by

$$\frac{\dot{Q}}{L}=\frac{2\pi(T_1-T_3)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1}+\frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2}}$$

$$\text{or, } 500=\frac{2\pi(250.491-50)}{\frac{\ln\left(\frac{0.035}{0.030}\right)}{25}+\frac{\ln\left(\frac{r_3}{0.030}\right)}{0.05}}$$

$$\text{or, } 500\times\frac{\ln\left(\frac{r_3}{0.030}\right)}{0.05}=125664$$

$$\text{or } \ln\left(\frac{r_3}{0.03}\right)=0.1257$$

$\therefore r_3=34.017\text{ mm}$

Thus, minimum thickness of insulation, $x=r_3-r_2=34.017-30=4.017\text{ mm}$

35. **A 200 mm diameter 50 m long pipe carrying steam is covered with 40mm of high temperature insulation (k=0.1 W/m) and 30 mm of low temperature insulation (k= 0.05 W/m). The inner and outer surface of the insulating layer are at 400^{0}C and 40^{0}C respectively. Determine,**

(a) **The rate of heat loss from the pipe;**

(b) **The temperature at the interface of two insulating layer,**

(c) **The rate of heat transfer from unit area of the pipe surface, and**

(d) **The rate of heat transfer from unit area of the outer surface of the composite insulation**

Solution:

Given, Outer radius of pipe $(r_1)=100\text{ mm}=0.1\text{ m}$

Outer radius of high temperature insulation $(r_2)=100+40=140\text{ mm}=0.14\text{ m}$

Outer radius of low temperature insulation $(r_3)=140+30=170\text{ mm}=0.17\text{ m}$

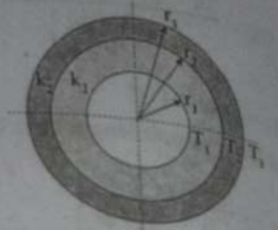
Inner surface temperature $(T_1)=400^0C$

Outer surface temperature $(T_3) = 40^\circ C$

Thermal conductivity of high temperature insulation $(k_1) = 0.1$ W/m

Thermal conductivity of low temperature insulation $(k_2) = 0.05$ W/ m

Rate of heat loss for the composite cylinder is then given by



$$\dot{Q} = \frac{2\pi L (T_1 - T_3)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2}} = \frac{2\pi \times 50\,(400 - 40)}{\frac{\ln\left(\frac{0.14}{0.1}\right)}{0.1} + \frac{\ln\left(\frac{0.17}{0.14}\right)}{0.05}}$$

$= 156042.756$ W

$$\frac{\dot{Q}}{L} = \frac{156042.758}{500} = 312.086 \text{ W/m}$$

Again, rate of heat loss between pipe and high temperature insulation is given by

$$\dot{Q} = \frac{2\pi k_1 L\,(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

$$\text{or, } 156042.785 = \frac{2\pi \times 0.1 \times 50\,(400 - T_2)}{\ln\left(\frac{0.14}{0.1}\right)}$$

$\therefore T_2 = 232.874^\circ C$

Again, rate of heat transfer from unit area of the pipe surface is given by

$$\frac{\dot{Q}}{A_{pipe}} = \frac{156042.758}{2 \times \pi \times 0.1 \times 50} = 496.7 \text{ W/m}^2$$

Also, rate of heat transfer from unit area of the outer surface of the composite insulation is given by

$$\frac{\dot{Q}}{A_{insulation}} = \frac{156042.758}{2\pi \times 0.17 \times 50} = 292.177 \text{ W/m}^2$$

36. A steel pipe (k=45.5 W/mK) with outer diameter of 90 mm and thickness 3 mm is used for flow of brine at -22° C. The pipe may be insulated by any one of the two types of insulation. Insulation I has k_1=0.037 W/mK and insulation II has k_2=0.047 W/mK. If one of these insulations has to be used for pipe insulation so that maximum heat transfer is to be limited to 11.6 W/m of pipe and the temperature of insulation at the outer surface could be maintained not less than 15°C, determine the required thickness of insulation for each case.

Solution:

Given, Inner radius of steel pipe $(r_1) = 45$ mm $= 0.045$ m

Outer radius of steel pipe $(r_2) = 45 + 3 = 48$ mm $= 0.048$ m

Thermal conductivity of Insulation I $(k_1) = 0.037$ W/mK

Thermal conductivity of Insulation II $(k_2) = 0.047$ W/ mK

Thermal conductivity of steel pipe (k) = 45.5 W/mK

Maximum heat transfer $\left(\frac{\dot{Q}}{L}\right) = 11.6$ W/m

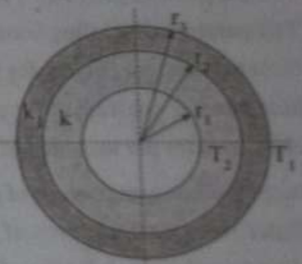
Temperature of insulation at the outer surface $(T_1) = 15^\circ C$

Inner surface temperature of pipe $(T_2) = -22^\circ C$

Let, outer radius of insulation be r_3.

Heat transfer per unit length for a composite cylinder is given by

$$\frac{\dot{Q}}{L} = \frac{2\pi (T_2 - T_1)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_1}}$$



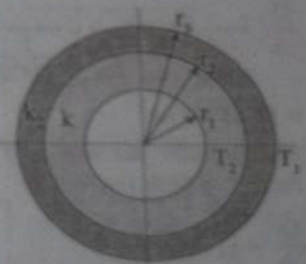
$$\text{or, } 11.6 = \frac{2\pi (15 + 22)}{\frac{\ln\left(\frac{0.048}{0.045}\right)}{45.5} + \frac{\ln\left(\frac{r_3}{0.048}\right)}{0.037}}$$

$\therefore r_3 = 100.75$ mm

Hence, Thickness of the insulation I = 100. 75 - 48

= 52.75 mm

Heat transfer per unit length when insulation II is used for composite cylinder is given by



$$\frac{\dot{Q}}{L} = \frac{2\pi (T_1 - T_2)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_1}}$$

or, $11.6 = \dfrac{2\pi(15+22)}{\dfrac{\ln\left(\frac{0.048}{0.045}\right)}{45.5} + \dfrac{\ln\left(\frac{r_3}{0.048}\right)}{0.047}}$

$\therefore r_3 = 123.11$ mm

Thus, thickness of the insulation II = 123.11 - 48 = 75.11 mm

37. A flat plate (k=100W/mK) of 5 mm thick is exposed to a gas at a temperature of 150° C on one side and cooling water at 20° C on the opposite side. The heat transfer coefficient for inside and outside surfaces are 4000 W/m²K respectively . Determine the heat transfer per unit area of the plate and the temperature at inner and outer surfaces of the plate.

Solution:

Given, Thickness of flat plate (L) = 5 mm = 0.05 m

Thermal conductivity of flat plate (k) = 100 W/mK

Temperature of gas (T_A) = 150° C

Temperature of cooling water (T_B) = 20° C

Heat transfer coefficient for inside surface (h_A) = 4000 W/m²K

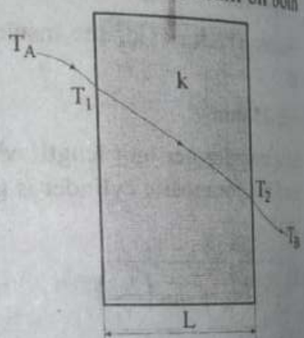
heat transfer coefficient for outside surface (h_B) = 2000 W/m²K

Heat transfer per unit area of plate ($\dot{q}$) = ?

Inner surface temperature of plate (T_1) = ?

Outer surface temperature of plate (T_2) = ?

Heat transfer per unit area of plane wall subjected to convective medium on both sides is then given by



$$\frac{\dot{Q}}{A} = \frac{T_A - T_B}{\frac{1}{h_A} + \frac{L}{k} + \frac{1}{h_B}} = \frac{150 - 20}{\frac{1}{4000} + \frac{0.005}{10} + \frac{1}{2000}}$$

= 162.5 kW/m²

Heat transfer per unit area from gas to inner surface of plate is given by

$$\frac{\dot{Q}}{A} = h_A (T_A - T_1)$$

or, $162.5 \times 10^3 = 4000\,(150 - T_1)$

$\therefore T_1 = 109.375°$ C

Also, heat transfer per unit area from outer surface of plate to cooling water is given by

$$\frac{\dot{Q}}{A} = h_B (T_2 - T_B)$$

$162.5 \times 10^3 = 2000\,(T_2 - 20)$

$\therefore T_2 = 101.125°$ C

38. A mild steel tank (k= 45W/mK) of wall thickness 15 mm contains water at 100° C. The heat transfer coefficients for the inside and outside surfaces of the tank wall are 2500 W/m²K and 20 W/m²K respectively. If the ambient air temperature is 20° C, determine:

(a) The rate of heat loss per unit area of wall .and

(b) The temperature at the inner and outer surface of the tank.

Solution:

Given, Thermal conductivity of mild steel tank (k) = 45 W/mK

Thickness of wall (L) = 15 mm = 0.015 m

Temperature of water (T_A) = 100° C

Heat transfer coefficient for the inside surface of the tank (h_A) = 2500 W/m²K

Heat transfer coefficient for the outside surface of the tank (h_B) = 20 W/m²K

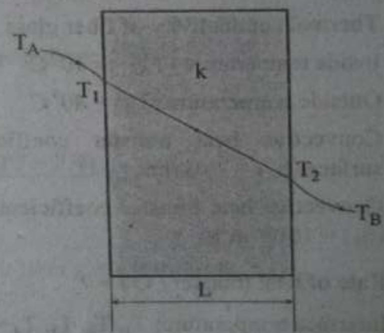
Ambient air temperature . (T_B) = 20° C

Rate of heat loss per unit area of the wall ($\dot{q}$) = ?

Inner surface temperature of the tank (T_1) = ?

Outer surface temperature of the tank (T_2) = ?

Heat transfer per unit area of plane wall subjected to convective medium on both sides is given by



$$\frac{\dot{Q}}{A} = \frac{T_A - T_B}{\frac{1}{h_A} + \frac{L}{k} + \frac{1}{h_B}}$$

$$= \frac{100 - 20}{\frac{1}{2500} + \frac{0.015}{45} + \frac{1}{20}} = \frac{80}{0.0507}$$

= 1577.91 W/m²

Heat transfer per unit area from water to inner surface of tank is given by

$$\frac{\dot{Q}}{A} = h_A (T_A - T_1)$$

or, $1577.91 = 2500\,(100 - T_1)$

$\therefore\ T_1 = 99.396^\circ C$

Again, heat transfer per unit area from outer surface of tank to ambient air is given by

$$\frac{\dot{Q}}{A} = h_B (T_2 - T_B)$$

or, $1577.97 = 20\,(T_2 - 20)$

$\therefore\ T_2 = 89.896^\circ C$

39. The inner dimension of the freezer compartment are 50cm*50cm*40cm. its wall consist of two 4 mm thick enameled steel sheet (k= 45 W/mK) separated by 5cm layer of fiber glass insulation (k= 0.05 W/mK). The inside temperature is maintained at -10° C and the outside temperature on a hot summer day is 40° C. Calculate the rate at which heat should be thrown out if convective heat transfer coefficients for inner and outer surface are 20 W/m²K and 10 W/m²K respectively. Also calculate interface temperatures.

Solution:

Given, Inner Dimensions of a freezer compartment = 50 cm × 50 cm × 40 cm

Thickness of enameled steel (L_1) = 4 mm = 0.004 m

Thickness of fiber glass insulation (L_2) = 5 cm = 0.05 m

Thickness of next enameled steel (L_3) = 4 mm = 0.004 m

Thermal conductivity of enameled steel (k_1) = 45 W/mK

Thermal conductivity of fiber glass insulation (k_2) = 0.05W/mK

Inside temperature (T_A) = - $10^\circ C$

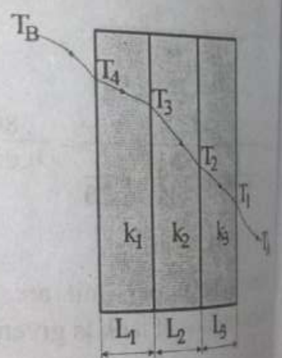
Outside temperature (T_B) = $40^\circ C$

Convective heat transfer coefficient for inner surface (h_A) = 20W/mK

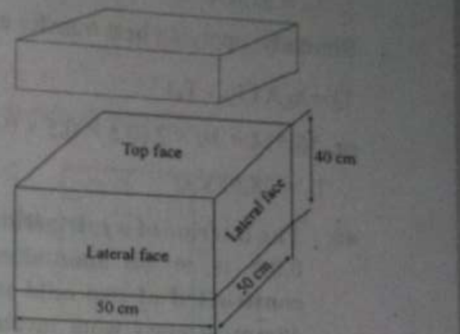
Convective heat transfer coefficient for out surface (h_B) = 10 W/m^2K

Rate of heat transfer ($\dot{Q}$) = ?

Interface temperature: T_1, T_2, T_3, T_4 = ?



Rate of heat transfer through top and bottom face of plane wall subjected to convective medium on both sides is given by



$$\dot{Q}_{tb} = 2 \times \frac{A\,(T_B - T_A)}{\frac{1}{h_A} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_1}{k_1} + \frac{1}{h_B}}$$

$$= 2 \times \frac{0.5 \times 0.5 \times (40 + 10)}{\frac{1}{20} + \frac{0.004}{45} + \frac{0.05}{0.05} + \frac{0.004}{45} + \frac{1}{10}}$$

$= 21.736$ W

Similarly, rate of heat transfer through faces of plane wall subjected to convective medium on both sides is given by

$$\dot{Q}_{lateral} = 4 \times \frac{(T_B - T_A) \times A}{\frac{1}{h_A} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_1}{k_1} + \frac{1}{h_B}}$$

$$= 4 \times \frac{0.5 \times 0.4 \times (40 + 10)}{\frac{1}{20} + \frac{0.004}{45} + \frac{0.05}{0.05} + \frac{0.004}{45} + \frac{1}{10}} = 34.778 \text{ W}$$

$\therefore$ Total rate of heat transfer, $\dot{Q} = \dot{Q}_{tb} + \dot{Q}_{lateral} = 21.736 + 34.778 = 56.514$ W

For interface temperatures:

Applying heat transfer equation for inner surface only,

$$\dot{Q} = A\,h_A\,(T_1 - T_A)$$

$$56.514 = 2 \times (0.5 \times 0.5 + 0.5 \times 0.4 + 0.5 \times 0.4)\,(T_1 + 10)$$

$\therefore\ T_1 = -7.826^\circ C$

Applying heat transfer equation for first enameled steel only

$$\dot{Q} = \frac{kA}{L_1}(T_2 - T_1)$$

or, $56.514 = \dfrac{45 \times 2\,(0.5 \times 0.5 + 0.5 \times 0.4 + 0.5 \times 0.4)}{0.004}\,(T_2 + 7.826)$

$\therefore\ T_2 = 7.822^\circ C$

Similarly, applying heat transfer equation for fiber glass insulation only,

$$\dot{Q} = \frac{K_2 A}{L_2}(T_3 - T_2)$$

or, $56.514 = \frac{0.05 \times 2 (0.5 \times 0.5 + 0.5 \times 0.4 \times 0.5 \times 0.4)}{0.05} (T_3 + 7.822)$

$\therefore T_3 = 35.650^0 C$

Similarly, applying heat transfer equation for outer surface only,

$\dot{Q} = h_B A (T_B - T_4)$

or, $56.514 = 10 \times 2 (0.5 \times 0.5 \times 0.5 \times 0.4 + 0.5 \times 0.4) (40 - T_4)$

$\therefore T_4 = 35.653^0 C$

40. The interior of a refrigerator having inside dimensions of 0.4m × 0.4m × 0.8m is to be maintained at 0° C. The wall of the refrigerator constructed of two mild steel sheet (k= 50 W/mK) 2.5 mm thick with 40mm of glass wool insulation (k= 0.05 W/mK) between them. If the heat transfer coefficient at the inner and outer surfaces are 10 W/m²K and 20 W/m²K respectively. Determine

(a) The rate of heat removal from the interior of the refrigerator when the kitchen temperature is 26° C,

(b) The temperature at both sides of the glass wool insulation

Solution:

Given, Thickness of mild steel: $L_1 = L_3 = 2.5$ mm $= 0.0025$ m

Thickness of glass wool insulation $(L_2) = 40$ mm $= 0.04$ m

Thermal conductivity of mild steel: $k_1 = k_3 = 50$ W/mK

Thermal conductivity of glass wool insulation $(k_2) = 0.05$ W/mK

Heat transfer coefficient at the inner surface $(h_A) = 10$ W/m^{2}K

heat transfer coefficient at outer surface $(h_B) = 20$ W/m^{2}K

Total area through which heat is coming into the refrigerator

$(A) = 2 (0.4 \times 0.4 + 2 \times 0.4 \times 0.8) = 1.6$ m^2

Inside temperature $(T_A) = 0^0 C$

Outside temperature $(T_B) = 26^0 C$

Rate of heat removal from the interior of the refrigerator is given by

$$\dot{Q} = \frac{A (T_B - T_A)}{\frac{1}{h_A} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_B}}$$

$$= \frac{1.6 \times (26 - 0)}{\frac{1}{10} + \frac{0.0025}{50} + \frac{0.04}{0.05} + \frac{0.0025}{50} + \frac{1}{20}} = 43.78 \text{ W}$$

Now, rate of heat transfer from outside to outer surface of glass wool insulation by convection is given by

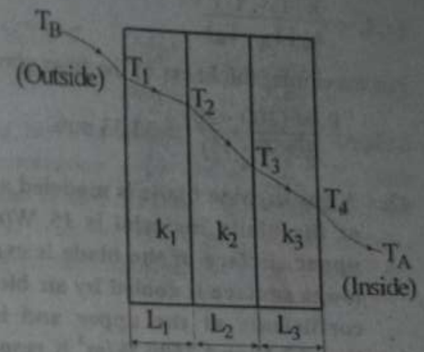
$\dot{Q} = h_B A (T_B - T_1)$

or, $43.78 = 20 \times 1.6 (26 - T_1)$

$\therefore T_1 = 24.632^0 C$

Rate of heat transfer through glass wool insulation only is given by

$\dot{Q} = \frac{k_1 A (T_1 - T_2)}{L_1}$



or, $43.78 = \frac{50 \times 1.6 \times (24.632 - T_2)}{0.0025}$

$\therefore T_2 = 24.631^0 C$

Similarly,

$\dot{Q} = h_A A (T_4 - T_A)$

or, $43.78 = 10 \times 1.6 (T_4 - 0)$

$\therefore T_4 = 2.736^0 C$

Similarly,

$\dot{Q} = \frac{k_3 A (T_3 - T_4)}{L_3}$

or, $43.78 = \frac{50 \times 1.6 (T_3 - 2.736)}{0.0025}$

$\therefore T_3 = 2.737^0 C$

41. The maximum operating temperature of the kitchen oven is set at 300° C. Due to seasonal variation, the kitchen temperature varies from 5° C to 35° C. If the average heat transfer coefficient between the oven outer surface and the kitchen air is 20 W/m²K, determine the required thickness of the fiber glass (k= 0.04 W/mK) insulation to ensure that the outside surface temperature of the oven does not exceed 50° C.

Solution:

Given, Temperature inside the oven $(T_1) = 300^0 C$

Temperature of the kitchen air $(T_B) = 5^0 C$ to $35^0 C$

Heat transfer coefficient between the oven outer surface and kitchen air $(h_B) =$ 20W/m^{2}K

Thermal conductivity of fiber glass $(k) = 0.04$ W/mK

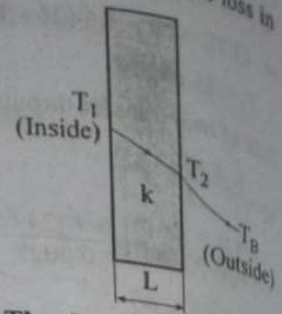
Outside surface temperature of oven $(T_2) = 50^0 C$

Thickness of fiber glass (L) = ?

Under steady state condition,

Rate of heat transfer through the fiber glass by conduction = Rate of heat loss in kitchen air by convection

$$\frac{kA\,(T_1 - T_2)}{L} = h_B A\,(T_2 - T_B)$$



$$\text{or, } L = \frac{k\,(T_1 - T_2)}{h_B\,(T_2 - T_B)}$$

For maximum thickness of the fiber glass, $T_B = 35^0$ C

$$\therefore L = \frac{0.04\,(300 - 50)}{20(50 - 35)} = 33.33 \text{ mm}$$

42. A gas turbine blade is modeled as a flat plate. The thermal conductivity of the blade material is 15 W/mK and its thickness is 1.5 mm. The upper surface of the blade is exposed to hot gases at 1000 ^{0} C and the lower surface is cooled by air bled of the compressor. The heat transfer coefficients at the upper and lower surfaces of the blade are 2500 W/m^2 K and 1500 W/m^2 K respectively. Under steady state conditions, the temperature, at the upper surface of the blade is measured as 850 ^{0} C, determine the temperature of the coolant air. ***(IOE 2069 Chaitra)***

Solution:

Given, Thermal conductivity of blade material (k) - 15 W/mk

Thickness of blade (L) = 1.5 mm = 1.5×10^{-3} m

Temperature of hot gas (T_A) = 100^0C

Temperature of upper surface of blade (T_1) = 850^0 C

Temperature of the coolant air (T_B) = ?

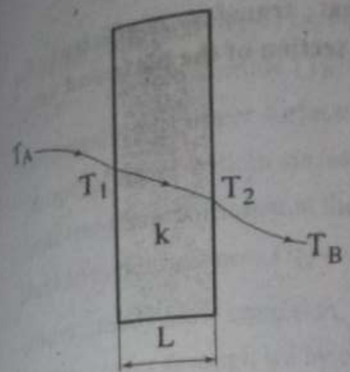
Heat transfer coefficient at the upper surface of blade (h_A) = 2500 W/m^{2}K

Heat transfer coefficient at the lower surface of blade (h_B) = 1500 W/m^{2}K

Heat transfer per unit area from hot gas to the upper surface of blade is given by

$$\frac{\dot{Q}}{A} = h_A\,(T_A - T_1) = 2500\,(100 - 850) = 375000 \text{ W}$$

- Also, heat transfer per unit area of wall subject to convective medium on both sides is given by



$$\frac{\dot{Q}}{A} = \frac{T_A - T_B}{\frac{1}{h_A} + \frac{L}{k} + \frac{1}{h_B}}$$

Under steady state condition,

$$\text{or, } 375000 = \frac{1000 - T_B}{\frac{1}{2500} + \frac{0.5 \times 10^{-3}}{15} + \frac{1}{1500}}$$

$$\therefore T_B = 562.5^0 \text{C}$$

43. The inside surface of an insulating layer is at 300 ^{0} C and the outside surface is dissipating heat by convection into air at 25 ^{0} C. The insulating layer has a thickness 5 cm of and thermal conductivity of 0.8 W/mK. What is the minimum heat transfer coefficient at the outside surface if the outside surface temperature should not exceed 100 ^{0} C? ***(IOE 2068 Bhadra)***

Solution:

Given, Inside surface temperature of insulating layer 21(T_1) = 300^0 C

Outside surface temperature of insulating layer (T_2) = 100^0 C

Ambient air temperature (T_B) = 25^0 C

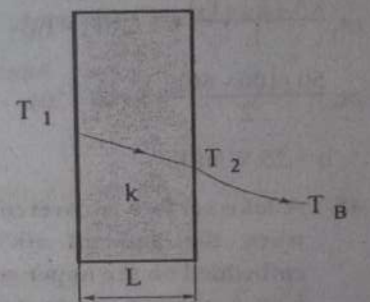
Thickness of insulating layer (L) = 5 cm = 0.05 m

Thermal conductivity of insulating layer (k) = 0.8 W/mK

Heat transfer coefficient at the outside surface (h) = ?

Under steady state condition,

Heat transfer through insulating layer by conduction = heat lost by convection in air



$$\text{or, } \frac{kA\,(T_1 - T_2)}{L} = hA\,(T_2 - T_B)$$

$$\text{or, } \frac{0.8\,(300 - 100)}{0.05} = h\,(100 - 25)$$

$$\therefore h = 42.667 \text{ W/m}^2\text{K}$$

44. A 2 m long steel pipe (k= 50 W/mK) is well insulated on its sides, while its left section is maintained at 100^0 C and the right section is exposed to ambient air at 20^0 C. Under steady state conditions, a thermocouple inserted at the middle of the plate gives a temperature of 80^0 C.

Determine the value of convection heat transfer coefficient for convection heat transfer between the right section of the plate and air.

Solution:

Given, Thickness of plate (L) = 2 m

Thermal conductivity of plate (k) = 50 W/mK

Temperature of left section of plate $(T_1) = 100^0 C$

Ambient air temperature $(T_B) = 20^0 C$

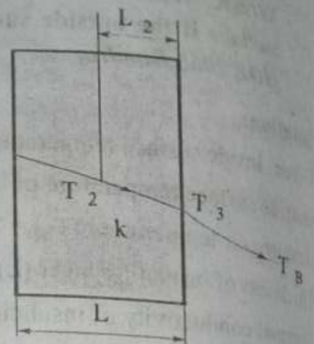
Temperature at the middle of the plate $(T_2) = 80^0 C$

Convection heat transfer coefficient (h) = ?

Under steady state condition,

Heat transfer through a plate between temperature T_1 and T_2 = Heat transfer through plate between temperature T_2 and T_3

or, $\frac{kA(T_1 - T_2)}{L_1} = \frac{kA(T_2 - T_3)}{L_2}$



At the middle of plate, $L_1 = L_2 = 1$ m

or, $\frac{100 - 80}{1} = \frac{80 - T_2}{1}$

$\therefore T_2 = 160 - 100 = 60^0 C$

Also, under steady state condition,

Heat transfer through a plate by conduction = heat transfer by convection between right section of plate and air

or, $\frac{kA(T_1 - T_2)}{L} = hA(T_3 - T_B)$

or, $\frac{50(100 - 60)}{2} = h(60 - 20)$

$\therefore h = 25 \text{ W/m}^2\text{K}$

45. A lake surface is covered by a 8 cm thick layer of ice (k=2.23 W/mK) when the ambient air temperature is -12.5^0 C. A thermocouple embedded on the upper surface of the layer indicates a temperature of -5^{0}C. Assuming steady state conduction in ice and no liquid subcooling at the bottom surface of the ice layer, find the heat transfer coefficient at the upper surface. Also work out the heat loss per unit area.

Solution:

Given, Thickness of ice (L) = 8 cm = 0.8 m

Thermal conductivity of layer of ice (k) = 2.23 W/mK

Ambient air temperature $(T_B) = -12.5^0 C$

Temperature of upper surface of the layer $(T_2) = -5^0 C$

Temperature of bottom surface of the layer $(T_1) = 0^0 C$

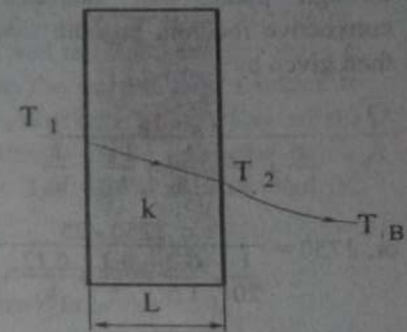
Heat transfer coefficient at the upper surface (h) = ?

Heat loss per unit area $(\dot{q})$ = ?

Under steady state condition,

Heat transfer through ice by conduction = heat loss in air by convection

or, $\frac{kA(T_1 - T_2)}{L} = hA(T_2 - T_B)$



or, $2.23(0 + 5) = h(-5 + 12.5)$

$\therefore h = 18.583 \text{ W/m}^2\text{K}$

Now, heat loss per unit area in air by convection is given by

$\dot{q} = \frac{\dot{Q}}{A} = h(T_2 - T_3) = 18.583 \times (-5 + 12.5)$

$= 139.375 \text{ w/m}^2$

46. A composite wall is made up of three layer of thickness 200 mm, 100 mm, 120 mm with thermal conductivities of 1.5 W/mK, 3 W/mK, k W/mK and respectively. The inside surface is exposed to hot gas at 1250^0 C and the outside surface is at 300^0 C which is exposed to ambient air at 250^0 C. The heat transfer coefficient for inside and outside surfaces are 20 W/m^{2}K and 10 W/m^{2}K respectively. Determine:

(a) The unknown thermal conductivity, and

(b) The interface temperatures.

Solution:

Given, Thickness of first layer $(L_1) = 200$ mm = 0.2 m

Thickness of second layer $(L_2) = 100$ mm = 0.1m

Thickness of third layer $(L_3) = 120$ mm = 0.12 m

Inside surface temperature of composite wall $(T_A) = 1250^0 C$

Ambient air temperature $(T_B) = 25^0 C$

Outside surface temperature of composite wall $(T_4) = 300$ C

Heat transfer coefficient for inside surface $(h_A) = 20 \text{ W/m}^2\text{K})$

Heat transfer coefficient for outside surface (h_B) = 10 W/m^{2}K

Thermal conductivity of first layer (k_1) = 1.5 W/mK

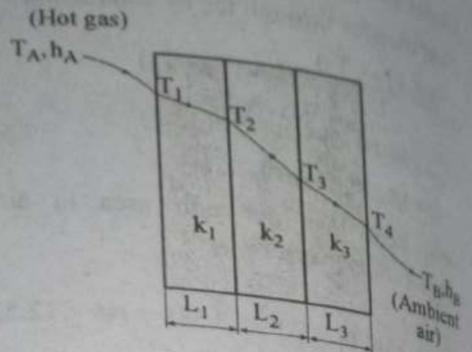
Thermal conductivity of second layer (k_2) = 3 W/mk

Thermal conductivity of third layer (k_3) = ?

Heat transfer per unit area in ambient air by convection is given by

$$\frac{\dot{Q}}{A} = h_B (T_4 - T_B) = 10 \times (300 - 25) = 2750 \text{ W/m}^2$$

Also, heat transfer per unit area through plane wall subjected to convective medium on both sides is then given by



$$\frac{\dot{Q}}{A} = \frac{T_A - T_B}{\frac{1}{h_A} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_B}}$$

$$\text{or, } 2750 = \frac{1250 - 25}{\frac{1}{20} + \frac{0.2}{1.5} + \frac{0.1}{3} + \frac{0.12}{k_3} + \frac{1}{10}}$$

$$\text{or} \left(0.31667 + \frac{0.12}{k_3}\right) \times 2750 = 1225$$

$$\therefore\ k_3 = 0.9318 \text{ W/m}^2\text{K}$$

Now, heat transfer per unit area from hot gas to outer surface of the composite cylinder is given as

$$\frac{\dot{Q}}{A} = h_A (T_A - T_1)$$

$$\text{or, } 2750 = 20\ (1250 - T_1)$$

$$\therefore\ T_1 = 1112.5^0\text{C}$$

Similarly, heat transfer per unit area through first layer of composite wall is given by

$$\frac{\dot{Q}}{A} = \frac{k_1 (T_1 - T_2)}{L_1}$$

$$\text{or, } 2750 = \frac{1.5\ (1112.5 - T_2)}{0.2}$$

$$\therefore\ T_2 = 745.833^0\text{C}$$

Similarly, heat transfer per unit area through the second layer of composite wall is given as

$$\frac{\dot{Q}}{A} = \frac{k_2 (T_2 - T_3)}{L_2}$$

$$\text{or, } 2750 = \frac{3\ (745.833 - T_3)}{0.1}$$

$$T_3 = 654.167^0\text{C}$$

7. In a coal-fired power plant, a furnace wall consists of a 125 mm wide refractory brick and a 125 mm wide insulating firebrick separated by an air gap. The outside wall is covered with 12mm thickness of plaster. The inner surface of the wall is at 1100° C, and the room temperature is at 10°C. The heat transfer coefficient from the outside wall surface to the air in the room is 17 W/m^{2}K, and the resistance to heat flow of the air gap is 0.16 K/W. The thermal conductivity of the refractor brick, insulating firebrick, and the plaster are 1.6, 0.3, and 0.14 W/mK, respectively. Calculate

(a) The rate of heat loss per unit area of wall surface.

(b) The temperature at each interface throughout the wall.

(c) The temperature at the outside surface of the wall.

Solution:

Given, Thickness of refractory brick (L_1) = 125 mm = 0.125 m

Thickness of insulating fire brick (L_2) = 125 mm = 0.125 m

Thickness of plaster (L_3) = 12 mm = 0.012 m

Heat transfer coefficient from the outside wall surface to the air (h_B) = 17 W/m^{2}K

Thermal conductivity of refractor brick (k_1) = 1.6 W/mK

Thermal conductivity of the insulating fire brick (k_2) = 0.3 W/mK

Thermal conductivity of the plaster (k_3) = 0.14 W/mK

Resistance to heat flow of the air gap (R_{th}) = $\frac{1}{h_A A}$ = 0.16 K/W

Inner surface temperature of the wall (T_1) = 1100^{0}C

Room temperature (T_B) = 10^{0}C

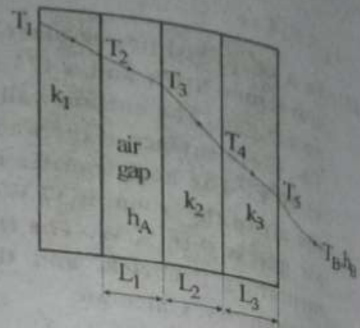
Rate of heat loss per unit area of wall surface is given by:

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_1 - T_B}{\frac{L_1}{k_1} + (R_{th})_{air\ gap} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_B}}$$

$$= \frac{1100 - 10}{\frac{0.125}{0.6} + 0.16 + \frac{0.125}{0.3} + \frac{0.012}{0.14} + \frac{1}{17}} = 1363.64\ W$$

Now, for interface temperatures:

Rate of heat transfer per unit area through the refractory brick only is given by



$$\dot{q} = \frac{\dot{Q}}{A} = \frac{k_1(T_1 - T_2)}{L_1}$$

$$\text{or, } 1363.64 = \frac{1.6 \times (1100 - T_2)}{0.125}$$

$$\therefore\ T_2 = 993.5^0 C$$

Similarly, rate of heat transfer per unit area through the air gap is given by

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{(T_2 - T_3)}{(R_{th})_{air\ gap}}$$

$$\text{or, } 1366.64 = \frac{993.5 - T_3}{0.16}$$

$$\therefore\ T_3 = 775.32^0 C$$

Similarly, rate of heat transfer per unit area through the insulating fire brick is given by

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{k_2(T_3 - T_4)}{L_2}$$

$$\text{or, } 1363.64 = \frac{0.3\ (775.53 - T_4)}{0.125}$$

$$\therefore\ T_4 = 207.35^0 C$$

Similarly, rate of heat transfer per unit area through plaster is given by

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{k_3(T_4 - T_5)}{L_2}$$

$$\text{or, } 1363.64 = \frac{0.14\ (207.35 - T_5)}{0.012}$$

$$\therefore\ T_5 = 90.45^0 C$$

6. **A square plate heater (10cm × 10 cm) is inserted between two slabs having the same cross-sectional areas. The left slab is 100mm thick (k= 50 W/mK) and the right slab is 50 mm thick (k= 0.25 W/mK). The heat transfer coefficients for left and right slab outer surface are 250W/m²K and 50 W/m²K respectively. The ambient air temperature is 25° C. If the rating of the heater is 1 kW, determine:**

(a) temperature at the heater surface, and

(b) outer surface temperatures of each slab.

Solution:

Given, Thickness of left slab (L_1) = 100 mm = 0.1 m

Thickness of right slab (L_2) = 50 mm = 0.05 m

Thermal conductivity of left slab (k_1) = 50 W/mK

Thermal conductivity of right slab (k_2) = 0.25 W/mK

Heat transfer coefficient for left outer surface (h_A) = 250 W/ m²K

Heat transfer coefficient for right outer surface (h_B) = 50 W/m²K

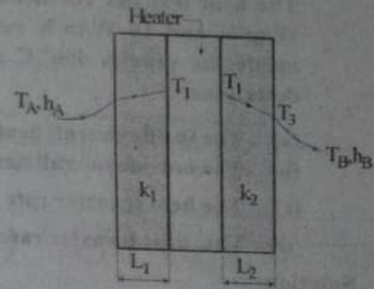
Ambient air temperature (T_B) = 25⁰ C

Rate of heat transfer ($\dot{Q}$) = 1 kW = 1000W

Area of plate (A) = 10 cm × 10 cm = 0.01 m²

For steady state condition,

Total heat transfer = Rate of heat flow through left slab + rate of heat flow through right slab.



$$\text{or, } \dot{Q} = \frac{(T_1 - T_A)A}{\frac{L_1}{k_1} + \frac{1}{h_A}} + \frac{(T_1 - T_A)A}{\frac{L_2}{k_2} + \frac{1}{h_B}}$$

$$\text{or, } 1000 = \frac{(T_1 - 25) \times 0.01}{\frac{0.1}{50} + \frac{1}{250}} + \frac{(T_1 - 25) \times 0.01}{\frac{0.05}{0.25} + \frac{1}{50}}$$

$$\text{or, } \frac{1000}{0.01} = (T_1 - 25)\left(\frac{1}{\frac{0.1}{50} + \frac{1}{250}} + \frac{1}{\frac{0.05}{0.25} + \frac{1}{5}}\right)$$

$$\therefore\ T_1 = 609.07^0 C$$

Now, rate of heat transfer through the left slab = rate of heat loss through convection

$$\dot{Q}_{left\ slab} = \frac{k_1 A (T_1 - T_2)}{L_1} = h_A A (T_2 - T_4)$$

or, $\frac{50\ (609.07 - T_2)}{0.1} = 250\ (T_2 - 25)$

or, $609.07 - T_2 = 0.5\ (T_2 - 25)$

or, $609.07 - T_2 = 0.5\ T_2 - 0.5 \times 25$

$\therefore\ T_2 = 414.38^0C$

Similarly, rate of heat transfer through the right slab = rate of heat loss through convection

$$\dot{Q}_{right\ slab} = \frac{k_2 A (T_1 - T_3)}{L_2} = h_3\ A\ (T_3 - T_4)$$

or, $\frac{0.25\ (609\ .\ 07 - T_3)}{0.05} = 50\ (T_3 - 25)$

or, $609.07 - T_3 = 10T_3 - 250$

$\therefore\ T_3 = 78.09^0C$

49. A 40 m long steel pipe (k= 50 W/mK) having an inside diameter 80mm and outside diameter 120 mm is covered with two layer of insulation. The layer in contact with the pipe is 30 mm thick asbestos (k= 0.15 W/mK) and the layer next to it is 20 mm thick magnesia (k= 0.1W/mK). The heat transfer coefficient for the inside and outside surfaces is 240 W/m²K and 10 W/m²K respectively. If the temperature of the steam inside the pipe is 400⁰ C and the ambient air temperature is 25⁰ C, determine:

(a) The inside overall heat transfer coefficient U_i,

(b) The outside overall heat transfer coefficient U_o,

(c) The heat transfer rate using U_i , and

(d) The heat transfer rate using U_o

Solution:

Given, Length of steel pipe (L) = 40 m

Thermal conductivity of steel pipe (k_1)= 50 W/mK

Inside radius of steel pipe (r_1) = 40 mm = 0.04 m

Outside radius of steel pipe (r_2) = 60 mm = 0.06 m

Outside radius of asbestos layer (r_3) = 60 + 30 = 90 mm = 0.09 m

Outside radius of magnesia layer (r_4) = 90 + 20 = 110 mm = 0.11 m

Thermal conductivity of asbestos layer (k_2) = 0.15 W/mK

Thermal conductivity of magnesia layer (k_3) = 0.1 W/mK

Heat transfer coefficient for the inside surface (h_A) = 240 W/m²K

Heat transfer coefficient for the outside surface (h_B) = 10 W/m²K

Temperature of the steam inside the pipe (T_A) = 400⁰ C

Ambient air temperature (T_B) = 25⁰ C

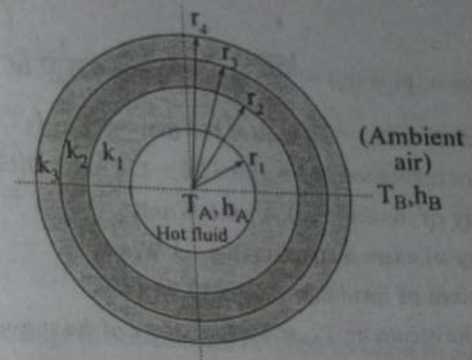
Inside overall heat transfer coefficient (U_i) = ?

Outside overall heat transfer coefficient (U_o) = ?

Inside curved surface area (A_1) = $2\pi r_1 L = 2\pi \times 0.04 \times 40$

= 10.053 m²

Outside curved surface area (A_2) = $2\pi r_4 L = 2\pi \times 0.11 \times 40 = 27.646$ m²



Inside overall heat transfer coefficient is given by

$$Ui = \frac{1}{\frac{1}{h_A} + \frac{A_1}{2\pi k_1 L}\ln\left(\frac{r_2}{r_1}\right) + \frac{A_1}{2\pi k_2 L}\ln\left(\frac{r_3}{r_2}\right) + \frac{A_1}{2\pi k_3 L}\ln\left(\frac{r_4}{r_3}\right) + \frac{A_1}{A_2}\frac{1}{h_B}} =$$

$$\frac{1}{\frac{1}{240} + \frac{2\pi\times0.04\times L}{2\pi\times50\times L}\ln\left(\frac{0.06}{0.04}\right) + \frac{2\pi\times0.04\times L}{2\pi\times0.15\times L}\ln\left(\frac{0.09}{0.06}\right) + \frac{2\pi\times0.04\times L}{2\pi\times0.1\times L}\ln\left(\frac{0.11}{0.09}\right) + \frac{2\pi\times0.04\times L}{2\pi\times0.11\times L}\times\frac{1}{10}}$$

$= 4.3621 W/m^2K$

Now, Heat transfer rate using U_i is given by

$\dot{Q} = Ui\ A\ (T_A - T_B)$

$= 4.3621 \times 10.053 \times (400 - 25)$

$= 16444.58$ W

Since, $U_i A_i (T_A - T_B) = U_o A_o (T_A - T_B)$

$\therefore$ Outside overall heat transfer coefficient is given by

$$U_o = \frac{U_i A_i (T_A - T_B)}{A_o (T_A - T_B)} = \frac{1.5862 \times 10.053}{27.646} = 1.5862 \text{ W/m}^2\text{K}$$

Heat transfer rate using U_o is given by

$$Q = U_o A_2 (T_A - T_B) = 1.5662 \times 27.66 \times (400 - 25)$$

$$= 16444.53 \text{ W}$$

50. A 140 mm diameter pipe carrying steam is covered by a layer of insulation (k= 0.5 W/mK) of 30mm thick. Later, an extra layer of another insulation (k= 1 W/mK) having a thickness 20mm is added. If the surrounding temperature remains constant and heat transfer coefficient for both insulating layer is 10 W/m²K, determine the percentage change in heat transfer rate due to extra insulation.

Solution:

Given, Outer Radius of pipe $(r_1) = \frac{140}{2}$ mm = 70 mm = 0.07 m

Outer radius of insulation (r_2) = 70 + 30 = 100 mm = 0.1 m

Outer radius of extra insulation (r_3) = 100 + 20 = 120 mm = 0.12 m

Thermal conductivity of insulation (k_1) = 0.5 W/mK

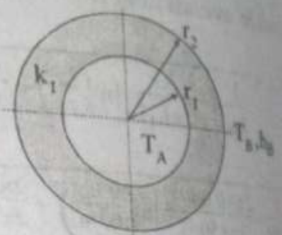
Thermal conductivity of extra insulation (k_2) = 1 W/mK

Heat transfer coefficient of insulation $(h_B) = 10 \text{ W/m}^2\text{K}$

Let, temperature of the steam be T_A, and temperature of the surrounding be T_B.

Case I : Without extra layer of insulation,

Heat transfer rate through pipe is given by



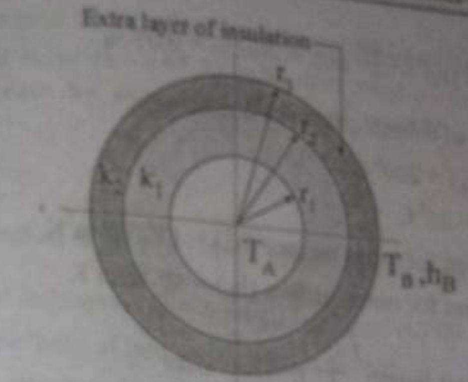
$$\dot{Q}_1 = \frac{2\pi L(T_A - T_B)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{1}{h_B r_2}} = \frac{2\pi L (T_A - T_B)}{\frac{\ln\left(\frac{0.10}{0.07}\right)}{0.5} + \frac{1}{10 \times 0.1}}$$

$$= \frac{2\pi L (T_A - T_B)}{1.71335}$$

Case II: With extra layer of insulation

Heat transfer coefficient of extra layer of insulation $(h_B) = 10 \text{ W/m}^2\text{K}$

Heat transfer rate through pipe is given by



$$\dot{Q}_2 = \frac{2\pi L(T_A - T_B)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{1}{h_B r_3}}$$

$$= \frac{2\pi L (T_A - T_B)}{\frac{\ln\left(\frac{0.1}{0.07}\right)}{0.5} + \frac{\ln\left(\frac{0.12}{0.1}\right)}{1} + \frac{1}{10 \times 0.12}}$$

$$= \frac{2\pi L (T_A - T_B)}{1.729}$$

Now, the percentage change in heat transfer rate due to extra insulation is

$$\frac{\dot{Q}_1 - \dot{Q}_2}{\dot{Q}_1} = 1 - \frac{\dot{Q}_2}{\dot{Q}_1} = 1 - \frac{1.729}{1.71335} = 1 - \frac{1.72335}{1.729}$$

$$= 0.00905 \times 100\% = 0.905\%$$

51. A steam pipe (k= 45 W/mK) has inside diameter of 100 mm and outside diameter of 140mm. it is insulated at the outside with asbestos (k= 1 W/mK). The steam temperature is 200° C and the air temperature is 25° C. The heat transfer coefficient for inner and outer surface are 120 W/m²K and 40 W/m²K respectively. Determine the required thickness of the asbestos in order to limit the heat losses to 1250 W/m.

Solution:

Given, Thermal conductivity of pipe (k_1) = 45 W/mk

Inside radius of pipe $(r_1) = \frac{100}{2}$ mm = 50 mm = 0.05 m

Outside radius of pipe (r_2) = $\frac{140}{2}$ mm = 70mm = 0.07 m

Thermal conductivity of asbestos (k_2) = 1 W/mK

Steam temperature (T_A) = 200°C

Air temperature (T_B) = 25°C

Heat transfer coefficient for inner surface (h_A) = 120 W/ m²K

Heat transfer coefficient for outer surface (h_B) = 40 W/m²K

Rate of heat loss per unit length $\left(\frac{\dot{Q}}{L}\right)$ = 1250 W/m

Let x be the thickness of the asbestos,

Outer radius of asbestos (r_3) = $r_2 + x = 0.07 + x$

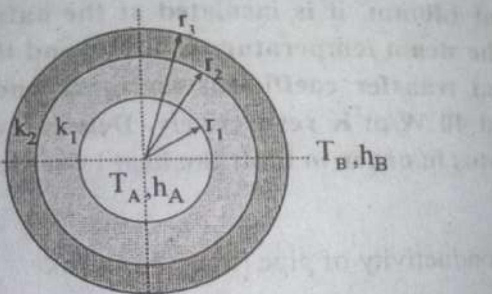
Rate of heat loss per unit length for the composite cylinder is given by

$$\dot{Q} = \frac{2\pi L\,(T_A - T_B)}{\frac{1}{h_A r_1} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{1}{h_B r_3}}$$

$$\frac{\dot{Q}}{L} = 1250 = \frac{2\pi\,(200 - 25)}{\frac{1}{120 \times 0.05} + \frac{\ln\left(\frac{0.07}{0.05}\right)}{45} + \frac{\ln\left(\frac{0.07 + x}{0.07}\right)}{1} + \frac{1}{40 \times (0.07 + x)}}$$

$$\text{or,}\ 1250 = \frac{2\pi \times 175}{\frac{1}{6} + \frac{\ln\left(\frac{0.07}{0.05}\right)}{45} + \frac{\ln\left(\frac{0.07 + x}{0.07}\right)}{1} + \frac{1}{40 \times (0.07 + x)}}$$

$\therefore$ x = 43.78 mm



52. A 100 mm diameter pipe carrying steam is covered by a layer of insulation (k= 0.05 W/mK) having a thickness of 40 mm. The heat transfer coefficient between the outer surface of insulation and the ambient air is 20 W/m²K. Determine the required thickness of another insulating layer (k= 0.08 W/mK) that must be added to reduce the heat transfer rate by 40% assuming heat transfer coefficient remains the same.

Solution:

Given, Outer radius of pipe (r_1) = $\frac{100}{2}$ mm = 50 mm = 0.05 m

Outer radius of insulation (r_2) = 50 + 40 = 90 mm = 0.09 m

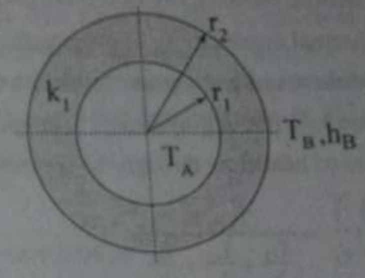
Thermal conductivity of insulation (k_1) = 0.05 W/mK

Thermal conductivity of another insulating layer (k_2) = 0.08 W/mK

Heat transfer coefficient between outer surface of insulation and ambient air (h_B) = 20 W/m²K

Case I: Without another layer of insulation

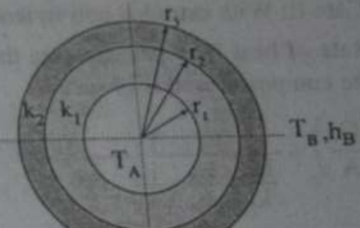
Heat transfer rate through the composite pipe is given by



$$\dot{Q}_1 = \frac{2\pi L\,(T_A - T_B)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{1}{h_B . r_2}}$$

Case II: with another layer of insulation

Heat transfer rate through the composite pipe is given by



$$\dot{Q}_2 = \frac{2\pi L\,(T_A - T_B)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{1}{h_B r_3}}$$

According to questions

$\dot{Q}_2$ = 60 % of $\dot{Q}_1$

$$\frac{2\pi L\,(T_A - T_B)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{1}{h_B r_3}} = 0.6 \times \frac{2\pi L\,(T_A - T_B)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{1}{h_B . r_2}}$$

$$\frac{\ln\left(\frac{0.09}{0.05}\right)}{0.05} + \frac{1}{20 \times 0.09} = 0.6 \times \left(\frac{\ln\left(\frac{0.09}{0.05}\right)}{0.05} + \frac{\ln\left(\frac{0.09 + x}{0.09}\right)}{0.08} + \frac{1}{20(0.09 + x)}\right)$$

$\therefore$ x = 87.38 mm

7.2 IOE Solutions

1. An exterior wall of a residential building of 25 cm thick brick [k = 0.7 W/m°C] on both sides. What thickness of extruded polystyrene insulation [k = 0.035 W/m° C] should be added to reduce the heat loss (or gain) through the wall by 55 percent? *(IOE 2070 Magh)*

Solution:

Given, Thickness of brick (L_2) = 25 cm = 0.25 m

Thermal conductivity of brick (k_2) = 0.7 W/m°C

Thickness of cement plaster on one side (L_1) = 2 cm = 0.02 m

Thickness of cement plaster on other side of brick (L_3) = 2 cm = 0.02 m

Thermal conductivity of cement plaster: $k_1 = k_3 = 0.48$ W/m°C

Thermal conductivity of polystyrene insulation (k_4) = 0.035 W/m°C

Thickness of polystyrene insulation (L_4) = ?

Case I: Without polystyrene insulation,

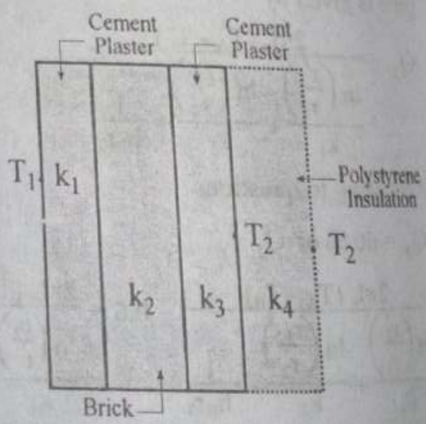
Rate of heat flow through the composite wall per unit are is given by

$$\left(\frac{\dot{Q}}{A}\right)_1 = \frac{T_1 - T_2}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}}$$

Case II: With extruded polystyrene insulation,

Rate of heat flow per unit area through the composite wall is given by

$$\left(\frac{\dot{Q}}{A}\right)_2 = \frac{T_1 - T_2}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{L_4}{k_4}}$$



According to the question,

$$\left(\frac{\dot{Q}}{A}\right)_2 = 45\% \text{ of} \left(\frac{\dot{Q}}{A}\right)_1$$

or, $$\frac{(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{L_4}{k_4}} = 0.45 \times \frac{(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}}$$

or, $$\frac{k_1}{L_1} + \frac{k_2}{L_2} + \frac{k_3}{L_3} = 0.45\left(\frac{k_1}{L_1} + \frac{k_2}{L_2} + \frac{k_3}{L_3} + \frac{k_4}{L_4}\right)$$

or, $$\frac{0.48}{0.02} + \frac{0.7}{0.25} + \frac{0.48}{0.02} = 0.45\left(\frac{0.48}{0.02} + \frac{0.7}{0.25} + \frac{0.48}{0.02} + \frac{0.035}{L_4}\right)$$

or, $$\frac{0.035}{L_4} = 62.089$$

$\therefore L_4 = 0.5637$ mm

2. A steam main of 8 cm inside diameter and 9.5 mm outside diameter is lagged with two successive layers of insulation. The layer in contact with pipe is 3.75 cm asbestos with thermal conductivity 0.11 W/mK and the asbestos layer is covered with 1.5 cm thick magnesia insulation with thermal conductivity of 0.067 W/mK. The inside film heat transfer co-efficient is 290 W/m²K and the outside film heat transfer co-efficient is 7.0 W/m²K. Conductivity of pipe material is 45 W/mK. Calculate the inside and outside overall heat transfer of co-efficient for 50 m length if the steam is passing at 350° C and the ambient temperature is 30° C. *(IOE 2070 Bhadra)*

Solution:

Given, Inside radius of pipe (r_1) = $\frac{8}{2}$ cm = 4 cm = 0.04 m

Outside radius of pip (r_2) = $\frac{9.5}{2}$ = 4.75 cm = 0.0475 m

Outside radius of magnesia insulation (r_4) = 8.5 + 1.5 = 10 cm = 0.1 m

Thermal conductivity of asbestos layer (k_2) = 0.11 W/mK

Thermal conductivity of magnesia (k_3) = 0.067 W/mK

Thermal conductivity of pipe material (k_1) = 45 W/mK

Heat transfer coefficient for the inside film (h_A) = 290 W/m²K

Heat transfer coefficient for outside film (h_B) = 7.00 W/m²K

Temperature of the steam inside the pipe (T_A) = 400°C

Ambient air temperature (T_B) = 30°C

Length of the pipe (L) = 50 m

Inside overall heat transfer coefficient (U_i) = ?

Outside overall heat transfer coefficient (U_o) = ?

Inside curved surface are (A_i) = $2\pi r_1 L = 2\pi \times (0.04) \times 50$

Outside curved surface area (A_o) = $2\pi r_4 L = 2\pi \times (0.1) \times 50$

Inside overall heat transfer coefficient is given by

Ui s

$$= \frac{1}{\frac{1}{h_A} + \frac{Ai}{2\pi k_1 L}\ln\left(\frac{r_2}{r_1}\right) + \frac{Ai}{2\pi k_2 L}\ln\left(\frac{r_3}{r_2}\right) + \frac{Ai}{2\pi k_3 L}\ln\left(\frac{r_4}{r_3}\right) + \frac{Ai}{A_o}\frac{1}{h_B}} =$$

$$\frac{1}{\frac{1}{290} + \frac{2\pi\times0.04\times L}{2\pi\times45\times L}\ln\left(\frac{0.0475}{0.04}\right) + \frac{2\pi\times0.04\times L}{2\pi\times0.11\times L}\ln\left(\frac{0.085}{0.0475}\right) + \frac{2\pi\times0.04\times L}{2\pi\times0.067\times L}\ln\left(\frac{0.1}{0.085}\right) + \frac{2\pi\times0.04\times L}{2\pi\times0.1\times L}\times\frac{1}{7}}$$

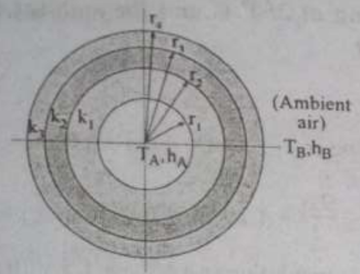
$= 2.70725 W/m^2 K$

Heat transfer rate through the composite cylinder is given by

$$\dot{Q} = U_o A_o (T_A - T_B) = U_i A_i (T_A - T_B)$$

∴ Outside overall heat transfer coefficient is given by

$$U_o = \frac{U_i A_i (T_A - T_B)}{A_o (T_A - T_B)} = \frac{2.70725 \times \pi \times 0.04 \times 50}{2\pi \times 0.1 \times 50} = 1.0829 \text{ W/m}^2\text{K}$$



3. **A brick work of a furnace is built up of layers laid of fire clay [$k_1 = 0.93$ W/m° C] and red brick [$k_3 = 0.7$ W/m° C] and the space between the two is filled with crushed diatomite brick [$k_2 = 0.13$ W/m° C]. The thickness of fire clay, diatomite filling and red brick are 12 cm, 5 cm and 25 cm respectively. What should be thickness of the red brick layer if the brick-work is to be laid without diatomite filling between the two layers, so that the heat flux through the brick-work remains constant? *(IOE 2069 Poush)***

Solution:

Given, Thickness of fire clay (L_1) = 12 cm = 0.12 m

Thermal conductivity of fire clay (k_1) = 0.92 W/m°C

Thickness of diatomite brick (L_2) = 5 cm = 0.05 m

Thermal conductivity of diatomite brick (k_2) = 0.13 W/m°C

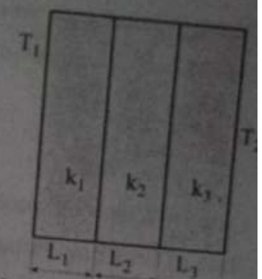
Thickness of red brick (L_3) = 25 cm = 0.25 m

Thermal conductivity of red brick (k_3) = 0.7 W/m°C

Case I: With diatomite filling.

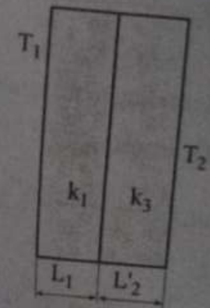
Rate of heat flux through the composite wall is given by

$$\dot{q}_1 = \frac{T_1 - T_2}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}}$$



Case II: Without diatomite filling Rate of heat flux through the composite wall is given by

$$\dot{q}_2 = \frac{T_1 - T_2}{\frac{L_1}{k_1} + \frac{L'_2}{k_3}}$$



According to the question, heat flux through the brick work remains constant i.e.

$\dot{q}_1 = \dot{q}_2$

or, $$\frac{(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}} = \frac{(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L'_2}{k_3}}$$

or, $$\frac{L_1}{k_1} + \frac{L'_2}{k_3} = \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}$$

$$\therefore L'_2 = \left(\frac{L_2}{k_2} + \frac{L_3}{k_3}\right)k_3 = \left(\frac{0.05}{0.13} + \frac{0.25}{0.7}\right) \times 0.7 = 0.5192 \text{ m} = 51.91 \text{ cm}$$

4. **An exterior wall of a house may be approximated by a 10 cm layer of common brick [k = 0.7 W/m° C] followed by a layer of a 3.8 cm layer of cement plaster [k = 0.48 W/m° C]. What thickness of loosely packed rock-wool insulation [k = 0.065 W/m° C] should be added to reduce heat loss (or gain) through the wall by 80%? *(IOE 2069 Ashad, IOE 2068 Baishakh)***

Solution:

Thickness of common brick (L_1) = 10 cm = 0.1 m

Thermal conductivity of common brick (K_1) = 0.7 W/m°C

Thickness of cement plaster (L_2) = 3.8 cm = 0.038 m

Thermal conductivity of gypsum plaster (k_2) = 0.48W/m°C

Thermal conductivity of rock wool insulation (k_3) = 0.065 W/m°C

Thickness of rock wool insulation (L_3) = ?

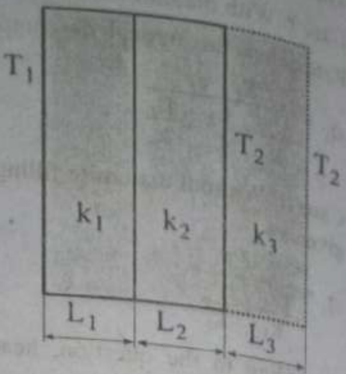
Case I: Without rock wool insulation

Rate of heat flow through the composite wall is given by

$$\dot{Q}_1 = \frac{A(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

Case II: With rock wool insulation

Rate of heat flow through the composite wall is given by



$$\dot{Q}_2 = \frac{A(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}}$$

According to the question,

$$\dot{Q}_2 = 20\% \text{ of } \dot{Q}_1$$

$$\text{or, } \frac{A(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}} = 0.2 \times \frac{A(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

$$\text{or, } \frac{L_1}{k_1} + \frac{L_2}{k_2} = 0.2\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)$$

$$\text{or, } \frac{0.1}{0.7} + \frac{0.038}{0.48} = 0.2\left(\frac{0.1}{0.7} + \frac{0.038}{0.48} + \frac{L_3}{6.065}\right)$$

$$\therefore L_3 = 0.8881 \times 0.065 = 0.5773 \text{ m} = 5.773 \text{ cm}$$

5. **A 3 cm thick 50 cm × 75 cm plate (k = 50W/mK) has inner surface temperature of 310^0 C. Heat is lost from the plate surface by convection and radiation to ambient air at 20^0 C. If the emissivity of the surface is 0.85 and convection heat transfer coefficient is 20 W/m^2 K, determine outer surface temperature of the plate.** *(IOE 2068 Chaitra)*

Solution:

Thickness of plate (L) = 2 cm = 0.02 m

Area of plate (A) = 50 cm × 75 cm = 0.375 m^2

Thermal conductivity of plate (k) = 50 W/mK

Inner surface temperature of plate (T_1) = 310°C = 310 + 273 = 583 K

Ambient air temperature (T_B) = 20°C = 20 + 273 = 293 K

Convection heat transfer coefficient (h) = 20 W/m^{2}K

Emissivity of the surface ($\in$) = 0.85

Outer surface temperature of plate (T_2) = ?

Rate of heat flow by conduction = Rate of heat dissipated by convection and radiation i.e.,

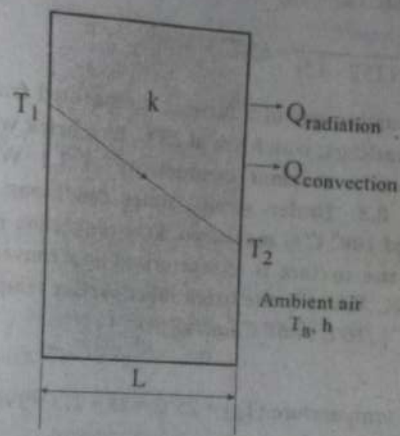
$$\dot{Q}_{conduction} = \dot{Q}_{convection} + \dot{Q}_{radiation}$$

$$\text{or, } \frac{kA(T_1 - T_2)}{L} = hA(T_2 - T_B) + \sigma \epsilon A(T_2^4 - T_B^4)$$

$$\text{or, } \frac{50 \times (583 - T_2)}{0.02} = 20(T_2 - 293) + 5.67 \times 10^{-8} \times 0.85 \times (T_2^4 - 293^4)$$

Solving for T_2, we get

T_2 = 587.695 K



6. **A 2 m long, 0.3 cm diameter electrical wire extends across a room at 15^0 C. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152^0 C in the steady operation. Also, the voltage drop and electric current through the wire are measured to be 60 V and 1.5 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.** *(IOE 2067 Ashad)*

Solution:

Given, Length of the wire (L) = 2 m

Diameter of wire (D) = 0.3 cm = 0.3 × 10^{-2} m

Room temperature (T_B) = 15°C

Surface temperature of the wire (T_1) = 152° C

Voltage drop (V) = 50 V

Electric current through the wire (I) = 1.5 A

Convection heat transfer coefficient (h) = ?

Electric power developed in the wire = Rate of heat loss from the outer surface of wire to the air in the room i.e.,

$$\dot{Q} = VI = hA\,(T_2 - T_B)$$

∴ Convection heat transfer coefficient is given by

$$h = \frac{VI}{A\,(T_1 - T_B)} = \frac{VI}{\pi DL\,(T_1 - T_B)}$$

$$= \frac{60 \times 1.5}{\pi \times 0.3 \times 10^{-2} \times 2\,(152 - 15)} = 69.703\ W/m^2K$$

7. **The hot combustion gas of a furnace is separated from the ambient air and its surroundings, which are at 25 C, by a brick wall of 0.15 m thick. The brick has a thermal conductivity of 1.2 W/mK and surface emissivity of 0.8. Under steady states conditions an outer surface temperature of 100^0 C is measured. Free convection heat transfer to the air adjoining the surface is characterized by a convention coefficient of $h = 20\ W/m^2 K$. What is the brick inner surface temperature? [$\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$]** ***(IOE 2067 Chaitra)***

Solution:

Given, Ambient air temperature (T_B) = 25°C = 25 + 273 = 298 K

Thickness of brick wall (L) = 0.15 m

Thermal conductivity of brick wall (k) = 1.2 W/mK

Outer surface temperature of brick wall (T_2) = 100°C = 100 + 273 = 373 K

Surface emissivity ($\in$) = 0.8

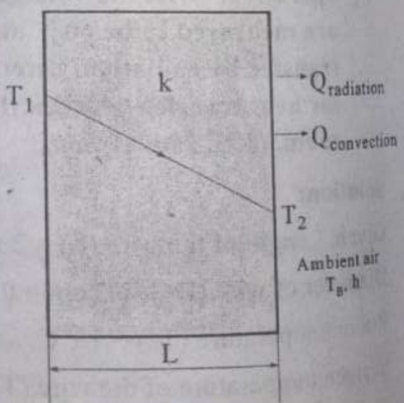
Convection coefficient (h) = 20w/m^{2}K

$\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$

Inner surface temperature of brick (T_1) = ?

Under steady state condition,

Rate of heat transfer by conduction = Rate of heat dissipated by convection and radiation i.e.,



$$\dot{Q}_{conduction} = \dot{Q}_{convection} + \dot{Q}_{radiation}$$

or, $$\frac{kA\,(T_1 - T_2)}{L} = hA\,(T_2 - T_B) + \in\sigma A\,(T_2^4 - T_B^4)$$

or. $$\frac{1.2\,(T_1 - 373)}{0.15} = 20\,(100 - 25) + 0.8 \times 5.67 \times 10^{-8} \times (373^4 - 298^4)$$

∴ $T_1 = 252.539 + 373 = 625.539\ K = 352.539°C$

8. **A thick walled tube of stainless steel [$k = 19\ W/m^0\ C$] with 2 cm inside diameter and 1 cm thickness is covered with a 3 cm layer of asbestos insulation [$k = 0.2\ W/m^0\ C$]. If the inside wall temperature of the pipe is maintained at 600^0 C and outside wall temperature of the insulation is maintained at 100^0 C, calculate the heat loss per meter of length. Also, calculate the tube-insulation interface temperature.** ***(IOE 2067 Mangsir)***

Solution:

Given, Inside radius of tube (r_1) = $\frac{2}{2}$ cm = 1cm = 0.01 m

Outside radius of tube (r_2) = 1 + 1 = 2 cm = 0.02 m

Outside radius of layer of asbestos (r_3) = 2 + 3 = 5 cm = 0.05 m

Thermal conductivity of stainless steel (k_1) = 19 W/m°C

Thermal conductivity of layer of asbestos (k_2) = 0.2 W/m°C

Inside wall temperature (T_1) = 600°C

Outside wall temperature (T_3) = 100°C

Tube insulation interface temperature (T_2) = ?

Rate of heat loss per meter of length $\left(\frac{\dot{Q}}{L}\right)$ = ?

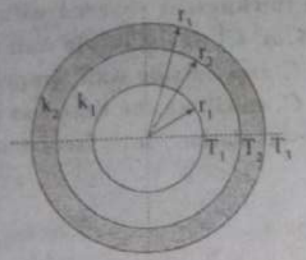
Rate of heat loss per unit of length through the composite wall is given by

$$\frac{\dot{Q}}{L} = \frac{2\pi\,(T_1 - T_3)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2}} = \frac{2\pi\,(600 - 100)}{\frac{\ln\left(\frac{0.02}{0.01}\right)}{19} + \frac{\ln\left(\frac{0.05}{0.02}\right)}{0.2}} = 680.302\ W/m$$

Applying heat transfer equation for tube only, $\frac{\dot{Q}}{L} = \frac{2\pi\,k_1\,(T_1 - T_2)}{\ln\left(\frac{r_2}{r_1}\right)}$

$\therefore$ Outside wall temperature, $T_2 = T_1 - \dfrac{\left(\dfrac{\dot{Q}}{L}\right)\times \ln\left(\dfrac{r_2}{r_1}\right)}{2\pi k_1}$

$$= 600 - \frac{680.302 \times \ln\left(\frac{0.02}{0.01}\right)}{2\pi \times 19} = 596.05^{\circ}C$$



9. **The inside surface of an insulating layer is at 270° C, and the outside surface is dissipating heat by convention into air at 20° C. The insulation layer is 4 cm thick and has thermal conductivity of 1.2 W/mK. What is the minimum value of the heat transfer coefficient at the outside surface if the outside temperature is not to exceed 70° C?** ***(IOE 2067 Mangsir)***

Solution:

Given, Inside surface temperature of insulating layer $(T_1) = 270^{\circ}C$

Outside surface temperature of insulating layer $(T_2) = 70^{\circ}C$

Ambient air temperature $(T_B) = 20^{\circ}C$

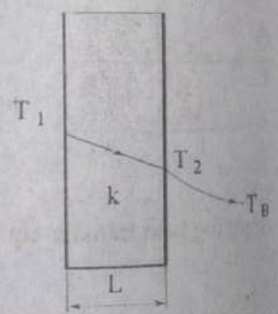
Thickness of insulating layer $(L) = 4cm = 0.04$ m

Thermal conductivity of insulating layer $(k) = 1.2$ W/mK

Heat transfer coefficient $(h) = ?$

Under steady state condition,

Heat transfer through insulating layer by conduction = heat lost by convection in air



$$\dot{Q}_{conduction} = \dot{Q}_{convection}$$

or, $\dfrac{kA(T_1 - T_2)}{L} = hA(T_2 - T_B)$

or, $\dfrac{1.2(270-70)}{0.04} = h(70-20)$

$\therefore h = 120\ W/m^2K$

7.3 Some Important Extra Questions

1. **A furnace wall is made of 20 cm of magnesite brick and 20 cm of common brick. The magnesite brick is exposed to hot gases at 1200° C and common brick outer surface is exposed to 35° C room air. The surface heat transfer coefficient of the inside wall is 40 W/m²K and that of the outside wall is 20 W/m²K respectively. Thermal conductivities of magnesite and common brick are 4 and 0.5 W/mK respectively. Determine:**

   (a) heat loss per m² area of the furnace wall and

   (b) maximum temperature to which common brick is subjected

Solution:

Given, Thickness of magnesite brick $(L_1) = 20$ cm $= 0.2$ m

Thickness of common brick $(L_2) = 20$ cm $= 0.2$ m

Thermal conductivity of magnesite brick $(k_1) = 4$ W/mK

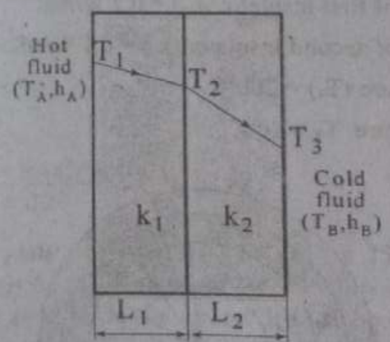
Thermal conductivity of common brick $(k_2) = 0.5$ W/mK

Heat transfer coefficient of the inner surface $(h_A) = 40\ W/m^2K$

Heat transfer coefficient of the outer surface $(h_B) = 20\ W/m^2K$

Hot gas temperature $(T_A) = 1200^{\circ}C$

Room air temperature $(T_B) = 35^{\circ}C$



Heat transfer per unit area for a composite plane wall subjected to convection on both sides is given as

$$\frac{\dot{Q}}{A} = \frac{T_A - T_B}{\frac{1}{h_A}+\frac{L_1}{k_1}+\frac{L_2}{k_2}+\frac{1}{h_B}} = \frac{1200-35}{\frac{1}{40}+\frac{0.2}{4}+\frac{0.2}{0.5}+\frac{1}{20}} = 2219.048\ W/m^2$$

The maximum temperature within the common brick is T_2. Therefore, considering heat transfer by convection on hot gas and a magnesite brick layer,

$$\frac{\dot{Q}}{A} = \frac{T_A - T_2}{\frac{1}{h_A} + \frac{L_1}{k_1}}$$

$$\therefore T_2 = T_A - \frac{\dot{Q}}{A}\left(\frac{1}{h_A} + \frac{L_1}{k_1}\right) = 1200 - 5825\left(\frac{1}{40} + \frac{0.2}{4}\right) = 763.275°C$$

2. A steel pipe with ID and OD as 80 mm and 120 mm is covered with two layer of insulation, 25 mm and 40 mm thick. The thermal conductivities of insulating materials are 0.2 W/mK and 0.1 W/mK respectively while that of steel is 50 W/mK. The inner surface of the pipe is 200°C while surface temperature of insulation is 40°C. Determine the heat loss from the unit length of the pipe and layer contact temperatures.

Solution:

Given, Inner radius of pipe $(r_1) = 40$ mm

Outer radius of pipe $(r_2) = 60$ mm

Outer radius of first insulator $(r_3) = 60 + 25 = 85$ mm

Outer radius of second insulator $(r_4) = 85 + 40 = 125$ mm

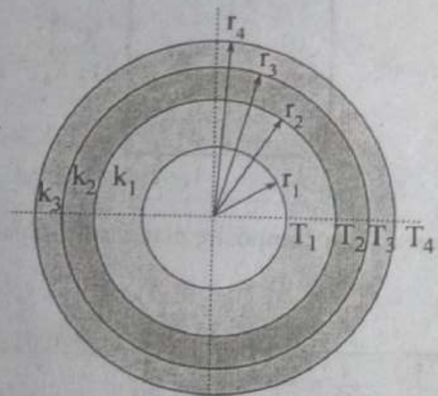
Thermal conductivity of pipe $(k_1) = 50$ W/mK

Thermal conductivity of first insulator $(k_2) = 0.2$ W/mK

Thermal conductivity of second insulator $(k_3) = 0.1$ W/mK

Inner surface temperature $(T_1) = 200°C$

Outer surface temperature: $T_2 = 40°C$



Heat transfer per unit length for the composite cylinder is then given by

$$\frac{\dot{Q}}{L} = \frac{2\pi (T_1 - T_4)}{\frac{\ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{k_3}}$$

$$= \frac{2\pi (200-40)}{\frac{\ln\left(\frac{60}{40}\right)}{50} + \frac{\ln\left(\frac{85}{60}\right)}{0.2} + \frac{\ln\left(\frac{125}{85}\right)}{0.1}}$$

$= 179.3189$ W/m

Applying heat transfer equation for steel only,

$$\frac{\dot{Q}}{L} = \frac{2\pi k_1 (T_1 - T_2)}{\ln (r_2/r_1)}$$

$$\therefore T_2 = T_1 - \frac{\frac{\dot{Q}}{L} \times \ln (r_2/r_1)}{2\pi k_1} = 200 - \frac{179.3189 \times \ln\left(\frac{60}{40}\right)}{2\pi \times 50} = 199.769°C$$

Again, applying heat transfer equation for first insulating layer,

$$\frac{\dot{Q}}{L} = \frac{2\pi k_2 (T_2 - T_3)}{\ln\left(\frac{r_3}{r_2}\right)}$$

$$\therefore T_3 = T_2 - \frac{\frac{\dot{Q}}{L} \times \ln (r_3/r_2)}{2\pi k_2}$$

$$= 199.769 - \frac{179.3189 \times \ln (85/60)}{2\pi \times 0.2} = 150.067°C$$

3. A 2.5 cm thick plate (k= 50W/mK) 50 cm by 75 cm is maintained at 300°C. Heat is lost from the plate surface by convection and radiation to the ambient air at 20°C. If the emissivity of the surface is 0.9 and the convection heat transfer coefficient is 20 W/m²K, determine the inside plate temperature.

Solution:

Given, Thickness of plate (L) = 2.5 cm

Cross sectional area of plate (A) = 50 × 75 cm²

Thermal conductivity of plate (k) = 50 W/mK

Outside surface temperature of plate (T_2) = 300°C

Ambient air temperature (T_B) = 20°C

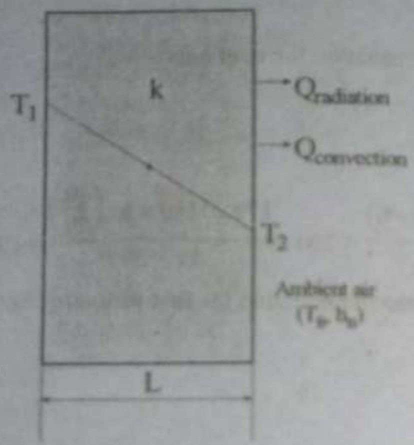
Emissivity of plate surface ($\in$) = 0.9

Convection coefficient (h) = 20 W/m²K

For steady state heat transfer,

$$\dot{Q}_{conduction} = \dot{Q}_{convection} + \dot{Q}_{radiation}$$

$$\text{or, } \frac{kA}{L}(T_1 - T_2) = h_A(T_2 - T_B) + \in\sigma A(T_2^4 - T_B^4)$$



$$\text{or, } \frac{k}{L}(T_1 - T_2) = h(T_2 - T_B) + \in\sigma(T_2^4 - T_B^4)$$

$$\therefore T_1 = T_2 + \frac{L}{k}[h(T_2 - T_B) + \in\sigma(T_2^4 - T_B^4)]$$

$$= 300 + \frac{2.5 \times 10^{-2}}{50}[20(300 - 20) + 0.9 \times 5.67 \times 10^{-8} \times (573^4 - 293^4)]$$

$$= 305.3625°C$$

4. A steel pipe having an outside diameter of 2 cm is to be covered with two layers of insulation, each having a thickness of 1 cm. The average conductivity of one material is 5 times that of the other. Assuming that the inner and outer surface temperatures of the composite insulation are fixed, calculate by what percentage the heat transfer will be reduced when the better insulating material is next to the pipe than it is away from the pipe. ***(IOE 2070 Chaitra, IOE 2068 Shrawan)***

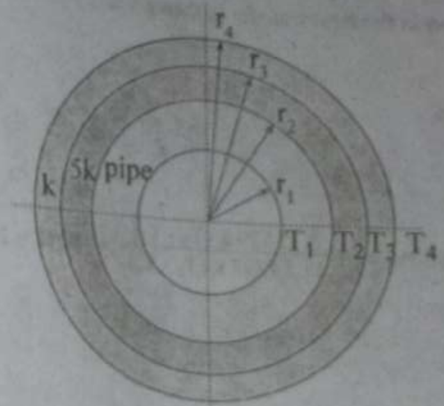
Solution:

Given, Outside radius of the pipe (r_2) = 1cm

Outside radius of the first insulator (r_3) = 1 + 1 = 2cm

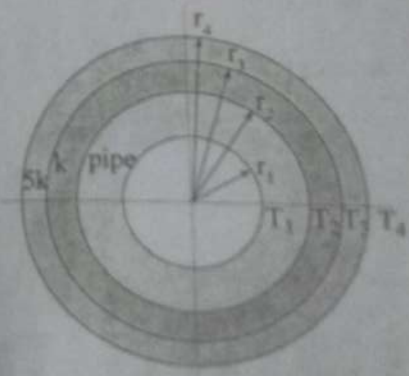
Outside radius of the second insulator (r_4) = 2 + 1 = 3cm

Let conductivity of insulating materials are k and 5k.



When better insulating material is away from the pipe as shown in figure, heat transfer is given by

$$\left(\frac{\dot{Q}}{L}\right)_1 = \frac{2\pi(T_2 - T_4)}{\frac{\ln\left(\frac{r_3}{r_2}\right)}{5k} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{k}} = \frac{2\pi(T_2 - T_4)}{\frac{\ln\left(\frac{2}{1}\right)}{5k} + \frac{\ln\left(\frac{3}{2}\right)}{k}} = 11.54797K(T_2 - T_4)$$



When better insulating material is next to the pipe as shown in figure, heat transfer is given by

$$\left(\frac{\dot{Q}}{L}\right)_2 = \frac{2\pi\,(T_2 - T_4)}{\frac{\ln\left(\frac{r_3}{r_2}\right)}{k} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{5k}} = \frac{2\pi\,(T_2 - T_4)}{\frac{\ln\left(\frac{2}{1}\right)}{k} + \frac{\ln\left(\frac{3}{2}\right)}{5k}} = 8.11529\,k\,(T_2 - T_4)$$

Percentage reduction in the heat transfer is then given by

$$\frac{\left(\frac{Q}{L}\right)_1 - \left(\frac{Q}{L}\right)_2}{\left(\frac{Q}{L}\right)_1} \times 100\%$$

$$= \left(1 - \frac{\left(\frac{\dot{Q}}{L}\right)_2}{\left(\frac{\dot{Q}}{L}\right)_1}\right) \times 100\% = \left(1 - \frac{8.11529\,k\,(T_2 - T_4)}{11.54797\,k\,(T_2 - T_4)}\right) \times 100\% = 29.725\%$$

APPENDIX 1

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